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ASSESSMENT OF APPROXIMATE ERROR VALUES USED IN ASTRONOMICAL NAVIGATION FOR POSITIONING

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Abstract: *Finding the vessel’s position on the terestrial sphere represents the most important activity carried out by the officer of the watch onboard, in order to ensure the safety of the vessel, crew and cargo. The ship’s position can be determined by several methods such as: coastal observations, astronomical observations, radar or through data provided by satellite global positioning systems. In order to determine the position of the vessel with astronomical observations, the officer of the watch uses a series of nautical tables and formulas of spherical trigonometry applied to the spherical triangle of position. The approximate values accuracy of the trigonometric functions used in computing can directly affect the position determined by astronomical observations. The purpose of this paper is to evaluate the errors of approximate values of the trigonometric functions used by the officer on watch for fix positioning with astronomical observations.*

Introduction

According to Standards of Training and Watchkeeping Code (STCW), the Officer in Charge with Navigational Watch (OOW) is the master’s representative on board the ship. He is primarly responsible at all times for the safe navigation of the ship for the prevailing circumstances and conditions until properly relieved.

In order to mantain a safe navigational watch, the navigational duties of the OOW has to execute the passage plan safely and to monitor the progress of the ship to that plan. For that, the OOW must know all the time the position of the ship by geographical coordinates.

The ship’s position can be determined by several methods such as: coastal observations, astronomical observations, radar or through data provided by satellite global positioning systems.

Another duty of the OOW is to make periodic checks on the navigational equipment in use. He must check and record gyro and magnetic compass errors at least once a watch, where possible, and after any major course alteration [1-2]. The gyro and magnetic compass error can be determined using coastal or astronomical methods.

The theoretical elements of navigation

Practicenavigationinvolves performinga series ofmathematical calculationsthat include the use oftrigonometric functions. Forthe results toshowa high degreeof trust it isrequiredto comply with certainrules based onmathematicalconsiderationsandthelong experience ofseafarers.

All calculationsfor solving problems ofnavigationare made withapproximatenumbersand always the precisioncalculationsmust match theprecisioncalculationsobservations. Increasing theaccuracy of the calculationssignificantly above theprecisionobservationsdoes not implyimproving the accuracyof the final result, buttheincreasingworkloadandtime requiredto solvethe problem.

Executed navigation observations are characterized by a limited accuracy imposed by the precision of the navigation instruments used and by the precision of their execution. A series of values used in navigation are determined through calculus, characterized by a certain degree of approximation. It is very important to know from the beginning the approximation degree of the calculus (rounded, truncated values) and to maintain the precision of the observations, in order to maintain the precision of the final result.

Determining thegeographical coordinates of theship - latitude (Lat.) and longitude(Long.) - is performed according to thesphericalcoordinatesofthe starsat which theobservations are made.Determination ofthe shipastronomical positioncan be achievedwithsimultaneous and successiveobservations.

The celestial point is determined at the intersection of two or three lines of position (LOP), according to the line of position theory [3-6]. The elements of the straight lines of position LOP (intercept and azimuth) are calculated for the same estimated point Ze (estimated latitude and longitude).

This method is most applicable when the stars are visible at same time, in positions that provide favorable conditions, namely:

- during twilight (when is visible the horizon and celestial bodies);
- during moonlight nights, depending on the horizon visibility;
- during daylight with the Sun using successive observations.

Determination of the errors of the gyro or magnetic compass with astronomical observations is made by comparing the azimuths and gyro or magnetic compass bearings of the observed stars.

Astronomical calculations are performed by applying trigonometry formulas in the spherical triangle position.

Calculating the intercept is made by using spherical trigonometric formula „sinh”. The azimuth angle (Zn) of a star can be achieved byusing spherical trigonometry from formulas „sinZ_C”, „cotZ_S” or with A.B.C. tables using Hydrographic Direction Tables (D.H.-90) or Norie’s Nautical Tables.

The spherical triangle of position arises through the intersection of three greatcircles:

- the observer’s celestial meridian;
- the vertical circle of the star;
- the hour circle of the star.

