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A TOOL FOR THE AIRCRAFTS ATTITUDE DETERMINATION

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Abstract: *The paper presents a Matlab implemented tool used to calculate the attitude angles of an aircraft starting from the strap-down gyros readings. The tool uses a dedicated method for the numerical integration of the quaternionic Poisson equation and allows the custom selection of one of the first six orders of the numerical method. In a first phase, the theoretical background of the attitude determination is shown, and the equations to be implemented are extracted in discretized form. Further, the obtained tool is shortly described. Subsequently, the validation of the tool is performed. Firstly, a numerical simulation validation step is achieved; finally, an experimental validation step is used to evaluate the tool performances. The obtained tool can be used booth in numerical simulation of a strap-down inertial navigation system, but also in an aircraft post-flight debriefing stage to evaluate the roll, pitch, and yaw attitude angles.*
Keywords: *strap-down inertial navigation, attitude tool, quaternionic method, numerical simulation, experimental validation.*

1. THEORETICAL BACKGROUND

The determination of an aircraft flight attitude with an inertial navigation system involves the determination of the angles of roll, pitch and yaw from the navigation data acquired. There are two main types of inertial navigation systems: the stabilized platform and the strap-down system. The determination of the inertial system attitude with a stabilized platform is relatively easy; being accomplished using angular transducers, which give the position of the stabilized platform relative to the vehicle's frame axes. The use of a platform involves the use of large, heavy equipment, such as a force gyro-stabilizer, making it virtually impossible to use in special applications with light aircraft**s**, and increasingly less in traditional applications [1]-[4].

The strap-down inertial system has the advantage of miniaturization, which makes it ideal for use in all types of applications, but involves the use of a high-performance navigation processor and of optimal algorithms to process the information received from the sensors The problem of the attitude determination with a strap-down system is slightly more complicated than in the case of the stabilized platform, sometimes requiring complex mathematical calculations and therefore corresponding numerical algorithms. There are two methods for determining the attitude in the strap-down systems, however both lead in the end to the achievement of the attitude matrix: the attitude quaternion method and the matrix method. Obtaining the final attitude matrix involves the numerical integration of the differential Poisson attitude equation, in one of its two forms, the matrix or the quaternionic one [2], [4], [5].

The two attitude representations are absolutely equivalent, but the numerical integration of the quaternionic form has several advantages related to the presence of only four parameters linked by a single constraint condition in the ortho-normalization process, compared with the matrix parameterization, which involves the presence of nine parameters bound by three orthogonality conditions and three conditions normality. Moreover, the attitude matrix ortho-normalization involves the existence of an iterative algorithm, which complicates to some degree the numerical calculation. Therefore, the usual recourse is made to the numerical integration of the Poisson quaternionic attitude equation.

To determine the aircraft' flight attitude, both the attitude quaternion is used as well as the direction cosine matrix that switches between the local horizontal frame and the vehicle frame R_l^v. The values known at each time point are the angular velocity components measured using the strap-down inertial system gyros. The quaternion equation to be integrated is [2], [4]-[6]:

$$\dot{Q} = \frac{1}{2}\begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_0 \end{bmatrix}. \quad (1)$$

According to the integration algorithm proposed by Wilcox, the elements of attitude quaternion Q at the time moment t_{n+1} are calculated from the ones at time t_n using the matrix relation [4]-[6]:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_0 \end{bmatrix}_{t_{n+1}} = \begin{bmatrix} C_m & S_m\Delta\phi_z(t_n) & -S_m\Delta\phi_y(t_n) & S_m\Delta\phi_x(t_n) \\ -S_m\Delta\phi_z(t_n) & C_m & S_m\Delta\phi_x(t_n) & S_m\Delta\phi_y(t_n) \\ S_m\Delta\phi_y(t_n) & -S_m\Delta\phi_x(t_n) & C_m & S_m\Delta\phi_z(t_n) \\ -S_m\Delta\phi_x(t_n) & -S_m\Delta\phi_y(t_n) & -S_m\Delta\phi_z(t_n) & C_m \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_0 \end{bmatrix}_{t_n}, \quad (2)$$

