THE CALCULATION OF THE MAXIMUM RATED VOLTAGE FOR DIFFERENT TYPES OF LOADING

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Abstract: In the first part of this work, we examined the determination of the stresses in the cross sections with a closed fracture. This placement of fracture is typical for objects with stress concentrator. In the second part we consider the possibility of determining the maximum rated voltage for various configurations of the location of the test sample zone, which has a closed fracture. The maximum rated voltage \( \sigma_{\text{max}} \) is also proposed to determine within fraktographical analysis result, and therefore the method of finding these voltages is valid for objects with a clearly defined zone. Features of this technique are mainly related to the configuration of breaks for different types of loading. Given the lack of such solutions in reference books we are about to review in details the nature of methods for determining the maximum rated voltage at the example of the two most common types of loading: tension with a coefficient of asymmetry of the cycle \( R > 0 \) and cyclic bending / 9 /.

**Keywords:** tension; fatigue fracture; crack form

### The Maximum Nominal Voltage in Tension

Under cyclic tension, crack front Lo break is usually a non-closed form /5,7,8,14,15,19/. The maximum rated voltage at the time of the destruction of the area because of the fatigue crack will differ from the initial stress of \( \sigma \), which leads to disruption of the linear relationship between stress \( \sigma \) and strain \( \varepsilon \).

### The Maximum Nominal Voltage in Tension

The study of fatigue fracture of machine parts, \( E_1 \) - hardening modulus.

\[ \varepsilon_T \text{ is strain corresponding to yield strength } \sigma_T; \]

\[ - \varepsilon \text{ is modulus of elasticity of the first kind;} \]

\[ \varepsilon \text{ is strain corresponding to yield strength } \sigma_T; \]

\[ E_1 \text{ - hardening modulus.} \]

The study of fatigue fracture of machine parts, destroyed in the high-cycle fatigue, showed that the final destruction of the area is not very different from the brittle fracture even for parts made of ductile metals / 3, 5, 37, 11, 13, 15, 16, 17, 18, 19 /. From this we can assume that at the time of the fatigue destroying of the object its diagram of deformation can be described by the equation of Hooke, i.e., determination of stress \( \sigma_{\text{max}} \) news from the assumption of a linear relationship between stress \( \sigma \) and strain \( \varepsilon \).

Then the maximum rated voltage in the area is defined by the formula of materials resisting for eccentric tension rod / 2,11 /.

\[ \sigma_{\text{max}} = P / F + Mx y / iy + My x / vx \] (2.5)

where \( iy \) and \( ix \) are the moments of inertia zone, respectively, relative to the axes \( y \) and \( x \), and \( x \) - \( y \) - coordinates of point B, the farthest from the neutral line \( n-n \), position of which is determined by the intercepts on the coordinate axes \( x \) and \( y \) /11, 12/.

\[ x_B = -j fry / x_B \] (2.6)

\[ y_B = -j fury / y_B \] (2.7)

where \( fry = \sqrt{iy / vx} \) - inertia radiuses related to the axes \( y \) and \( x \) respectively.

If we draw perpendicular from point A onto the neutral line, the position of point B is determined by the maximum interval length received by projection of points of the zone profile onto perpendicular and measured from the line \( n-n \). Expressions (2.5) - (2.7) will get considerably simplified for parts having a symmetrical cross section.

Let us apply formula (2.5) for circular objects most often encountered in practice. Depending on the level of existing nominal voltages and the nature of the voltage concentration, fatigue fracture of objects of that form is shown in figure (a and b are objects without voltage concentrator at high and low load), the \( v \) and \( g \) are objects with a significant concentration of stress under high and low level of loading, \( d \) and \( e \) are objects with low stress concentration at high and low load) / 5, 7, 14, 15, 18, 19 /. The cross section of the object with diameter \( d \) at break has a concave crack of length \( L \) and depth \( H \). Center of gravity of the cross section is located at point \( O \), and the zone is at the point \( A \). The coordinates of the latter is defined by formulas (2.1) - (2.4), and because of the symmetry of the zone \( x_B = 0 \) (the coordinate axis is chosen so that it is symmetrically located on zone of crack propagation) and, therefore,

\[ My = 0 \text{, so formula (2.5) will get simpler and will look like} \]

\[ \sigma_{\text{max}} = P / F + PyA \text{ umax / 1x} \] (2.8)

Here is the coordinate of the most distant point from the central axes equals: for the concave crack front (Figure 2.6)

\[ y_{\text{max}} = yA + 0,5d + H \] (2.9)

for a convex crack front (Figure 2.7)

\[ y_{\text{max}} = yA + 0,5d - H \] (2.10)

where the distance from the crack tip to the straight line connecting the points of intersection of the crack front with a circle of area cross-section

\[ K = (H - 0,5d + 0,5j \sqrt{d^2 - L^2}) j \] (2.11)

where \( j = -1 \) for the case where the distance from point \( E \) to the line B-B is more than 0,5 \( d \), in other cases, \( j = 1 \), \( i = 1 \) - for concave, \( i = -1 \) - a convex crack front.
Let us represent the zone in the form of two segments BCB and BMB, the area where F1 and F2 (hereafter the subscript 1 refers to the geometric characteristics BCB segment, and the index 2 – to those of segment BMB), are respectively equal

\[ F_1 = 0.25d^2(0.57\pi + \varphi + 0.56\sin2\varphi) \quad (2.12) \]

\[ F_2 = \rho^2(0.5\pi - \varphi + 0.5\sin2\varphi) \quad (2.13) \]

Where

\[ \varphi = \arcsin[(0.5d - H + K)/0.5d], \]

\[ \rho = 0.5(\pi' + 0.25d^2)/K; \]

angle \( \gamma = \arcsin[(\rho - \varphi)/\rho] \).