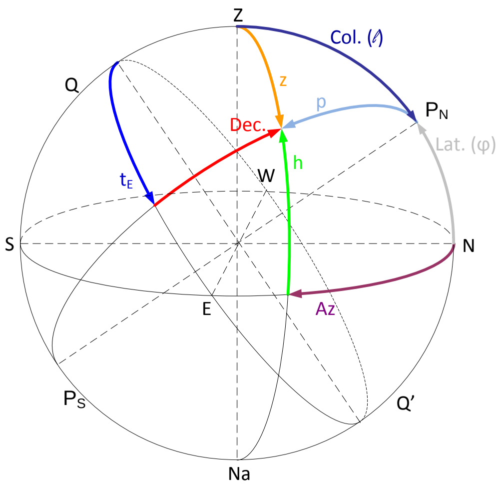

Figure 1. The celestial sphere

The elements of a spherical triangle are:

- the triangle’s peaks;
- the triangle’s sides;
- the triangle’s angles.

The peaks of spherical triangle are:

- the zenith (Z);
- the high celestial pole N_P (S_P);
- the star A.

Spherical triangle's sides are great arcs resulted by combining horizontal and equatorial coordinates at the intersection of the three great circles:

- the colatitude: Col. = 90° - Lat.;
- the zenith distance: z = 90° - Hc;
- the polar distance: = 90° - Dec.;

Spherical triangle's peaks are as follows:

- the zenith angle (Zn);
- the meridian angle ($t_{E/W}$);
- the parallactic angleA.

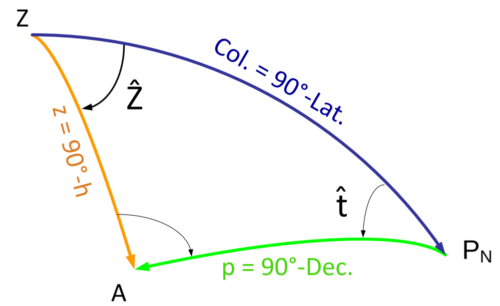

Figure 2. The spherical triangle

In the spherical triangle, by appling the cosin of a side, the sines or the four consecutive elements formula, the altitude and the azimuth from the cuadrantal or secircular zenith angle is obtained:

$$\sin(Hc) = \sin(Lat.)\sin(Dec.) + \\ + \cos(Lat.)\cos(Dec.)\cos(t) \quad (1)$$

$$\frac{\sin(Zc)}{\sin(Dec.)} = \frac{\sin(t)}{\cos(Hc)} \Rightarrow$$

$$\sin(Zc) = \sec(Hc)\cos(Dec.)\sin(t) \quad (2)$$

$$\cot(Zs) = \frac{\tan(Dec.)\cos(Lat.)}{\sin(t)} - \frac{\sin(Lat.)}{\tan(t)} \quad (3)$$

where:

- Calculated altitude (Hc)is an angular distance above the horizon measured along a vertical circle, from 0° at the horizon to 90° at the zenith. It is positive in the visible hemisphere and negative in the invisible hemisphere;
- Latitude (Lat.)is the angle between a line in the direction of gravity at a station and the plane of the equator with values from 0° to 90°, north or south of the celestial equator;
- Declination (Dec.) is an angular distance north or south of the celestial equator. It is measured along an hour circle, from 0° at the celestial equator to 90° at the celestial poles. It is labeled N or S to indicate thedirection of measurement;
- Meridian angle (t) is an angular distance west or east of thelocal celestial meridian, or the arc of the celestial equator, betweenthe upper branch of the local celestial meridian in eitheran easterly or westerly direction; values from 0° to 180°.
- Quadrantal azimuth angle (Zc) is an arc of the horizon measured either clockwise or counterclockwise with values from 0° to 90°, starting at the north or south point of the horizon.
- Semicircular azimuth angle (Zs) it is an arc of the horizon measured either clockwise or counterclockwise with values from 0° to180°, starting at the north point of the horizon in north latitude and the south point of the horizon in south latitude.

The azimuth (Zn) is an arc of the horizon measured clockwise starting from the north point on the horizon. It takes values from 0° to 360°.

The azimuth is determined from the quadrantal or semicircular azimuth angle.

The quadrantal azimuth angle (Zc) it is expressed as follow:

Zc= NEα° → Zn = α°
Zc = NWα° →Zn = 360° - α°
Zc= SEα° → Zn = 180° - α°
Zc = SWα° → Zn = 180° + α°

The semicirculat azimuth angle (Zs) is expressed as follow:

- for an observer situated at northern latitudes
  Zs= N α° E → Zn = α°
  Zs = N α° W →Zn = 360° - α°
- for an observer situated at southern latitudes
  Zs = S α° E → Zn = 180° - α°
  Zs = S α° W → Zn = 180° + α°

where α° represent the angular value of the quadrantal or semicircular azimuth angle.

