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Steering of WIG on stationary longitudinal movement with small disturbances

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Abstract: The main problem in the design of the ekranoplan is the ensuring acceptable stability and controllability. An ekranoplan is, in a broad sense, a dynamically unstable and complex control object. Therefore, it is customary to determine the particular characteristics of stability and controllability by individual parameters, for example, by the angle of attack, speed. Also, it is necessary to ensure its acceptable performance properties in the process of their implementation in practice. Since an ekranoplan is the object of research, the following factors must be taken into account: the distance from the screen is one of the defining aspects directly affecting the aerodynamic characteristics. The quantitative result of such modeling as described in this article will allow for each specific device to evaluate its behavior at the time intervals of the calculation and to obtain a qualitative analysis of the operation of the system with a given geometry and the nature of disturbances to assess the tendencies of the ekranoplan's behavior. The method makes it possible to fairly strictly take into account the effect of changing the standing from the surface on the aerodynamic characteristics of the ekranoplan at the stage for solution of the system of differential equations of motion dynamics. Unlike from previously used approaches, the distance from the surface is considered as determining parameter for all aerodynamic coefficients of the ekranoplan.

Key words: Static and dynamic stability, steering of ekranoplan, aerodynamic characteristics

1. Introduction

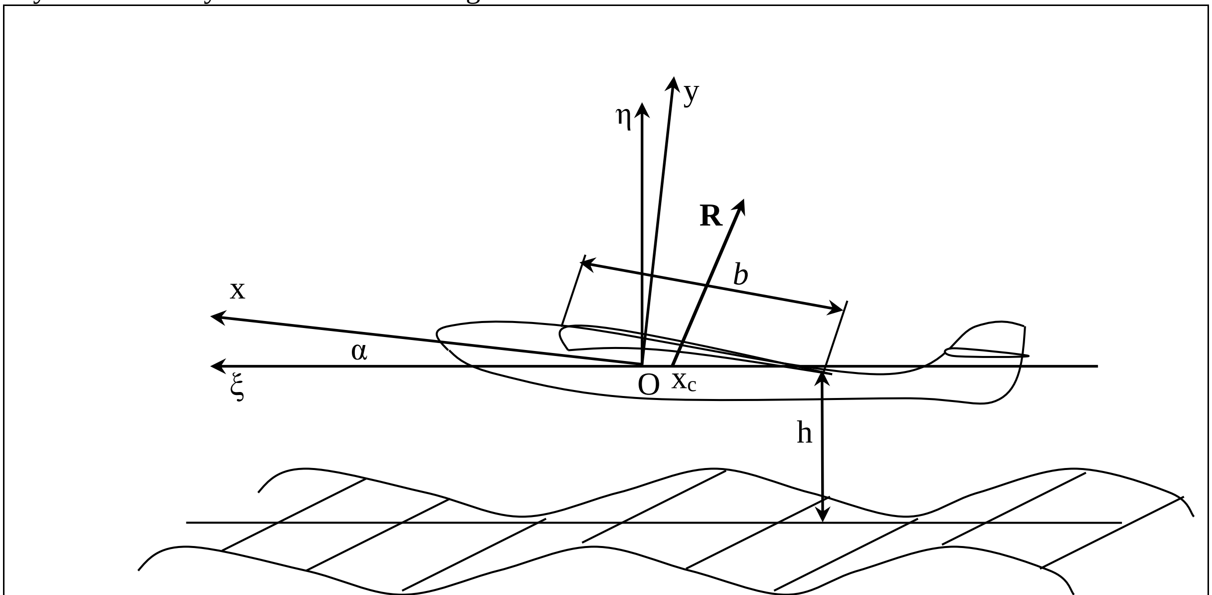
The nonlinear dependence of the aerodynamic characteristics on such kinematic parameters as: angle of attack, angle of deflection of aerodynamic control surfaces.

The emerging non-stationary effects have a significant impact on the dynamics of motion, that is, the values of the aerodynamic coefficients constantly change over time. Secondly, due to the proximity of the underlying surface, the presence of irregularities on it, the requirements for the stability and control accuracy of the ekranoplan increase. WIG craft represent a new class of aircraft, as a rule, aircraft aerodynamic configuration, focused on using the advantages of flight in the zone the influence of the screen effect.The screen effect is manifested in an increase in the coefficient of lift and aerodynamic quality of the aircraft at distances from the trailing edge of the wing to the water surface that are less than the length of the wing chord. The aerodynamic quality of an ekranoplan at a relative cruising altitude above the screen can be the same as that of modern passenger aircraft. This level of quality is achieved at significantly lower wing aspect ratios than in aircraft. Flying in close proximity to the water surface in a narrow operating altitude range, measured by fractions of the wing chord length, imposes significant restrictions on the permissible maneuvers of the aircraft and forces otherwise look at the requirements for the characteristics of stability, controllability and the principles of ekranoplan control. The experience in the development and operation of the constructed marine ekranoplanes proved the need to use specific piloting techniques and the development of control automation aimed at improving stability and controllability characteristics, ensuring flight safety and reducing the workload on pilots.