where C_m, S_m are the m-order development coefficients of the Wilcox algorithm (Table 1 [4]-[5]), and $\Delta\phi_x$, $\Delta\phi_y$, $\Delta\phi_z$ are the angular increments of the roll, pitch and yaw axes, having the expressions:

$$\begin{aligned} \Delta\phi_x(t_n) &= \int_{t_n}^{t_{n+1}}\omega_x(t_n)\mathrm{d}t = \omega_x(t_n)(t_{n+1}-t_n) = \omega_x(t_n)\Delta t, \\ \Delta\phi_y(t_n) &= \int_{t_n}^{t_{n+1}}\omega_y(t_n)\mathrm{d}t = \omega_y(t_n)(t_{n+1}-t_n) = \omega_y(t_n)\Delta t, \\ \Delta\phi_z(t_n) &= \int_{t_n}^{t_{n+1}}\omega_z(t_n)\mathrm{d}t = \omega_z(t_n)(t_{n+1}-t_n) = \omega_z(t_n)\Delta t. \end{aligned} \quad (3)$$

Table 1. Coefficients of the Wilcox algorithm

n	C_n	S_n
1	1	1/2
2	$1-\phi_0^2/8$	1/2
3	$1-\phi_0^2/8$	$1/2-\phi_0^2/48$
4	$1-\phi_0^2/8+\phi_0^4/384$	$1/2-\phi_0^2/48$
5	$1-\phi_0^2/8+\phi_0^4/384$	$1/2-\phi_0^2/48+\phi_0^4/3840$
6	$1-\phi_0^2/8+\phi_0^4/384-\phi_0^6/46080$	$1/2-\phi_0^2/48+\phi_0^4/3840$

In the relations set (3) it was considered that the angular velocities are approximately constant during a Δt cycle of data acquisition from gyros into the computer [7]. As can be seen from Table 1, the expressions C_m and S_m depend on the norm ϕ_0, which can be calculated with formula:

$$\phi_0(t_n)=\sqrt{\Delta\phi_x^2(t_n)+\Delta\phi_y^2(t_n)+\Delta\phi_z^2(t_n)}. \tag{4}$$

Thus, the new attitude quaternion parameters result as [4]-[6]:

$$\begin{aligned}
q_1(t_{n+1})&=C_m q_1(t_n)+S_m\Delta\phi_z(t_n)q_2(t_n)-S_m\Delta\phi_y(t_n)q_3(t_n)+S_m\Delta\phi_x(t_n)q_0(t_n),\\
q_2(t_{n+1})&=-S_m\Delta\phi_z(t_n)q_1(t_n)+C_m q_2(t_n)+S_m\Delta\phi_x(t_n)q_3(t_n)+S_m\Delta\phi_y(t_n)q_0(t_n),\\
q_3(t_{n+1})&=S_m\Delta\phi_y(t_n)q_1(t_n)-S_m\Delta\phi_x(t_n)q_2(t_n)+C_m q_3(t_n)+S_m\Delta\phi_z(t_n)q_0(t_n),\\
q_0(t_{n+1})&=-S_m\Delta\phi_x(t_n)q_1(t_n)-S_m\Delta\phi_y(t_n)q_2(t_n)-S_m\Delta\phi_z(t_n)q_3(t_n)+C_m q_0(t_n).
\end{aligned} \tag{5}$$

According to the quaternion ortho-normalization algorithm, results the norm

$$|Q(t_{n+1})|=\sqrt{q_0^2(t_{n+1})+q_1^2(t_{n+1})+q_2^2(t_{n+1})+q_3^2(t_{n+1})} \tag{6}$$

and the new quaternion components values are [2], [4]-[6]:

$$q_1(t_{n+1})=\frac{q_1(t_{n+1})}{|Q(t_{n+1})|},\ q_2(t_{n+1})=\frac{q_2(t_{n+1})}{|Q(t_{n+1})|},\ q_3(t_{n+1})=\frac{q_3(t_{n+1})}{|Q(t_{n+1})|},\ q_0(t_{n+1})=\frac{q_0(t_{n+1})}{|Q(t_{n+1})|}. \tag{7}$$

