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MODELLING THE ECLIPSE REGION FOR GPS SATELLITES

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Abstract: *There are many benefits of space technologies and practical applications of satellite navigation systems have penetrated deep into the everyday life, fundamentally changing people's lifestyle. Thus, the need of a more precise position on Earth's surface has appeared. GPS satellites enter Earth's umbra once a year for an interval of maximum 1-2 minutes. During this time the satellites aren't affected at all by the solar radiation pressure emitted by the Sun. In this thesis we present a new method for modeling the eclipse region of GPS satellites. Through numerical integration, using the Runge-Kutta of 4th order method, we determined the relative positions of the Sun and a GPS satellite to Earth. Using this values we verified the conditions of a fully illuminated satellite, a satellite in penumbra and a satellite in the Earths umbra.*
Keywords: *GPS satellite, eclipse, numeric integration, solar radiation.*

INTRODUCTION
MODELLING THE ECLIPSE REGION

Satellites orbiting the Earth, periodically enter areas of total or partial eclipse when they pass through regions known as umbra and penumbra.

Penumbra region represents the area partially occulted by the Earth while the umbra can be defined as the region opposite to the Sun from then Earth, region which is completely devoided by solar radiation.

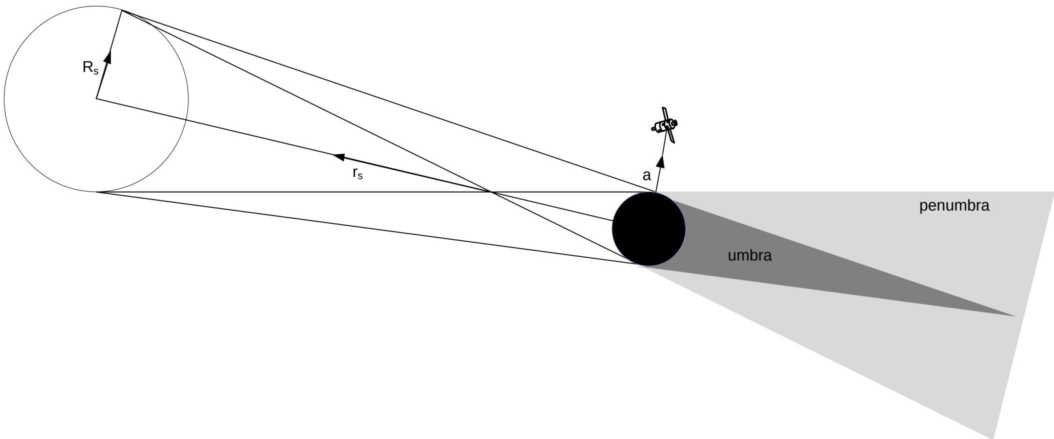


Fig. 1 The eclipse area of a GPS satellite

In the above figure we represented the umbra and penumbra regions for the spherical shape of the Earth. In reality, Earth is flat at its poles and the edges of umbra and penumbra will suffer some modifications.

According to the modeling of umbra region literature it seems that these methods are either precise or effective but not both at the same time. Thus in this thesis we will present a new method for determining the eclipses (when the satellite is fully illuminated or when it is in the penumbra or umbra region), taking into consideration Earth's true shape. The aim is to develop a new method in which we can combine precision with effectiveness. This new method has its roots in the method developed in the works of Adhya et al.(2004) and in Adhya's PhD thesis (2005).

Determining the satellites eclipses

We will now present the method developed by Adhya in his PhD thesis (2005).

The method is based on geometry and determines if the lines coming from the Suns edges to the satellite intersect the Earth or not. If we have an intersection and the distance from the Sun to this point in smaller than the Sun-satellite distance then the satellite is in penumbra or umbra area.