2. The problem of steady movement of the ekranoplan at stationary longitudinal displacement at height and small perturbations in pitch, roll and flight altitude

In the theory of flight stability, static and dynamic stability are traditionally considered. The problem of the static stability of the aircraft is reduced to the analysis of the conditions for the occurrence of forces and moments on the body, preventing the growth of the mismatch between the disturbed and reference characteristics of the motion. The study of dynamic stability is reduced to the analysis of the process of bringing the disturbed motion parameters to the reference ones. It makes no sense to pose the problem of dynamic stability without a positive answer about static. The decision on static stability requires solving the problem of the distribution of forces and moments on the body of the aircraft and is, in general, an aerodynamic problem, which is described by a part of the specified dynamic system, namely "EKRANOPLAN - AIR ENVIRONMENT".

To solve the problem of dynamic stability, it is necessary to analyze the integral mathematical model of the dynamic system "EKRANOPLAN - AIR - MOVING BORDER". Thisarticle presents a mathematical model of this system component-wise and a method of integrating these components to analyze the stability of the motion of the ground effect vehicle.



Picture1 – The describtion of movement of the ekranoplan on the height “h” above the water surface with small disturbances

Consider the accompanying coordinate system $O\xi\eta\varsigma$ and rigidly connected with the ekranoplan $Oxyz$ .The matrix for converting coordinates (ξ, η, ς) in (x, y, z) at small angles has the form:

$$A_{\xi}^{x} = \begin{Vmatrix} 1 & \varphi_z(t) & -\varphi_y(t) \\ -\varphi_z(t) & 1 & \varphi_x(t) \\ \varphi_y(t) & -\varphi_x(t) & 1 \end{Vmatrix} \qquad (1)$$

The horizontal motion of an ekranoplan above the water surface at a height h with a translational speed $V_э$ parallel to the plane of the water is investigated. Perturbations of the flow caused by the movement of the water surface cause changes in the aerodynamic forces acting on the ekranoplan, which affects the kinematic characteristics of the vehicle's motion, namely, the angles of roll φ_x, yaw φ_y and pitch φ_z, and the meanings of projections v_x, v_y, v_z of the translational velocity on the axis of the coordinate system $Oxyz$, associated with the ekranoplan.

Assuming the perturbations to be small, we will compose a system of linear equations of motion of the ground effect vehicle in the coordinate system $Oxyz$. Let us introduce the notation:

$\bar{V} = (v_x, v_y, v_z) = (\dot{q}_1, \dot{q}_2, \dot{q}_3)$; $\bar{\Omega} = (\dot{\varphi}_x, \dot{\varphi}_y, \dot{\varphi}_z) = (\dot{q}_4, \dot{q}_5, \dot{q}_6)$;

– $\mathbf{J} = \left| J_{ij} \right|$ - the tensor of inertia of the ekranoplan;

– $\mathbf{\Lambda} = \left| \lambda_{pr} \right|$ - the tensor of the added masses of the ekranoplan.

– m – the weight of ekranopan.

Then, in a moving system, the system of equations of motion has the form:

$$\begin{cases} m\dot{\bar{V}} + \sum_{r=1}^{6} \lambda_{pr}\ddot{q}_r + \bar{\Omega} \times \left(m\bar{V} + \sum_{r=1}^{6} \lambda_{pr}\dot{q}_r \right) = \bar{F},\ p = 1..3 \\ \mathbf{J}\dot{\bar{\Omega}} + \sum_{r=1}^{6} \lambda_{pr}\ddot{q}_r + \bar{\Omega} \times \left(\mathbf{J}\bar{\Omega} + \sum_{r=1}^{6} \lambda_{pr}\dot{q}_r \right) + \bar{V} \times \left(m\bar{V} + \sum_{r=1}^{6} \lambda_{pr}\dot{q}_r \right) = \bar{M},\ p = 4..6 \end{cases} \tag{2}$$