These equations are obtained due to the following considerations:

Let us express the area of BCB segment:

\[ F_{BCB} = F_{a1} \cdot F_{a2} \]

Under the reference book:

\[ F_{BCB} = d^2 \left( \frac{\alpha}{2} - \frac{\sin\alpha}{2} \right) \]

We transform

\[ F_{BCB} = d^2 \left( \frac{\alpha}{2} - \frac{\sin\alpha}{2} \right) = \frac{d^2}{4} \left(0.5\pi - \varphi - 0.5\sin2\varphi\right) \]

Then on substituting into the formula for the area:

\[ F_{BCB} = \frac{md^2}{4} \left(0.5\pi - \varphi - 0.5\sin2\varphi\right) = \frac{d^2}{4} \left(0.5\pi + \varphi + 0.5\sin2\varphi\right) \]

Then the area of the zone

\[ F = F_1 + F_2 \]

Presentation of the area segment in the form (2.12) and (2.13) differs from the conventional in engineering /1, 11/, but it is convenient for calculations. Using the formulas of materials resisting /11, 12/ as well as the solution of definite integrals by means of tables by G.B. Dwight /6/ we define the geometric characteristics of the segments of a circle with diameter \( d \) (area F, the static moment Sx and moment of inertia Ix, relatively to the x-axis). The figure shows segments 3 and 4, defined by the angle \( \alpha \). Let us select an element of area \( dF = dx\;dy \), therefore

\[ F_3 = 2 \int_0^{0.5d} \int_0^{0.5d} \sqrt{0.25d^2 - y^2} \; dy \; dx = \frac{d^2}{4} \int_0^{0.5d} \sqrt{0.25d^2 - y^2} \; dy \]

The final result is

\[ F_3 = 0.25d^2(0.5\pi - \varphi - 0.5\sin2\alpha) \quad (2.15) \]

\[ F_4 = \frac{md^2}{4} + F_3 = 0.25d^2(0.5\pi + \varphi - 0.5\sin2\alpha) \quad (2.16) \]

\[ S_{a1} = 2 \int_0^{0.5d} \int_0^{0.5d} \sqrt{0.25d^2 - y^2} \; dy \; dx \]

\[ S_{a1} = (\cos\alpha)^2 \quad (2.17) \]

\[ S_{a2} = (-\cos\alpha)^2 \quad (2.18) \]

\[ I_{a1} = 2 \int_0^{0.5d} \int_0^{0.5d} \sqrt{0.25d^2 - y^2} \; dy \; dx \]

On having submitted the integration limits, we get

\[ I_{a1} = d^4(0.5\pi - \varphi + 0.25\sin4\alpha)/64 \]

\[ I_{a2} = \frac{md^4}{4} + I_{a1} = d^4(0.5\pi + \varphi - 0.25\sin4\alpha)/64 \]

Using formulas (2.15) - (2.20), we define the geometric characteristics needed to calculate the maximum rated voltage of \( \sigma_{\text{max}} \). The position of the central axis \( Ax \) of the zone, defined by the coordinate \( y_{Ax} \) is to be found by (2.2) and (2.4), on having presenting a static point on the central axes of the cross section as

\[ S_{a1} = S_{a1} - S_{a2} \]

where

\[ S_{a1} = \frac{(\cos\varphi)^2}{12}; \]

\[ S_{a2} = \frac{y_{Ax}^2}{2}. \]

The distance from gravity center of BCB segment to axis \( Ox_0 \)

\[ y_{Ax} = \frac{2}{3}(\rho \cos\gamma^\prime) / F_3 \]

This a distance between axes \( Ox_0 \) and \( O'x' \).

Inertia moment \( I_{l1} \) related to axis \( Ox \) equals

\[ I_{l1} = I_{a1} - I_{a2} \]

where \( I_{l1} \) and \( I_{l2} \) are inertia moments related to axis \( Ax \),

\[ I_{l1} = I_{a1} + (y_1 + y_2)^2 F_1; \]

\[ I_{l2} = I_{a2} + (y_1 + y_2)^2 F_2. \]

We find exact moments of inertia \( I_{l1} \) and \( I_{l2} \), with the help of formulas (2.19) and (2.20) and formulas for the transfer of inertia moments to parallel axes /11, 12/. This is what we have for the exact inertia moment \( I_{l1} \)

\[ I_{l1} = I_{l1} + y_{Ax}^2 F_1 \]

where \( I_{l1} \) is inertia moment of BCB segment related to axis \( Ox_0 \),

\[ I_{l1} = d^4(0.5\pi + \varphi - 0.25\sin4\alpha)/64 \]

\[ y_1 \]

is distance from segment BCB gravity center to axis \( Ox_0 \),

\[ y_{Ax} = S_{a1}/F_3. \]

This is what we have for the exact inertia moment \( I_{l2} \)

\[ I_{l2} = F_{a2} + y_{Ax}^2 F_2 \]

where \( F_{a2} \) is a BCB segment inertia moment related to axis \( O'x' \),

\[ I_{l2} = 0.25d^4(0.5\pi - \varphi + 0.25\sin4\alpha) \]

To calculate the voltage \( \sigma_{\text{max}} \) by formulas (2.8) - (2.25), the algorithm developed can quickly determine the desired values. According to the results of calculation on the charts for concave and convex crack fronts, the relative maximum rated voltage \( \sigma_{\text{max}} / \sigma \) the relative length \( L / d \) crack within different values of its relative \( H / d \) depth are presented. Such a presentation of the calculation results pursuant to formula (2.8) makes it possible to quickly determine the voltage \( \sigma_{\text{max}} \) if you know the size and location of the zone.
REFERENCES