Having in view the form of the matrix equivalent to the attitude quaternion [2], [4]-[6],

$$R_l^v=\begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2+q_0q_3) & 2(q_1q_3-q_0q_2)\\ 2(q_1q_2-q_0q_3) & q_0^2+q_2^2-q_1^2-q_3^2 & 2(q_2q_3+q_0q_1)\\ 2(q_1q_3+q_0q_2) & 2(q_3q_2-q_0q_1) & q_0^2+q_3^2-q_1^2-q_2^2 \end{bmatrix}, \tag{8}$$

then the resulting matrix elements are

$$\begin{aligned}
r_{11}(t_{n+1})&=q_0^2(t_{n+1})+q_1^2(t_{n+1})-q_2^2(t_{n+1})-q_3^2(t_{n+1}),\\
r_{12}(t_{n+1})&=2[q_1(t_{n+1})q_2(t_{n+1})+q_0(t_{n+1})q_3(t_{n+1})],\\
r_{13}(t_{n+1})&=2[q_1(t_{n+1})q_3(t_{n+1})-q_0(t_{n+1})q_2(t_{n+1})],\\
r_{21}(t_{n+1})&=2[q_1(t_{n+1})q_2(t_{n+1})-q_0(t_{n+1})q_3(t_{n+1})],\\
r_{22}(t_{n+1})&=q_0^2(t_{n+1})+q_2^2(t_{n+1})-q_1^2(t_{n+1})-q_3^2(t_{n+1}),\\
r_{23}(t_{n+1})&=2[q_3(t_{n+1})q_2(t_{n+1})+q_0(t_{n+1})q_1(t_{n+1})],\\
r_{31}(t_{n+1})&=2[q_1(t_{n+1})q_3(t_{n+1})+q_0(t_{n+1})q_2(t_{n+1})],\\
r_{32}(t_{n+1})&=2[q_2(t_{n+1})q_3(t_{n+1})-q_0(t_{n+1})q_1(t_{n+1})],\\
r_{33}(t_{n+1})&=q_0^2(t_{n+1})+q_3^2(t_{n+1})-q_1^2(t_{n+1})-q_2^2(t_{n+1}).
\end{aligned} \tag{9}$$

Because the direction cosine matrix, which makes the passage between the vehicle frame and the local horizontal frame, can be obtained with three successive rotations (e.g. yaw with angle ψ, pitch with angle θ and roll with angle φ), its elements have the expressions [2]:

$$\begin{aligned}
r_{11}&=\cos\theta\cos\psi,\\
r_{12}&=\cos\theta\sin\psi,\\
r_{13}&=-\sin\theta,\\
r_{21}&=\sin\varphi\sin\theta\cos\psi-\cos\varphi\sin\psi,\\
r_{22}&=\sin\varphi\sin\theta\sin\psi+\cos\varphi\cos\psi,\\
r_{23}&=\sin\varphi\cos\theta,\\
r_{31}&=\cos\varphi\sin\theta\cos\psi+\sin\varphi\sin\psi,\\
r_{32}&=\cos\varphi\sin\theta\sin\psi-\sin\varphi\cos\psi,\\
r_{33}&=\cos\varphi\cos\theta.
\end{aligned} \tag{10}$$

Knowing the values of the matrix elements at the current time moment and from the previous one, from relations (10) can be determined the roll, pitch and yaw angles with the formulas [4]:

$$\varphi = \mathrm{arctg}(r_{23} / r_{33}),$$
$$\theta = \arcsin(-r_{13}),$$
$$\psi = \mathrm{arctg}(r_{12} / r_{11}), \quad (11)$$

which take into account the quadrants in which the trigonometric angles are localized.

2. ATTITUDE TOOL SHORT DESCRIPTION

By implementing the previously presented algorithm in an S-function in Matlab/ Simulink, the “Wilcox Attitude” subsystem from Fig. 1 was obtained. The subsystem has as inputs the components of the angular speed read by the three strap-down gyros ω_x, ω_y, ω_z (“omx”, “omy” and “omz” in Fig. 1), and as outputs the attitude angles φ, θ, ψ.