Where $\bar{F}$ and $\bar{M}$ are vectors of external forces and moments.Assuming the values $\dot{q}_p$ to be small and neglecting the products $\dot{q}_p\dot{q}_r$ in (1),we obtain a system of linear equations of motion for the ekranoplan in the usual notation:

$$\begin{cases} (m+\lambda_{11})\dot{v}_x + \lambda_{12}\dot{v}_y + \lambda_{13}\dot{v}_z + \lambda_{14}\ddot{\varphi}_x + \lambda_{15}\ddot{\varphi}_y + \lambda_{16}\ddot{\varphi}_z = F_x \\ \lambda_{21}\dot{v}_x + (m+\lambda_{22})\dot{v}_y + \lambda_{23}\dot{v}_z + \lambda_{24}\ddot{\varphi}_x + \lambda_{25}\ddot{\varphi}_y + \lambda_{26}\ddot{\varphi}_z = F_y \\ \lambda_{31}\dot{v}_x + \lambda_{32}\dot{v}_y + (m+\lambda_{33})\dot{v}_z + \lambda_{34}\ddot{\varphi}_x + \lambda_{35}\ddot{\varphi}_y + \lambda_{36}\ddot{\varphi}_z = F_z \\ \lambda_{41}\dot{v}_x + \lambda_{42}\dot{v}_y + \lambda_{43}\dot{v}_z + (J_{xx}+\lambda_{44})\ddot{\varphi}_x + (J_{xy}+\lambda_{45})\ddot{\varphi}_y + (J_{xz}+\lambda_{46})\ddot{\varphi}_z = M_x \\ \lambda_{51}\dot{v}_x + \lambda_{52}\dot{v}_y + \lambda_{53}\dot{v}_z + (J_{yx}+\lambda_{54})\ddot{\varphi}_x + (J_{yy}+\lambda_{55})\ddot{\varphi}_y + (J_{yz}+\lambda_{56})\ddot{\varphi}_z = M_y \\ \lambda_{61}\dot{v}_x + \lambda_{62}\dot{v}_y + \lambda_{63}\dot{v}_z + (J_{zx}+\lambda_{64})\ddot{\varphi}_x + (J_{zy}+\lambda_{65})\ddot{\varphi}_y + (J_{zz}+\lambda_{66})\ddot{\varphi}_z = M_z \end{cases} \tag{3}$$

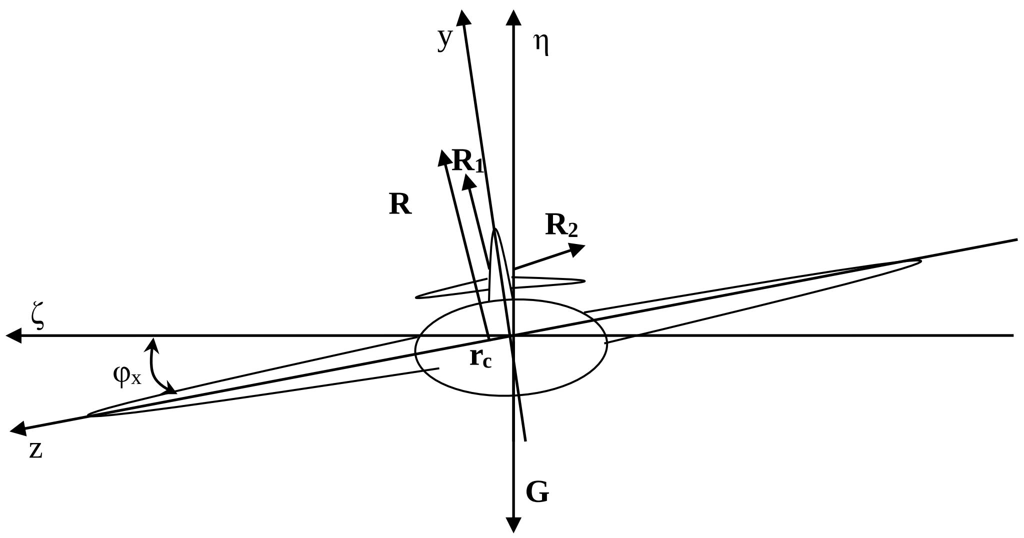
Consider the forces acting on the ekranoplan. First of all, this is the thrust force of the engines, the effect of the air environment on the fuselage and wings, and the force of gravity. The effect of the medium on the motion of the wing-effect vehicle is due to the viscosity and its shape. The coefficient of viscous resistance depends on the value of the dimensionless velocity, area and surface roughness of the body and is practically not subject to changes caused by small disturbances of the medium. The coefficients of forces and moments due to the shape of the apparatus are subject to changes to a greater extent, since perturbations affect the distribution of pressure over the surface of the body. The nature of the mutual influence of the environment and the apparatus is very complex: the environment changes the position and movement of the ekranopaln, which in turn creates additional aerodynamic forces and moments, which are either compensatory in nature and make the movement stable, or increase the mismatch between the reference and current parameters of the movement, making it unstable. In the second case, it is necessary to create additional forces and moments on the body that return the motion parameters to the permissible limits of change. This is achieved by using control means - flaps, rudder, etc., that is, by changing the shape of the ekranoplan. This change, in turn, will affect the aerodynamic force and moment on the hull.