A plane is defined in the geocenter, $\vec{r}_s$ being the Earth-Sun vector and $\vec{a}$ the Earth-satellite vector. In this bidimensional space, the Sun is represented as a circle. In plane, the sunlight that leaves the Sun edges tangent the Earth and form the edges of penumbra and umbra regions. In order to determine the points of intersection we will use the direct approximation of an elipsoid for the shape of the Earth.

$$\frac{x^2+y^2}{p^2}+\frac{z^2}{q^2}=1 \tag{1}$$

where *x, y, z* are the coordinates of a point on the ellipsoid, *p* is the equatorial radius and *q* is Earth's polar radius. The straight line connecting the satellite with one of the Sun's edges has the following form:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{2}$$

where (a_1, a_2, a_3) are the coordinates of the satellites position vector and (b_1, b_2, b_3) are the coordinates of a Suns edge. Thus equation (2) can be written in the following form:

$$\frac{x-a_1}{b_1-a_1}=\frac{y-a_2}{b_2-a_2}=\frac{z-a_3}{b_3-a_3} \tag{3}$$

Vectors $\vec{a}$ and $\vec{b}$ can be obtained for a specific age from precise ephemerides or by numerical integration. From the equation (3) we can write *y* and *z* as a function of *x* having $\vec{a}$ and $\vec{b}$ as coefficients:

$$y=\frac{x(b_2-a_2)+a_2b_1-a_1b_2}{b_1-a_1}\qquad z=\frac{x(b_3-a_3)+a_3b_1-a_1b_3}{b_1-a_1}\tag{4}$$

replacing in equation (1) we obtain:

$$\frac{x^2+\left[\dfrac{x(b_2-a_2)+a_2b_1-a_1b_2}{b_1-a_1}\right]^2}{p^2}+\frac{\left[\dfrac{x(b_3-a_3)+a_3b_1-a_1b_3}{b_1-a_1}\right]^2}{q^2}=1$$

$$q^2p^2x^2+q^2\left[\frac{x(b_2-a_2)+a_2b_1-a_1b_2}{b_1-a_1}\right]^2+p^2\left[\frac{x(b_3-a_3)+a_3b_1-a_1b_3}{b_1-a_1}\right]^2=p^2q^2$$

moving on with calculus we will obtain a 2[nd] degree equation such as:

$$Ax^2+Bx+C=0\tag{5}$$

that has the following coefficients:

$$A=q^2\left[(b_1-a_1)^2+(b_2-a_2)^2\right]+p^2(b_3-a_3)^2$$

$$B=2q^2(b_1b_2a_2-a_1b_2^2-a_2^2b_1+a_1a_2b_2)+2p^2(b_1b_3a_3-a_1b_3^2-a_3^2b_1+a_1a_3b_3)$$

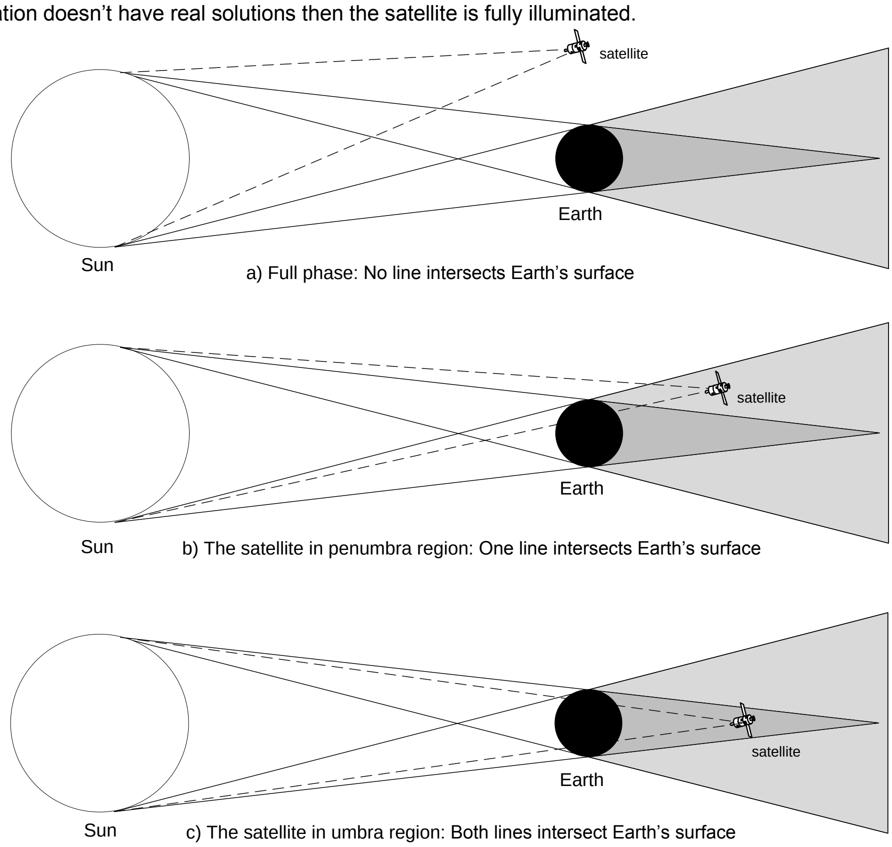
$$C=q^2\left(b_1a_2-b_2a_1\right)^2+p^2\left(b_1a_3-b_3a_1\right)^2-p^2q^2(b_1-a_1)^2$$