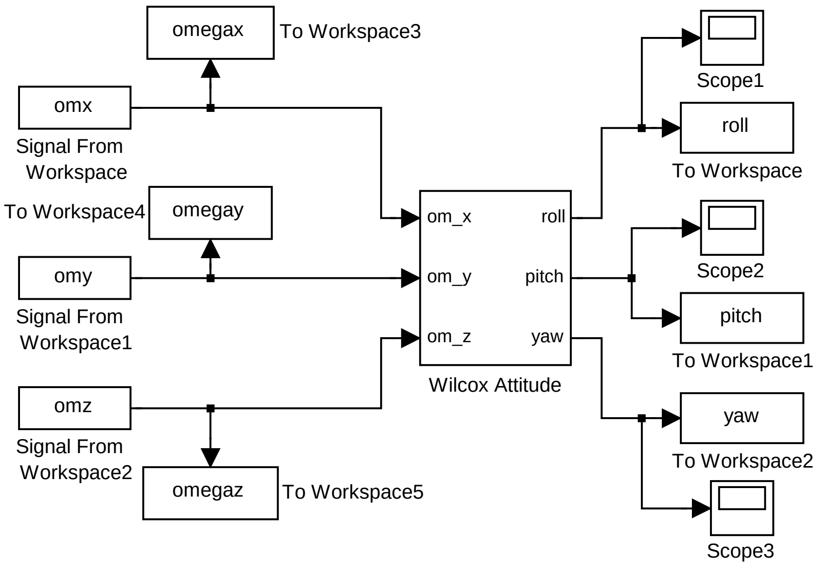

Fig. 1 The SIMULINK model of the algorithm

The user can set the values of the initial attitude angles and of the sample time by using the interface in Fig. 2. The same interface allows the user to choose one of the six orders of the Wilcox algorithm in order to be used in the gyro data processing. In order to obtain the initialization values of the quaternion components the tool uses the algorithm implemented in Fig. 3, where “roll_i”, “pitch_i” and “yaw_i” are the initial attitude angles values introduced by the user at the interface level, while “q0_i”, “q1_i”, “q2_i” and “q3_i” are the initial values of the quaternion components

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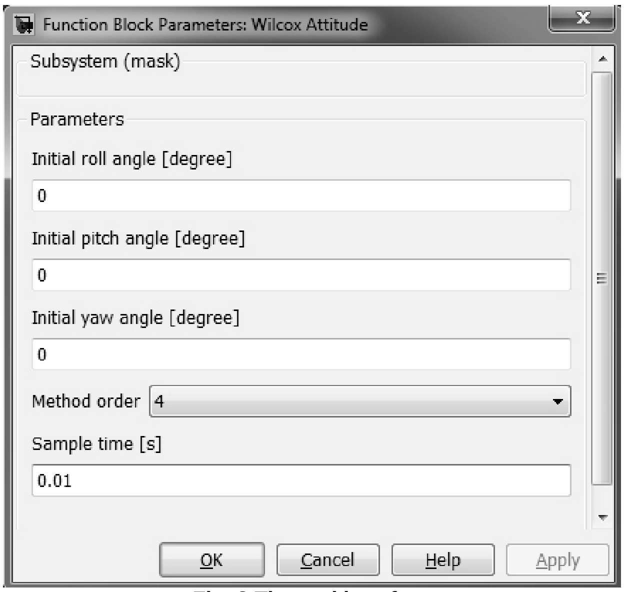

Fig. 2 The tool interface

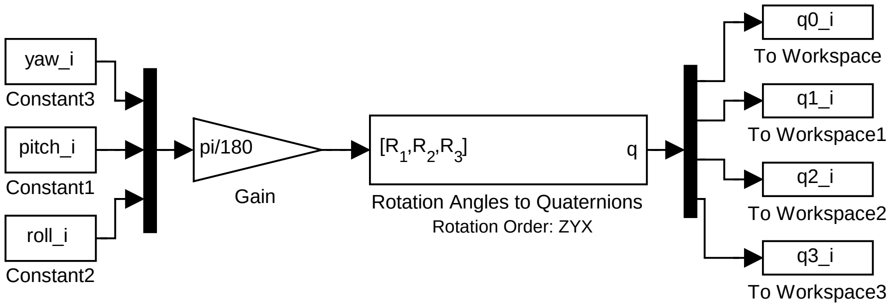

Fig. 3 Quaternion initialization algorithm

3. NUMERICAL SIMULATION AND EXPERIMENTAL VALIDATION OF THE TOOL

For testing the algorithm, at the system input there are applied signals of different forms and the correct angular position is calculated with a theoretical method. Subtracting the value resulted from the simulation it results the error for the respective test. Independent inputs on the three angular speed channels will be applied to facilitate the evaluation of the attitude angles errors.