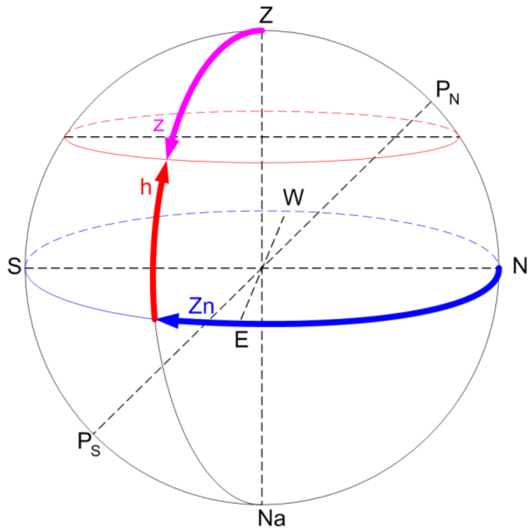

Figure 3. Thealtitude and azimuth representation

1. The practical elements of celestial navigation

The formulas (1-3) can be solved by two methods.
The first method is based on logarithms, as follows:

- formula (1)

$$a = \sin(lat.)\sin(Dec.) \quad b = \cos(Lat.)\cos(Dec.)\cos(t)$$
$$\sin(Hc) = a + b$$

Type ofcalculationwithlogarithms is:

Finding Hc
Lat. = … lg sin(Lat.) = … lg cos(Lat.) = …
Dec.= … + lg sin(Dec.)= … + lg cos(Dec.)= …
t = … + lg cos(t) =…
lg a = … lg b = …
a = …
+ b = …
sin(Hc)= …
Hc = …

- formula (2)

Type ofcalculationwithlogarithms is:

Finding Zc
Hc = ... lg Hc = ...
Dec.= … +lgcos(Dec.)=...
t = … +lgsin(t) =...
lgsin(Zc) =...
sin(Zc)= …
Zc = …

Formula (3):

$$m = \tan(Dec.)\cos(Lat.)\,cosec(t) \quad n = -\sin(Lat.)\cot(t)$$
$$\cot(Zs) = m + n$$

Type ofcalculationwithlogarithms is:

Finding Zs
Dec. = … lgtan(Dec.) = …
Lat. = … + lg cos(Lat.) = … lgsin(Lat.) = …
t = … + lg cosec(t) = …+ lg cot(t) = …
lg m = … lg n = …

m = ... n =...
+ n = …
cot(Zs) = …
Zs = …

The values of logarithms of trigonometrical functions can be found in tables: in D.H.-90 or Norie’s Nautical Tables. Its is expressed as numerical values with five decimal places.
The second method is based on direct application of formula (1-3). The D.H.-90 or Norie’s Nautical Tables contains tables with the values of trigonometrical functions, Its is expressed as numerical values with five decimal places.

The mathematical evaluation

Mathematicalprocessingof the observations resultis achievedwith approximate numbersandtheaccuracy of the calculationsmust alwaysmatchthe precisionobservations.
Mathematical calculationsmust be performedusingformulastestedinpractice andensure speedin obtaining thefinal output.
Calculation resultsshould not containerrorsandomissions.Therefore, the results must be provided with acontrol forintermediateoperations.
Calculationsmust be arrangedintypes ofrational calculationmadeto obtainthe necessarypromptnessinworkandavoid mistakesoromissions.
The accuracy of organizing operations and writing figures are of special importance given that, usually onboard ship, calculations are performed by one person and often, time does not allow to be repeated for verification.
The approximate value of a given quantity has as a basis:
- all observations made, are characterized bylimited accuracy, meaning that each of the observation represents a close value to the true one, depending on the precision of the observation;
- not all the values can be observed (measured) directly, meaning that their value do not result from observations but from mathematic relations and calculus characterized by a certain degree of approximation;
- in calculus, arithmetic operations are always present and accompanied by a certain.

Thus, along with the approximate value obtained from observations, in calculations inherent approximation appear due to arithmetic operations. This is the reason why it is very important to know from the beginning the degree of approximation for calculations. Thus the approximation from calculus (rounding) should not reduce the precision of the observations and so, the precision of the final result will not be reduced.
The notion ofapproximate value entails the existence of errors that characterizes the approximate values. These can be absolute or relative [7].
The absolute error of an approximate value is given by the difference between the precise value and the approximate value:

$$\Delta = X - A \quad (4)$$

where:
X – represent the precise value;
A – represent the approximate value.