2. Statistical stability of horizontal motion

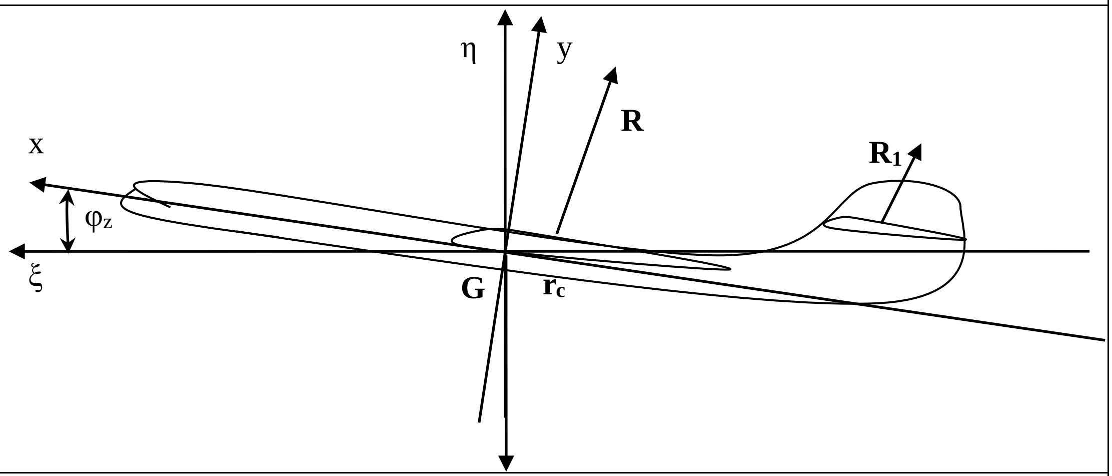
To analyze the static stability of the WIG craft, it is important to find out which changes in position and movement lead to the emergence of compensating forces and moments, and for which it is necessary to use controls.

Qualitatively the picture of the mutual influence of the elements of the "Ekranoplan - air- moving border" system looks as follows. Under the influence of disturbances, the ekranoplan changes its position relative to the screen, which, in turn, also leads to a change in the pressure distribution over

the surface of the body. Let us consider what effects arise with characteristic changes in the position of the apparatus.
When a roll occurs φ_x, the bearing properties of the wing located closer to the screen increase, while for the other wing, on the contrary, they decrease.The point of application of aerodynamic forces $\bar{R}$ on the wing and $\bar{R}_1$ on the tail rudder is shifted towards the roll.



Picture2 – Forces which make influence for ekranoplan during the pitching on the angle φ_x



Picture 3 - Forces which make influence for ekranoplan during the rolling on the angle φ_z

These forces create the moments $\bar{M}_R$ and $\bar{M}_{R1}$, providing a decrease in the roll angle and restoration of the straight position of the apparatus.The projections $\bar{R}$ and $\bar{R}_1$ on the axis $O\varsigma$ causes the displacement of the apparatus in the transverse direction. That is, the fuselage and tail leave the symmetrical flow mode, which causes the vehicle to rotate about the axis Oy, the so-called yaw φ_y. On the vertical rudder, a force is generated $\bar{R}_2$, in the direction of rotation.It creates a moment $\bar{M}_{R2}$, to compensate for the yaw.

The displacement of the points of application of forces $\bar{R}$ and $\bar{R}_1$ along the axis Ox contributes to the appearance of the pitch angle φ_z, which in turn affects the valuesof $\bar{R}$ and $\bar{R}_1$, and the points of their application along the axis Ox. Depending on the position of the centers of pressure relative to the center of gravity, the forces $\bar{R}$ and $\bar{R}_1$ generate either a restoring moment or a moment that increases the angle. As you know, an increase in the angle of attack of the wing beyond the permissible values leads to a catastrophic loss of its bearing capacity.

From the given qualitative picture of the behavior of the ekranoplan, the complexity of the interaction of forces and motion parameters is clear, which makes the problem significantly nonlinear and problematic for an analytical solution. Therefore, we will highlight the most significant factors: $\bar{R}$ – the force on the wing, $\bar{r}_R$ – radius vector of its application;

- $\bar{R}_1$ – horizontal tail rudder force, $\bar{r}_{R1}$ – radius vector of its application;
- $\bar{R}_2$ – vertical tail rudder force, $\bar{r}_{R2}$ – radius vector of its application;
- $\bar{G}$ – the gravity; $\bar{r}_G$ – radius vector of its application;
- $\bar{N}$ – the frictional resistance of the fuselage applied at the center of gravity;
- $\bar{P}$ – pulling force applied at the center of gravity.