Thus, on channel x it is applied the signal from Fig. 4 while on the rest of the channels the inputs are maintained null [4]. The simulation is made for the first 6 orders of the Wilcox method. The resulting graphical characteristics of the attitude angles for order 1 are the ones presented in Fig. 5. The final value of the roll angle for all the four orders of the Wilcox method and the error, are presented in Table 2 (we also have to take into consideration that the final roll angle is calculated using a theoretical method and it equals 120°).

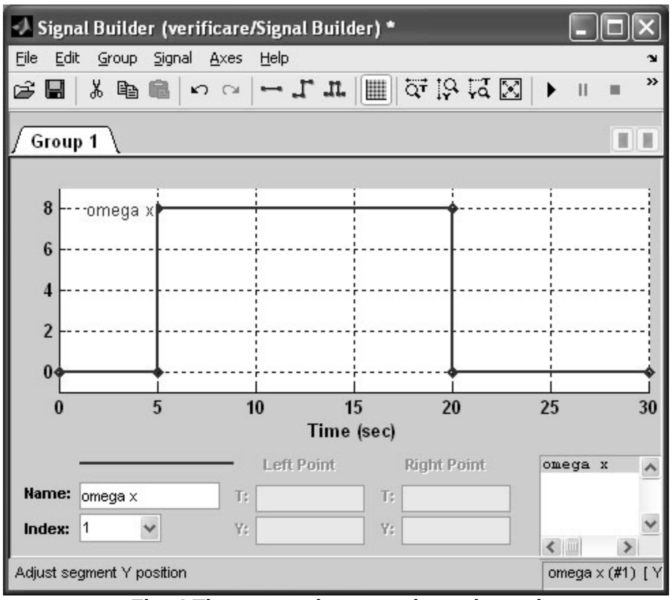

Fig. 4 The system input on the *x* channel

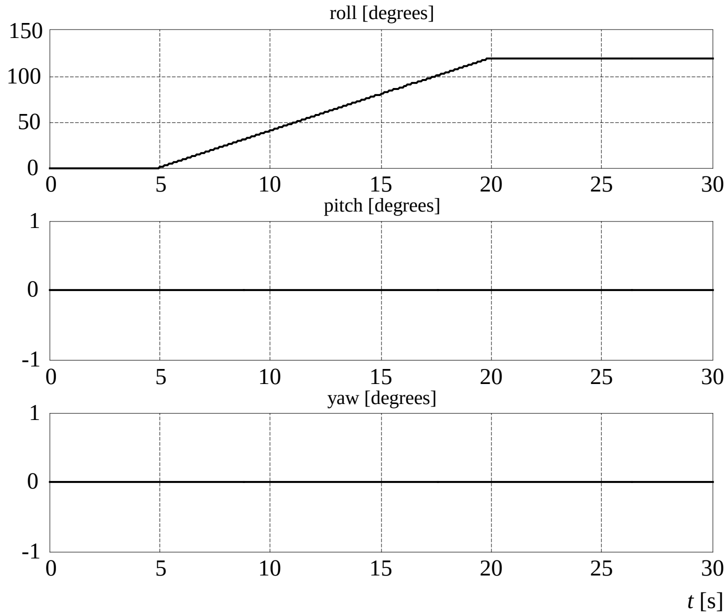

Fig. 5 Attitude angles for the *x* channel input

The simulation is repeated for the same conditions, but this time for input channels *y* and *z*. The graphic characteristics of the attitude angles are the ones in Fig. 6 for non-null input in *y* axis and the ones in Fig. 7 for non-null input in *z* axis. The pitch angle theoretical calculated for the input in Fig. 4 is 60°, and the yaw angle equals 120°.