In order to solve the system we need to know the coordinates for the Suns edges. The real solutions for this 2[nd] degree equation are the *x* coordinates of the intersecting points of the straight line that leaves from one of the Sun-satellite edge with the ellipsoid. The condition to have real solutions is:

$$B^2-4AC\geq 0$$

If this condition is fulfilled then the intersection will take place. The negative value of this inequality suggests that no intersection takes place, thus the satellite is fully illuminated. After obtaining the value of *x* we must find out *y* and *z* in order to calculate the distance to the Sun. If this distance is bigger than the Sun-satellite distance then the satellite is fully illuminated. Otherwise we must take into consideration the following conditions:

- if both straight lines have real solutions then the satellite is in the umbra region;
- if just one of the straight lines has real solutions then the satellite is in the penumbra region;
- if the equation doesn't have real solutions then the satellite is fully illuminated.



a) Full phase: No line intersects Earth's surface

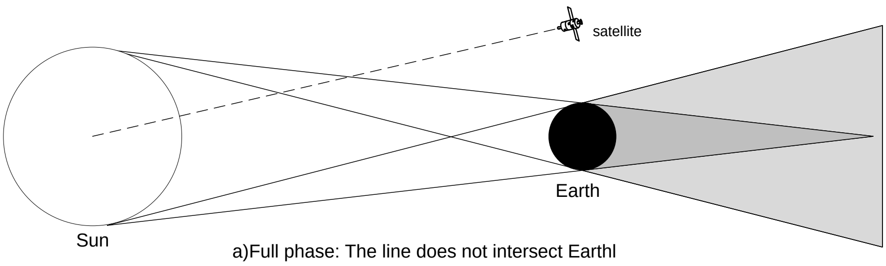
b) The satellite in penumbra region: One line intersects Earth's surface

c) The satellite in umbra region: Both lines intersect Earth's surface

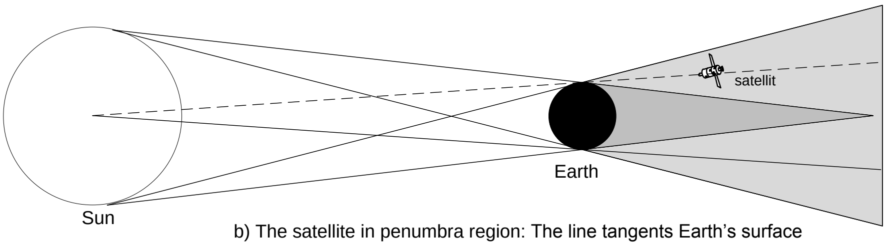
Fig.2 Determining the GPS satellite's positions

The method we propose consists in using a straight line that runs through the following points: Sun's center – satellite. We can obtain the Sun's coordinates center by numeric integration.