We neglect the resistance of the fuselage shape (well streamlined body). For the analysis of static stability, we formulate conditions.The force $\bar{R}$ will always create a recovery moment in pitch if the wing center of pressure moves beyond the vehicle's center of gravity at $\varphi_z > 0$, and in front, if $\varphi_z < 0$. The center of pressure is force $\bar{R}_1$ is always located behind the central of gravity of apparatus, therefore, the direction of the force itself will affect the sign of the moment $\bar{R}_1$. Structurally, this is achieved by a symmetrical profile of the horizontal tail rudder. When $\varphi_z > 0$ the projection $\bar{R}_1$ onto the axis Oy is positive, when $\varphi_z < 0$ – negative. In that case the force moment $\bar{R}_1$ restoring.That is, the sum of the projections of the moments $\bar{M}_{Rz} = \left(\bar{M}_R + \bar{M}_{R1}\right)\cdot\bar{k}$ of the axis Oz have a sign opposite to the sign φ_z

$$M_{Rz} = \left(x_c - x_G\right)R_y - \left(y_c - y_G\right)R_x + \left(x_c^1 - x_G\right)R_{1y} - \left(y_c^1 - y_G\right)R_{1x} = \\ = \left(x_c - x_G\right)(R_y + R_{1y}) - \left(y_c - y_G\right)(R_x + R_{1x}) \quad (4)$$

The vertical rudder design always generates a restoring torque that reduces yaw. Also, the forces $\bar{R}$ and $\bar{R}_1$ will always generate a moment that contributes to a decrease in the roll angle due to the fact that the bearing capacity of the wing located closer to the screen increases, and this causes the center of pressure to shift along the axis Oz towards the roll.From what has been said, it can be seen that the static stability of the horizontal flight of the vehicle without active control is ensured by the correct choice of the wing and its placement on the fuselage to ensure the required values of forces $\bar{R}$ and $\bar{R}_1$, and the correct location of their centers of pressure.

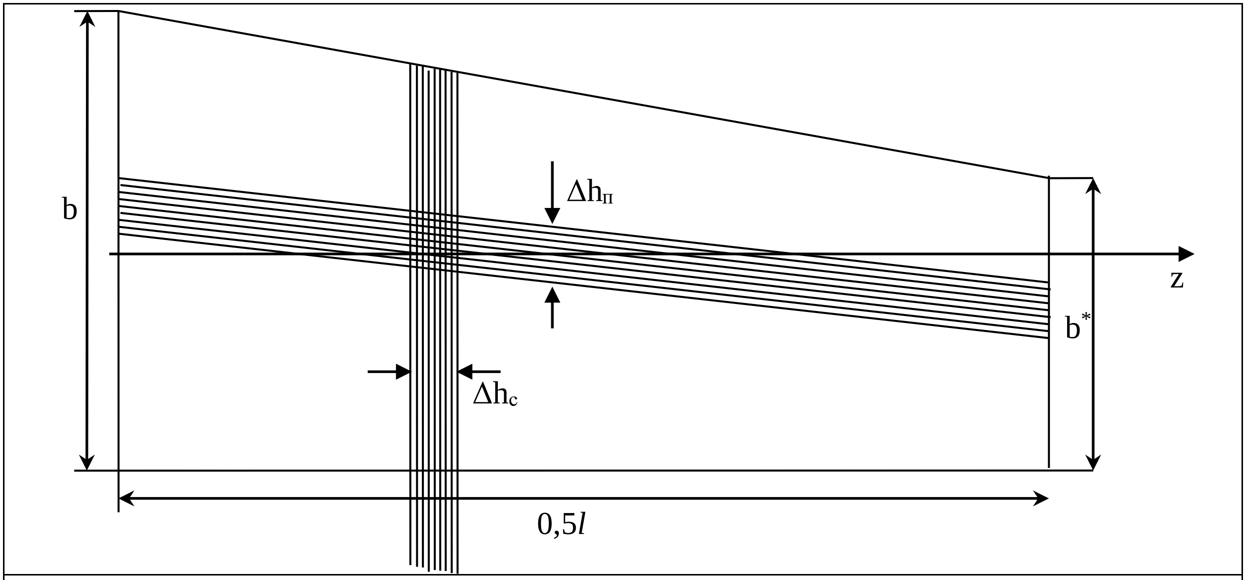
3. Dynamic stability of horizontal motion

Solution of the problem of dynamic stability of horizontal flight of an aircraft is reduced to clarifying the nature of the dependence of the kinematic parameters of the ground effect vehicle on time. The dynamic model of small deviations of the vehicle from horizontal flight is described by system (3) with initial conditions and specific forces acting on it. Aerodynamic forces are determined by the geometry of the ground effect vehicle moving in the disturbed air environment. Taken into consideration, within the framework of the simplifications made, the forces $\bar{R}$, $\bar{R}_1$, $\bar{R}_2$ and the