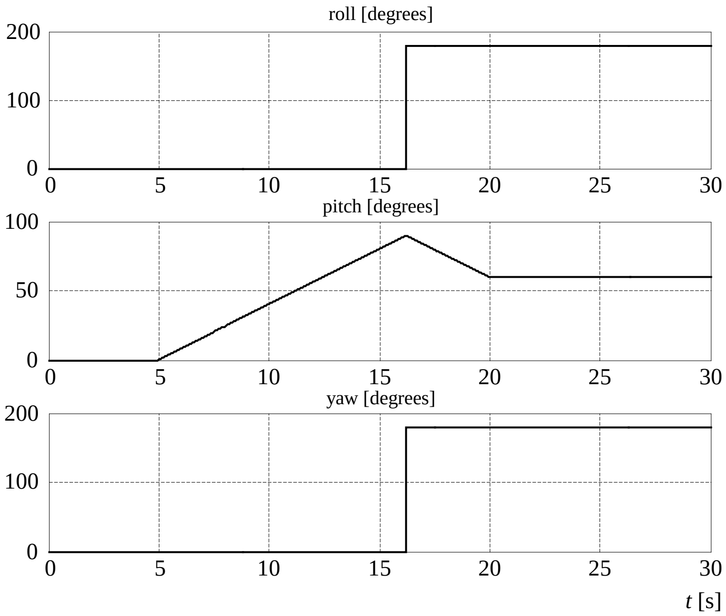

Fig. 6 Attitude angles for the *y* channel input

Table 2. Attitude angles and errors for inputs in Fig. 4

Order	Attitude angles [degrees]			Error [degrees]		
	Roll	Pitch	Yaw	Roll	Pitch	Yaw
1	119.9	60	119.9	$1.94 \cdot 10^{-5}$	$-1.94 \cdot 10^{-5}$	$1.94 \cdot 10^{-5}$
2	120	59.9	120	$-9.74 \cdot 10^{-6}$	$9.74 \cdot 10^{-6}$	$-9.74 \cdot 10^{-6}$
3	120	59.9	120	$-8.24 \cdot 10^{-13}$	$8.17 \cdot 10^{-13}$	$-8.24 \cdot 10^{-13}$
4	119.9	60	119.9	$2.84 \cdot 10^{-13}$	$-3.12 \cdot 10^{-13}$	$2.84 \cdot 10^{-13}$
5	119.9	60	119.9	$1.63 \cdot 10^{-19}$	$-2.34 \cdot 10^{-19}$	$1.63 \cdot 10^{-19}$
6	120	59.9	120	$-8.27 \cdot 10^{-19}$	$9.12 \cdot 10^{-19}$	$-8.27 \cdot 10^{-19}$

From the values presented in the previous table we notice that the absolute values for the errors on the three attitude channels are almost identical when applying the same inputs of angular speed. Also, there can be noticed a tendency to coincide for orders 1 and 2, for orders 3 and 4, as well as for orders 5 and 6. The difference between orders 1-2 and 3-4 is seven orders of magnitude, while between orders 3-4 and 5-6 is six orders of magnitude. It is worth mentioning the change in error sign from order 1 to 2, from order 3 to 4, and from order 5 to 6.

