

SOLVING CONCRETE PROBLEMS IN NAVAL FRAMEWORK THROUGH CANONICAL MATHEMATICAL MODELS

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Abstract: The paper aims to point out how good results in the management of shipping might be achieved through canonical mathematical models.

Key-words: mathematical model, North-west corner method, minimum element method, mixed differences method.

INTRODUCTION

Between June 2012 and February 2013, in the port of Okpo in South Korea, Daewoo and Marine Engineering worked to build the largest container shipping vessels, named Maersk Mc-Kinney Moller. It measures 400 meters length, 59 meters width and has a height comparable to a 20-storey building, while the transport vessel has a capacity of 18000 containers, each of 20 feet [2], [4].



Maersk Mc-Kinney Moller [12]

Note the fact that, a container of 20 feet has the length - 20 ft (5.9 m), the width - 8 ft (2.4 m), the height - 8 ft (2.4 m) and the volume - 33m³ [10], [11].

For its construction, steel was used such as for eight Eiffel Towers and a single transport capacity could bring 111 million pairs of sneakers (in a 20 feet container, around 6000 pairs can be carried on) or 182 million iPads (about 10100 iPads/container). Putting all the containers one over the other, an amazing height of 47 km would be reached [2], [5], [7].

Its purpose is to serve the trade route between Asia and Europe and bring in the ports of Europe millions of products manufactured in China, Malaysia, Taiwan and Korea [6], [7].

Fuel reduction is spectacular, from 214 tons per day to only 160 tons. The maximum speed of the vessel is 23 knots and has a large contribution to reduce the fuel consumption. The maximum speed would have been limited to 21 knots, but we opted for the flexibility of moving quickly in case of delay, for whatever reason, in order to follow the schedule and eliminate certain potential additional costs for delays [8].

PRESENTATION OF THE PROBLEM

To streamline costs more for Maersk Mc-Kinney Moller, we imagine that we can apply the mathematical model for a transportation problem for a shipment of 10 000 containers on the route Asia - Europe. We considered the following ports: Algeciras, Hamburg, Rotterdam and Marseilles and with the demand of vacuum cleaners, footwear, and TVs (LCD). These products are necessary for containers in the following amounts: $b_1 = 10$, $b_2 = 30$, $b_3 = 35$, and $b_4 = 25$ (to easier make calculations we understand that 10 means 1000 containers). We used the next notations for ports:

- Hamburg = P1
- Rotterdam = P2
- Algeciras = P3
- Marseille = P4

For the current transport, one container can hold 1000 vacuums or 6000 pairs shoes or 2000 LCD TVs.

Note suppliers:

- LG Electronics Inc. = F1
- Nike = F2
- Samsung Smart TV = F3

The transport costs per unit (note \$ 100 = 1u.m.) are: 4, 3, 2, 1; 3, 2, 1, 2; 3, 4, 1, 2; monetary units from F1, F2 and F3. The mathematical model of the transport problem can be solved by three methods: the North-west corner method; the Minimum element method; the Mixed differences method [1].

For the beginning we use the North-west corner method to find an initial solution. If x_{ij} denotes the amount of products that will be transported from the provider F_i , $i = 1, 2, 3$, to the port P_j , $j = 1, 2, 3, 4$, we obtain the following mathematical model:

$$\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &= 50 \\ x_{21} + x_{22} + x_{23} + x_{24} &= 20 \\ x_{31} + x_{32} + x_{33} + x_{34} &= 30 \\ x_{11} + x_{21} + x_{31} &= 10 \\ x_{12} + x_{22} + x_{32} &= 30 \\ x_{13} + x_{23} + x_{33} &= 35 \\ x_{14} + x_{24} + x_{34} &= 25 \end{aligned}$$

$$x_{ij} \geq 0, i = 1, 2, 3; j = 1, 2, 3, 4.$$

$$\begin{aligned} (\min) f &= 4x_{11} + 3x_{12} + 2x_{13} + x_{14} + 3x_{21} + 2x_{22} + x_{23} \\ &\quad + 2x_{24} + 3x_{31} + 4x_{32} + x_{33} + 2x_{34} \end{aligned}$$

Or in a table form:

$F_i \backslash P_j$	P ₁	P ₂	P ₃	P ₄	Avai lable
F ₁	4 x_{11} 1	3 x_{12}	2 x_{13}	1 x_{14}	50
F ₂	3 x_{21} 1	2 x_{22}	1 x_{23}	2 x_{24}	20
F ₃	3 x_{31} 1	4 x_{32}	1 x_{33}	2 x_{34}	30
Nec essa ry	10	30	35	25	100

To find the initial solution with the North-west corner method, the procedure consists in the next steps: we choose $x_{11} = \min(10, 50) = 10$; then $x_{21} = x_{31} = 0$. Then $x_{12} = \min(30, 40) = 30$ and $x_{22} = x_{32} = 0$. Next is $x_{13} = \min(35, 10) = 10$ so $x_{14} = 0$. Further $x_{23} = \min(25, 20) = 20$, and $x_{24} = 0$ and finally $x_{33} = \min(5, 30) = 5$ and $x_{34} = 25$.

Typically, the initial solution is determined in the table, each time decreasing availability and demand, and joining remaining writing. Thus we have:

$F_i \backslash P_j$	P_1	P_2	P_3	P_4	Ava ilab le
F_1	4 10	3 30	2 10	1 0	50; 40; 10
F_2	3 0	2 0	1 20	2 0	20; 0
F_3	3 0	4 0	1 5	2 25	30; 25; 0
Neces sary	10 0	30 0	35 25 5	25 0	100

The total cost function value for the initial solution is found:
 $f = 4 \cdot 10 + 3 \cdot 30 + 2 \cdot 10 + 1 \cdot 20 + 1 \cdot 5 + 2 \cdot 25 = 225$ u.m, that means that the price is $225 \cdot 10^4 = \mathbf{2\ 250\ 000\ \$}$

This method is very simple but less efficient because it does not account for costs c_{ij} but only for the available values and the required quantities.