corresponding moments depend on the lift and drag coefficients of the wing, tail horizontal and vertical rudders operating in the incoming flow at a speed $-\overline{V}_э$ with small perturbations.
Disturbances in the environment are caused by the excitement of the water surface.
The effect of the medium on the motion of the ekranoplan is due to its viscosity and its shape. The coefficient of viscous resistance depends on the value of the dimensionless velocity, area and roughness of the body surface and is practically not subject to changes caused by small disturbances of the medium. The coefficients of forces and moments due to the shape of the apparatus are subject to changes to a greater extent, since perturbations affect the distribution of pressure over the surface of the body. The complete picture of the homogeneous flow around the WIG over the moving boundary is realized by the method of superposition of the oncoming homogeneous flow and disturbances caused by the fuselage and wings of the WIG, as well as the disturbances of the medium by the moving boundary. To solve the problem of determining the IM characteristics, the method of superposition of potential flows is adopted in the work.
The potential of a homogeneous incoming flow in a connected coordinate system $Oxyz$ has the form:

$$\Phi_0 = -V_э\left(x + \varphi_z y - \varphi_y z\right) \qquad (5)$$

To calculate the inductive velocities caused by the influence of the "fuselage-wing" system on auniform flow, it is based on the method of distribution of features over the surface of the hull.
The wing and the vortex wake behind the trailing edge of the wing are modeled by a continuous vortex surface, namely by a system of attached vortices continuously distributed along the middle of the wing and by a continuous system of free vortices leaving the trailing edge of the wing to infinity - a vortex sheet. We assume that the plane of the vortex sheet is parallel to the plane of the undisturbed water surface. This assumption is fully justified, since the angle of attack α of the wing is small in the usual modes of motion of the ekranoplan.



Picture 4 - The bundle of the attached vortexes of thickness Δh_c and Δh_n

To ensure the condition of constant circulation along the vortex tube, certain conditions are imposed on the circulation distribution function. We assume that the bundle of the attached vortex of thickness Δh_n consists of many vortex filaments escaping from the attached wing along the span. These filaments, in turn, form a vortex cord of escaping vortices of thickness Δh_c, which leaves the trailing edge of the wing in a free vortex. It is clear that the intensity of the elementary attached vortex will vary along the wingspan due to the runaway of the vortex filaments. In the transverse direction of the wing, from the trailing edge to the leading edge, the intensity of the attached vortex decreases due to the depletion of free vortex filaments distributed between the attached ones at a

given cross section. Similarly, the intensity of the bundles of runaway vortices changes in the longitudinal and transverse directions of the wing.
To simplify the analytical expressions, we introduce a continuous parameter μ , indexing the attached vortices. For rectangular wings, this can be the x coordinate. For trapezoidal wings, this parameter can be entered as follows. If the attached vortices lie uniformly on the wing, then the axis of the vortex has the equation: $x = \mu(z - 0.5l) - 0.5b,\ 0 \le z \le 0.5l,\ -\frac{2b}{L} \le \mu \le 0$ on the right wing, and $x = \mu(z + 0.5l) - 0.5b,\ 0.5l \le z < 0,\ 0 < \mu \le \frac{2b}{L}$ on the left wing, $L = \frac{lb}{b - 2b}$

Let $\gamma(x(\mu), z)$ the density be the distribution of the circulation of the vortex filaments leaving the attached vortex μ . Then the distribution density of the circulation of the attached vortex μ at the point of the plane on the wing with coordinates $(x(\mu), z)$ is equal to the total circulation of all vortex filaments descending from this attached vortex on the interval $(z, 0.5l)$ for the right wing, and $(z, -0.5l)$ for the left.For points symmetrically located about the axis Ox fair that $\gamma_n(x(\mu), z) = \gamma_n(x(\mu), -z)$. Thus:

$$\gamma_n^+(x(\mu), z) = \int_z^{0.5l} \gamma(x(\mu), s)\, ds,\ \ z \ge 0$$
$$\gamma_n^-(x(\mu), z) = \int_{-0.5l}^{z} \gamma(x(\mu), s)\, ds,\ \ z < 0 \qquad (6)$$

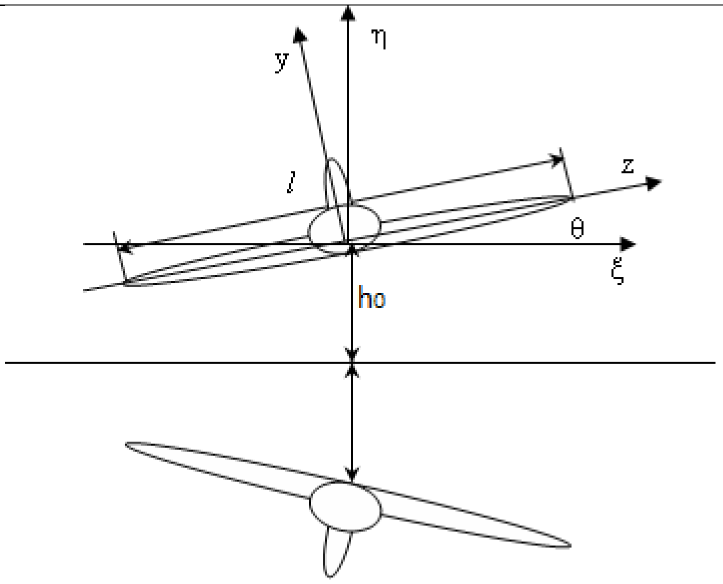
The distribution density of the circulation $\gamma_c(x(\mu), z)$ of the vortex rope formed by the vortex filaments escaping from the attached ones is equal to the total circulation of these filaments in the range $\left(\mu, \frac{2b}{L}\right)$ of parameter values μ for the right wing, and $\left(-\frac{2b}{L}, \mu\right)$ for the left.The same is true: $\gamma_c(x(\mu), z) = \gamma_c(x(\mu), -z)$.