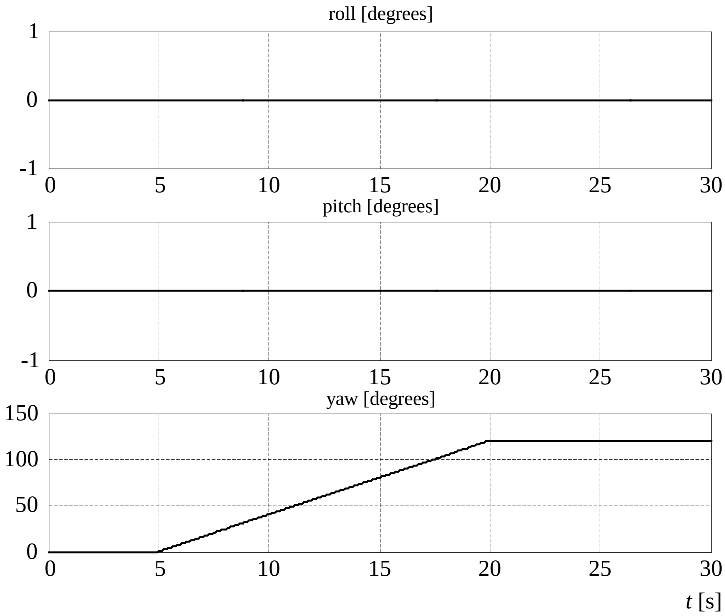

Fig. 7 Attitude angles for the *z* channel input

To experimentally validate the functionality of the developed tool, an integrated GPS-SDINS navigation system was used as a secondary system to calculate the attitude. The integrated system was placed on a bench tester, and an alignment system was used in order to obtain a good horizontality of the testing table. The test duration was of approximately 300 seconds, period in that the attitude angles were calculated by the GPS-SDINS and acquired in order to be compared with the ones calculated by our tool. The tool inputs were the outputs of the three gyros used by the SDINS, and is presented in Fig. 8. The test was a statically one (all of the three rotation rates was maintained zero), but the gyro sensors errors affected the measurements. The outputs of the three gyros presented in Fig. 8 were corrected from the point of view of the biases.

The comparative results of the GPS-SDINS obtained angle of attitude and of our tool are shown in Fig. 9 (roll angle), Fig. 10 (pitch angle) and Fig. 11 (yaw angle). From this characteristics can be easily observed that the GPS-SDINS and tool responses are very close one to each other, with the remark that the GPS-SDINS have the good results. The reason is the presence of GPS in the integration algorithm, which has an important influence in the SDINS sensors errors' correction; we made the correction of the gyros biases by using a classical method based on the means values, while the integration algorithm in the GPS-SDINS system estimates statistically the biases by using a Kalman filter. The initialization values for the roll, pitch and yaw angles used in tool were: 0.37 deg, -0.13 deg, and -64.49 deg respectively.