Using the minimal element method we will find an initial solution to this problem.

Because $X_{14} = X_{23} = X_{33} = 1$ is the minimum cost, we first determine the variable X_{33} , as we get the maximum value ($X_{14}=25$; $X_{23}=20$; $X_{33}=30$). So, we will take $X_{33} = 30$, which implies: $X_{31} = X_{32} = X_{34} = 0$. Then choose $X_{14} = 25$, as follows from the minimum value = 1 and the resulting $X_{24} = 0$. It has remained $X_{23} = 5$, and the resulting $X_{13} = 0$. Consider costs equal to 2 and choose $X_{22} = 15$, this implies $X_1 = 15$, $X_{21} = 0$, $X_{11} = 10$.

$F_i \backslash P_j$	P_1	P_2	P_3	P_4	Ava la ble
F_1	4 10	3 15	2 0	1 25	50;25; 10
F_2	3 0	2 15	1 5	2 0	20;15; 0
F_3	3 0	4 0	1 30	2 0	30;0
Neces sary	10 0	30 15 0	35 5 0	25 0	100

The value of the objective function is:

$f = 4 \cdot 10 + 3 \cdot 15 + 1 \cdot 25 + 2 \cdot 15 + 1 \cdot 5 + 1 \cdot 30 = 175$ and that means $\mathbf{1\ 750\ 000\ \$}$.

Next, we will use the mixed differences method. The variables have the same values as in the previous methods, but the order is made by another rule. For determination of the following, the smallest difference between two elements (costs) is calculated for each row, each column respectively. Then on the row or column with the maximum difference, variables are determined in the box with minimal cost.

Conclusion

We conclude that using the north-west corner method we got a less efficient solution than that obtained by the minimum element method, and that of the mixed differences method. These solutions can be improved and optimized by using the algorithm of potentials and the results can lead to the confirmation of the fact that mathematical methods could be used more in practice on board.

Bibliography

[1] Acu, A.M., Acu D., Acu M., Dicu P., Matematici aplicate în economie – Note de curs pentru învățământul la distanță, depmath.ulbsibiu.ro/.../MatematiciAplicateInEconomiePentruDD.pdf

Let's find out an initial solution with the maximum difference method for the transportation problem stated above.

We calculate the differences on lines and columns. We have the following table of differences:

4	3	2	1	1
3	2	1	2	1
3	4	1	2	1
1	1	1	1	

Calculated as follows: $2-1=1$ for the first line, $2-1=1$ the second line, $2-1=1$ for the third row, $4-3=1$ for the first column, $3-2=1$ in the second column, $2-1=1$ in the third column, and on the fourth column $2-1=1$. It is noted that the differences are all equal to 1. Choose $X_{33} = 30$ because it gives the largest allocation for minimum prices. Then $X_{31} = X_{32} = X_{34} = 0$.

Recalculating the differences on the remaining rows and columns, we get the table:

4	3	2	1	1
3	2	1	2	1
1	1	1	1	

All differences are equal. Given the minimal cost, choose $X_{14} = 25$, this implies $X_{24} = 0$. For rows and columns that are blank we recalculate the differences and get the table:

	3	2		1
3	2	1		1
1	1	1		

Choose $X_{23} = 5$, the last of the minimum cost is 1 and this implies $X_{13} = 0$.

4	3		1
3	2		1
1	1		

By arranging the values in a table, we have:

$F_i \backslash P_j$	P_1	P_2	P_3	P_4	Ava la ble
F_1	4 10	3 15	2 0	1 25	50;25; 10
F_2	3 0	2 15	1 5	2 0	20;15; 0
F_3	3 0	4 0	1 30	2 0	30;0
Neces sary	10 0	30 15 0	35 5 0	25 0	100

And the value of f to the initial solution is:

$f = 4 \cdot 10 + 3 \cdot 15 + 1 \cdot 25 + 2 \cdot 15 + 1 \cdot 5 + 1 \cdot 30 = 175$, that means $\mathbf{1\ 750\ 000\ \$}$.

It is noted that we obtained the same initial solution as we obtained using the minimum element method.

- [2] [http://en.wikipedia.org/wiki/M%C3%A6rsk_Mc-Kinney_M%C3%B8ller_\(ship\)https://www.google.ro/maps/@45.9146382,10.6812598,6z](http://en.wikipedia.org/wiki/M%C3%A6rsk_Mc-Kinney_M%C3%B8ller_(ship)https://www.google.ro/maps/@45.9146382,10.6812598,6z)
- [3] <http://maritime-connector.com/ship/maersk-mc-kinney-moller-9619907/>
- [4] <http://gcaptain.com/worlds-largest-ship-photos-maersk-moller/>
- [6] <http://www.largestships.com/maersk-mc-kinney-moller/>
- [7] http://www.containership-info.com/vessel_9619907.html
- [8] <http://www.vesselfinder.com/news/1472-Biggest-ship-in-the-World-and-Most-Fuel-Efficient-Maersk-Mc-Kinney-Moeller>
- [9] http://www.marinetraffic.com/ais/details/ships/shipid:155711/mmsi:219018271/imo:9619907/vessel:MAERSK_MC_KINNEY_MOLLER
- [10] <http://www.freightgate.com/resources/container.tet>
- [11] <http://www.searates.com/reference/equipment/1/>
- [12] <http://worldmaritimenews.com/archives/90755/video-triple-e-vessel-maersk-mc-kinney-moller-on-sea-trials/>