$$\gamma_c^+(x(\mu), z) = \int_{\mu}^{2b/L} \gamma(x(\mu), z)\, \dot{x}_\mu d\mu,\ \ z \ge 0$$
$$\gamma_c^-(x(\mu), z) = \int_{-2b/L}^{\mu} \gamma(x(\mu), z)\, \dot{x}_\mu d\mu,\ \ z < 0 \qquad (7)$$

The distribution density of the circulation $\hat{\gamma}_c(x(\mu), z)$ of a free vortex rope descending from the trailing edge of the wing is determined as follows:

$$\hat{\gamma}_c^+(x(\mu), z) = \int_0^{2b/L} \gamma(x(\mu), z)\, \dot{x}_\mu d\mu,\ \ z \ge 0$$
$$\hat{\gamma}_c^-(x(\mu), z) = \int_{-2b/L}^{0} \gamma(x(\mu), z)\, \dot{x}_\mu d\mu,\ \ z < 0 \qquad (8)$$

Next, we will apply the results obtained to describe the motion of the wing near the boundary. To do this, consider the problem of the flow around wings that are mirrored relative to the boundary. By virtue of symmetry, the condition of the absence of normal flow rates will be satisfied at the boundary. To apply the method of superposition of potential flows, it is necessary to distribute the vortex potential on the symmetrical wings and vortex sheets behind them. Again, due to the symmetry of the vortex density at symmetrical points P and P' both the main and auxiliary wings must coincide.



Picture 5 – The ekranoplan on coordinates of the wing points in the system $Oxyz$

Then the flow velocities induced by the vortices distributed over the wings are expressed by the following relation (Bio-Savarra formula):

$$\begin{aligned}\bar{V}_\kappa(\bar{r}_p) = &\int_{-0.5l}^{0}\int_{-2b/L}^{0}\left(\gamma_n^-(x(\mu),z)\cdot(\mu\bar{i}+\bar{k})+\gamma_c^-(x(\mu),z)\cdot\bar{i}\right)\times\left(\frac{(\bar{r}_p-\bar{r})}{|\bar{r}_p-\bar{r}|^3}+\frac{(\bar{r}_p-\bar{r}')}{|\bar{r}_p-\bar{r}'|^3}\right) z\,d\mu\,dz + \\ &\int_{0}^{0.5l}\int_{0}^{2b/L}\left(\gamma_n^+(x(\mu),z)\cdot(-\mu\bar{i}+\bar{k})+\gamma_c^+(x(\mu),z)\cdot\bar{i}\right)\times\left(\frac{(\bar{r}_p-\bar{r})}{|\bar{r}_p-\bar{r}|^3}+\frac{(\bar{r}_p-\bar{r}')}{|\bar{r}_p-\bar{r}'|^3}\right) z\,d\mu\,dz + \\ &\int_{-\infty}^{-0.5b}\int_{-0.5l}^{0}\hat{\gamma}_c^-(z)\cdot\bar{i}\times\left(\frac{(\bar{r}_p-\bar{r})}{|\bar{r}_p-\bar{r}|^3}+\frac{(\bar{r}_p-\bar{r}')}{|\bar{r}_p-\bar{r}'|^3}\right)dzdx + \int_{-\infty}^{-0.5b}\int_{0}^{0.5l}\hat{\gamma}_c^+(z)\cdot\bar{i}\times\left(\frac{(\bar{r}_p-\bar{r})}{|\bar{r}_p-\bar{r}|^3}+\frac{(\bar{r}_p-\bar{r}')}{|\bar{r}_p-\bar{r}'|^3}\right)dz\,dx\end{aligned} \quad (9)$$

Here it is necessary to express the coordinates of the wing points in the system $Oxyz$, of the wing points in the system associated with the upper (investigated) wing. The system $Oxyz$ is obtained from a stationary system $O\xi\eta\varsigma$ by rotation, which is described by matrix (1). The system $O'x'y'z'$, associated with the lower wing is obtained from the system $O\xi\eta\varsigma$ by rotation described by the matrix $Oxyz$, associated with the upper (investigated) wing. The system $O'x'y'z'$, associated with the lower wing is obtained from the system by rotation described by the matrix:

$$A_{\xi}^{x'} = \begin{Vmatrix} 1 & -\varphi_z(t) & -\varphi_y(t) \\ \varphi_z(t) & 1 & -\varphi_x(t) \\ \varphi_y(t) & \varphi_x(t) & 1 \end{Vmatrix} \qquad (10)$$