The relative error (δ) of the approximate value is given by the ratio between the absolute error and the precise value:

$$\delta = \frac{\Delta}{X} \quad (5)$$

The absolute error (Δ) in relation to X and A is a very small value and X and A are very close to each other. Thus, we can consider:

$$\delta = \frac{\Delta}{A} \quad (6)$$

Thus, relative of to the relative error, the relation for the precise value is:

$$X = A\left(1 + \frac{\Delta}{A}\right) = A(1 + \delta) \quad (7)$$

The relative values are usually expressed in percents ($\delta\%$):

$$\delta\% = 100\frac{\Delta}{A} \quad (8)$$

For approximate values in which a large number of decimals is known, the sign of the error can be determined.This can not be said for the values obtained from observations. If a approximate value of the absolute error can be determined, the sign of the error can not be determined.
The absolute errors and concretely numbers and have the same value as the value of the observation.The relative errors are abstract numbers without having any unit of measure.
The absolute error of the sum/difference between the approximate numbers is equal to the sum/difference between the absolute errors of the numbers of which this sum/difference is formed.
The absolute error of multiplication is the difference between the exact values and the multiplication of approximative values:

$$\Delta = X_1X_2 - A_1A_2 = \pm\Delta_1A_2 \pm \Delta_2A_1 \quad (9)$$

The absolute error of the division is:

$$\Delta = \frac{\pm\Delta_1A_2 \mp \Delta_2A_1}{A_2^2} \quad (10)$$

Assessment of approximate error values

May the ship be anchored in the estimated position: Lat.= $44°32'_0N$, Long.= $029°57'_0E$ on april 28, 2015.To determine the fix position of the ship, the officer ofthe watch use two sextant star sights: to Markab and Rasalhague.
At the chronometer time (C) C=$03^h47^m30^s$ the officer of the watch observesMarkab and notes the sextant altitude (Hs) Hs=$43°18'_5$
At the chronometer time (C) C=$03^h51^m34^s$ the officer of the watch observesRasalhague and notes the sextant altitude (Hs) Hs=$43°26'_0$

First of all, the he must determine the LOP.
For that, he will determine:

1. thevalue of the Universal Time (UT) for the moment of the sextant star sight. It is determined from the chronometer time (C) and the chronometer error (CE);
2. the values of the meridian angle ($t_{E/W}$) and the declination (Dec.) of the star at the UT of sight using astronomical ephemeris (Brown’s Nautical Almanac or The Nautical Almanac);
3. the value of the calculated altitude (Hc) using the mathematical formula (1);
4. the value of the azimuth (Zn) using the mathematical formula (3);
5. the value of the true altitude (Ho) from the sextant altitude sight.
6. the intercept (p) which is the difference between the true altitude and calculated altitude.

After these steps, the officer ofthe watch will plot on the paper chart that two lines of position to determin the fix position.
Knowing the values of the chronometer error (CE=$+1^m07^s$), the sextant index (I=$-1'._2$), and the height of eye(h=12m) , the officer of the watch will determine the lines of positions.

For Markab:
1. UT=$03^h48^m37^s$
2. t_E= $43°28'_1$ Dec.=$N15°17'_1$
3. Hc= $43°09'_1$
4. Zn=$114°._5$
5. Ho=$43°10'_1$
6. p=$+1'._0$

For Rasalhague:
1. UT=$03^h52^m41^s$
2. t_E= $40°00'_8$ Dec.=$N12°33'_0$
3. Hc= $43°15'_6$
4. Zn=$239°_5$
5. Ho=$43°17'_6$
6. p=$+2'._0$

The calculated altitude of the Markab was determined using the precise values of the trigonometric functions (Tab. 1).

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Tab. 1. The precise determination of the calculated altitude

The trigonometric function	The precisevalue
sin(Lat.)	0,701324097951209
cos(Lat.)	0,712842555991800
sin(Dec.)	0,263620520383821
cos(Dec.)	0,964626467205085
cos(t.)	0,725754705241152
a	0,184883423659612
b	0,499048382980609
sin(Hc)	0,683931806640221
Hc	43,1516575960571
Hc	43°09'5.97"

The accuracy of the calculated altitude is expressedindegrees, minutesanddecimal of minutes. Thus, the value of calculated altitude is Hc= $43°09'_{1}$, because 5.97'' (arcseconds) representsonetenth of aminute of arc.
Using onlyfivedecimals fortrigonometricfunctionsaccording tothe current practiceofnavigationit was determinedthe calculated altitude, absoluteandrelativeerrors (Tab. 2).