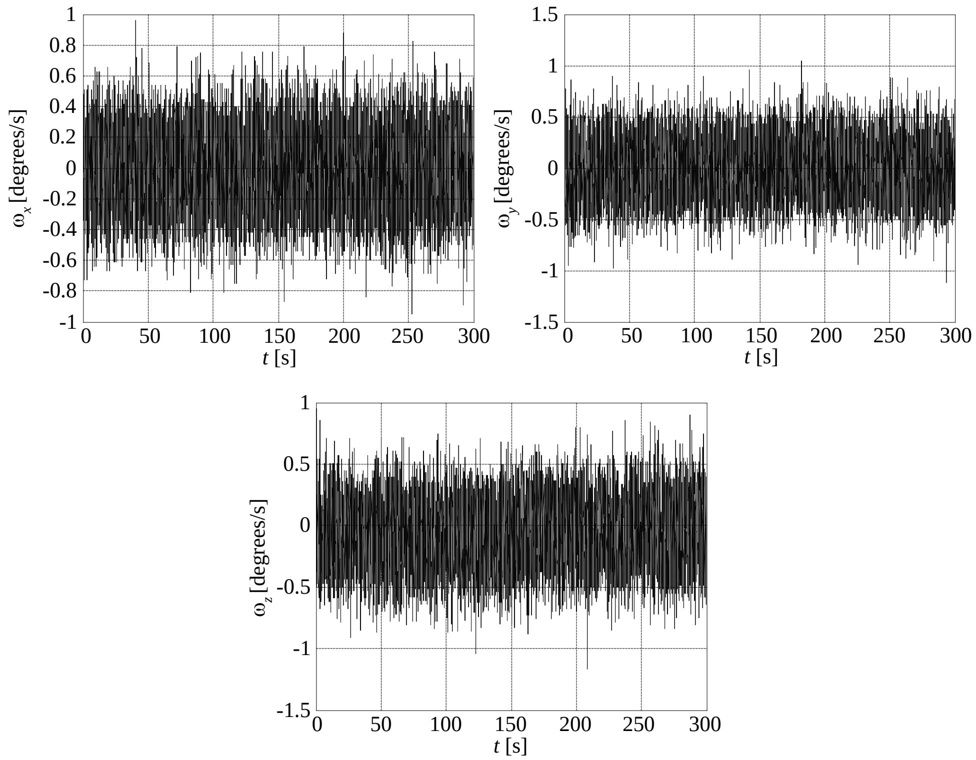

Fig. 8 Outputs of the SDINS’ three gyros

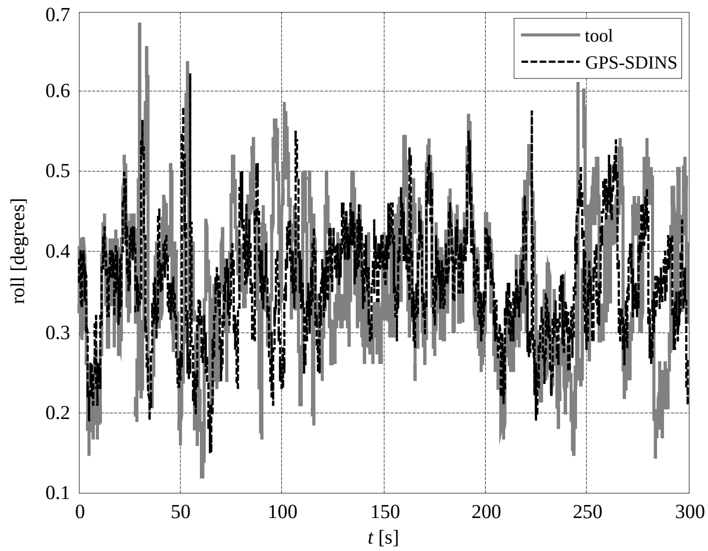

Fig. 9 Roll angle - tool versus GPS-SDINS

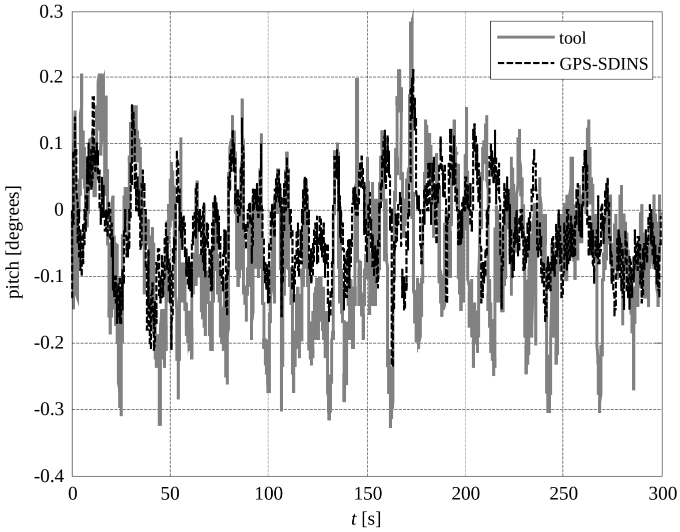

Fig. 10 Pitch angle - tool versus GPS-SDINS

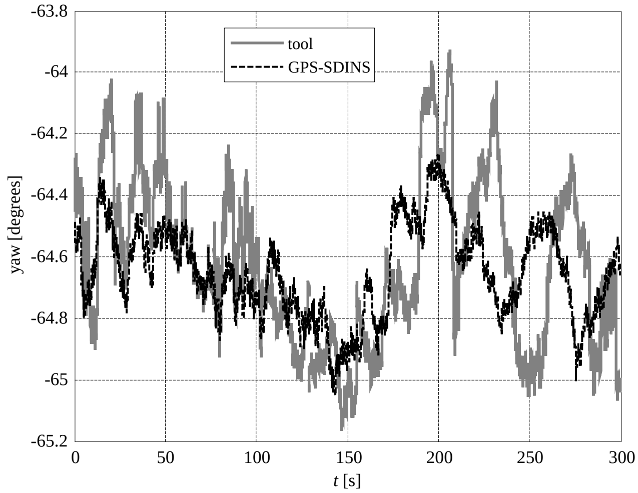

Fig. 11 Yaw angle - tool versus GPS-SDINS

4. CONCLUSIONS

The paper presented a Matlab implemented tool used to calculate the attitude angles of an aircraft starting from the strap-down gyros readings. It is based on the numerical integration of the Poisson equation in quaternionic form by using the Wilcox method. The tool allows user to select one of the first six orders of the integration method.

The numerical simulation and experimental validation obtained results confirm the validity of the developed tool and lead to the conclusion that the usage of the Wilcox method for the numerical integration of the Poisson quaternionic attitude equation is recommended in strap-down inertial navigation systems (SDINS) not only from the point of view of its velocity as an algorithm but also because of its small values of the attitude final errors.

ACKNOWLEDGMENT

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