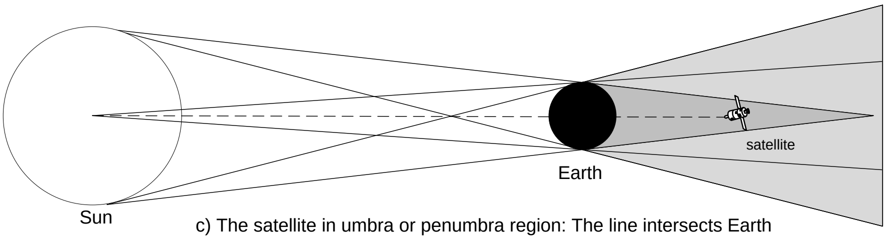
Next we will present the intersection situations of the straight line that runs through the following points: Sun's center – satellite and Earth's ellipsoid.



a)Full phase: The line does not intersect Earthl



b) The satellite in penumbra region: The line tangents Earth’s surface

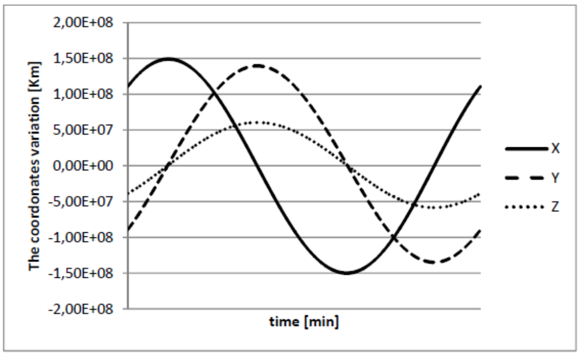


c) The satellite in umbra or penumbra region: The line intersects Earth

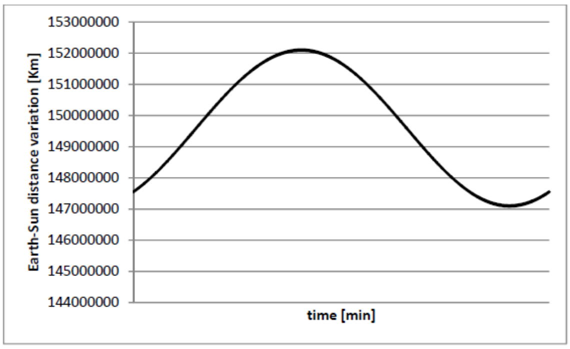
- if the system does not have real solutions then the satellite is fully illuminated;
- if the system has only one real solution then the line is tangent to the ellipsoid’s surface and the satellite is in the penumbra area;
- if the system has two real solutions then it is necessary to determine *y* and *z* intersection coordinates in order to calculate the distance to the Sun.

If this distance is smaller than the distance between the Sun and the satellite then the satellite is fully illuminated. Otherwise the satellite is in the penumbra or umbra area.

For this study the author has determined Sun’s coordinates by numeric integration, minute by minute for 365 days, using as a starting point the date 10.02.2011, 00:00:00 Universal Time (UTC).



Sun’s coordinates variation for a year



Earth-Sun distance variation for a year

In order to determine the geocentric coordinates of the satellite minute by minute for a period of 365 days we started from the initial conditions of position and speed for a GPS satellite on 10.02.2011, 00:00:00 UT.

xai = 2425.8676; vxai = 3.8327;
yai = -15215.1157; vyai = 0.4529;
zai = 21743.2188; vzai = -0.1055;

The satellite was considered to be perturbed by the J2 zonal harmonic. By numeric integration of the satellite movement using Runge-Kutta 4^{th} order method we determined the satellite’s coordinates minute by minute for a whole year.

Tab. 1 The number of solutions for the GPS satellite entering in the umbra or penumbra regions

Ellipsoid type	Number of real solutions	The period of time spent by the satellite in umbra or penumbra area
p = 6378,137 Km q = 6356,752 Km	26424	0
p =q = 6378,137 Km	26506	0
p =q = 6402 Km	26702	0

CONCLUSIONS

According to the observations made upon GPS satellites, the period of time spent in the eclipse region is 1-2 minutes.

For the situations when equation (5) had real solutions, the satellite did not enter Earth’s umbra or penumbra region at any time.

Numerical results regarding the passing of the satellite through a penumbra or umbra region are according to the real movement of GPS satellites, taking into consideration the inherent computational errors, more precisely truncation and rounding errors.

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