Tab. 2. *The absolute and relative error for calculated altitude determined with five decimal places*

The Trigonometric functions	Value with 5 decimals	The absolute error	The relative error
sin(Lat.)	0,70132	$40,979 \cdot 10^{-7}$	0,00058
cos(Lat.)	0,71284	$25,559 \cdot 10^{-7}$	0,00036
sin(Dec.)	0,26362	$5,203 \cdot 10^{-7}$	0,00020
cos(Dec.)	0,96463	$-35,327 \cdot 10^{-7}$	-0,00037
cos(t.)	0,72575	$47,052 \cdot 10^{-7}$	0,00065
a	0,18488	$34,236 \cdot 10^{-7}$	0,00185
b	0,49905	$-16,170 \cdot 10^{-7}$	-0,00032
sin(Hc)	0,68393	$-18,066 \cdot 10^{-7}$	0,00026
Hc	43,15166°	$-24,039 \cdot 10^{-7}$	-0,00001
Hc	43°09'5.98"		

Using different numbers of decimals for the trigonometric functions the absolute error and the error in position in meters for the calculated altitude was determined (Tab. 3).

Tab. 3. The absolute error and the error on position for different numbers of decimal places of the calculated altitude (Hc)

Numbers of decimal places	The Hc value	The absolute error	The error in position
Precisevalue	43°09'5.97"	-	-
5	43°09'5.98"	$-24,039 \cdot 10^{-7}$	0,3 m
4	43°09'6.12"	$-42,403 \cdot 10^{-6}$	4,6 m
3	43°09'7.2"	$-34,240 \cdot 10^{-5}$	37,9 m
2	43°09'0"	$16,575 \cdot 10^{-4}$	-184,2 m
1	43°12'0"	$-48,342 \cdot 10^{-3}$	5371,7m

It can be seenfrom the graph (Fig. 4)that the errorof calculated altitude (Hc) increases exponentiallywhen twodecimalsare used for its determination.

Conclusions

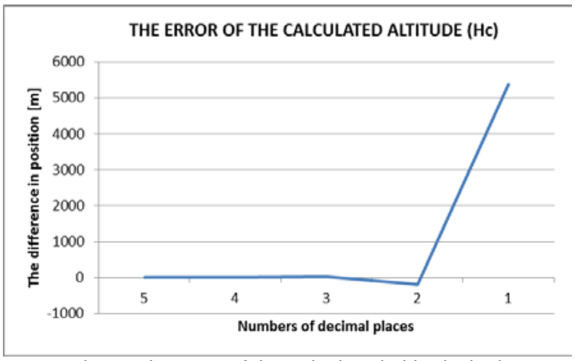

Fig. 4. The error of the calculated altitude (Hc)

The followingvalueshave been obtained for azimuth angle (Tab. 4).

Tab. 4. The precise determination of the azimuth angle

The trigonometric function	The precise value
tan(Dec.)	0,273287670768186
cos(Lat.)	0,712842555991800
cosec(t.)	1,453586478272300
sin(Lat.)	0,701324097951209
cot(t.)	1,054947226081040
m	0,283174754251492
n	0,739859911717416
cot(Zs)	-0,456685157465923
tan(Zs)	-2,189692359499590
Zs'	-65,454523667493400°
Zs=N(Zs'+180°)E	114,545476332507000°
Zn=Zs	114,545476332507000°
Zn	114°32'43.71''

For practice of navigation, the azimuth angle is expressedindegrees, andtenths of degrees. Thus, the value of azimuth angle is Zn=$114°._5$

Using different numbers of decimasl for the trigonometric functions the absolute error for the azimuth angle was determined (Tab. 5).

Tab. 5. The absolute error for different numbers of decimal places of the azimuth angle (Zn)

Numbers of decimal places	The Zn value	The absolute error
Precisevalue	114°32'43.71''	-
5	114°32'43.73''	$-36,374 \cdot 10^{-7}$
4	114°32'43.8''	$-23,667 \cdot 10^{-6}$
3	114°32'42''	$47,633 \cdot 10^{-5}$
2	114°33'00''	$45,236 \cdot 10^{-4}$
1	114°30'00''	$45,476 \cdot 10^{-3}$

From the expression of the azimuth angle in degreesanddecimal of degrees it can be seenthat the errorinits calculationis insignificant.

Inastronomicalnavigationproblems, in order to determine the calculated altitude or azimuth angle, the officer on watch can use a series of nautical publications or a scientific calculator.
Considering the precision of the calculated altitude thatthe officer of the watch operateson board the ship,thevaluesof thetrigonometricfunctionscanbe at least threedecimals.
To determine theazimuthangle the values of the trigonometric functions can be at least two decimals.

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