Considerapoint K ntheplaneofthemainwingwithcoordinates $(x,0,z)$ inthesystem $Oxyz$ and a point K', that is mirror-symmetric to it on the auxiliary wing with coordinates (x', y', z') in system $O'x'y'z'$. Then the coordinates of a point K' in system $Oxyz$ are determined by the ratio $(x',y',z') = (x,y,z)\cdot\left(A_{\xi}^{x}\right)^{-1}\cdot A_{\xi}^{x'}$ and have the form:

$$\begin{cases} x' = x \\ y' = 2(z\varphi_x - x\varphi_z + h) \\ z' = z \end{cases} \qquad (11)$$

4. Linearization of the aerodynamic characteristics of the wing

In (11), only the factor $\overline{\rho} = \left(\frac{(\overline{r}_p - \overline{r})}{\left|\overline{r}_p - \overline{r}\right|^3} + \frac{(\overline{r}_p - \overline{r}')}{\left|\overline{r}_p - \overline{r}'\right|^3} \right)$ depends on the parameters $\varphi_x, \varphi_y, \varphi_z, \Delta\tilde{h}$ and, due to the smallness of their values, it can be represented in the form $\overline{\rho} = \overline{\rho}_0 + \varphi_x\overline{\rho}_{\varphi_x} + \varphi_z\overline{\rho}_{\varphi_z} + \Delta\tilde{h}\overline{\rho}_h$, where:

$$\overline{\rho}_0 = \left(\frac{1}{\left((x_p - x)^2 + (z_p - z)^2\right)^{3/2}} + \frac{1}{\left((x_p - x)^2 + (z_p - z)^2 + 4h_0^2\right)^{5/2}} \right)\left((x_p - x)\overline{\mathbf{i}} + (z_p - z)\overline{\mathbf{k}}\right) - \\ - \frac{2h_0}{\left((x_p - x)^2 + (z_p - z)^2 + 4h_0^2\right)^{3/2}}\overline{\mathbf{j}};$$

$$\overline{\rho}_{\varphi_x} = -\frac{12h_0\left((x_p - x)\overline{\mathbf{i}} + (z_p - z)\overline{\mathbf{k}}\right) + 2\left((x_p - x)^2 + (z_p - z)^2 - 8h_0^2\right)\overline{\mathbf{j}}}{\left((x_p - x)^2 + (z_p - z)^2 + 4h_0^2\right)^{5/2}}z;$$

$$\overline{\rho}_{\varphi_z} = \frac{12h_0\left((x_p - x)\overline{\mathbf{i}} + (z_p - z)\overline{\mathbf{k}}\right) + 2\left((x_p - x)^2 + (z_p - z)^2 - 8h_0^2\right)\overline{\mathbf{j}}}{\left((x_p - x)^2 + (z_p - z)^2 + 4h_0^2\right)^{5/2}}x; \qquad (12)$$

$$\rho_h = -\frac{12h_0\left((x_p - x)\overline{\mathbf{i}} + (z_p - z)\overline{\mathbf{k}}\right) + 2\left((x_p - x)^2 + (z_p - z)^2 - 8h_0^2\right)\overline{\mathbf{j}}}{\left((x_p - x)^2 + (z_p - z)^2 + 4h_0^2\right)^{5/2}}$$

For simplicity and better visibility of expression (11), we introduce the notation:

$$\left(\gamma_n^-\left(x(\mu),z\right)\cdot\left(\mu\bar{i}+\bar{k}\right)+\gamma_c^-\left(x(\mu),z\right)\cdot\bar{i}\right)\times\bar{\rho}_0=\bar{\gamma}_0^-\left(x(\mu),z\right);$$
$$\left(\gamma_n^+\left(x(\mu),z\right)\cdot\left(-\mu\bar{i}+\bar{k}\right)+\gamma_c^+\left(x(\mu),z\right)\cdot\bar{i}\right)\times\bar{\rho}_0=\bar{\gamma}_0^+\left(x(\mu),z\right);$$
$$\hat{\gamma}_c^-(z)\cdot\bar{i}\times\bar{\rho}_0=\hat{\gamma}_0^-(z);\ \hat{\gamma}_c^+(z)\cdot\bar{i}\times\bar{\rho}_0=\hat{\gamma}_0^+(z);$$
$$\left(\gamma_n^-\left(x(\mu),z\right)\cdot\left(\mu\bar{i}+\bar{k}\right)+\gamma_c^-\left(x(\mu),z\right)\cdot\bar{i}\right)\times\bar{\rho}_{\varphi_x}=\bar{\gamma}_{\varphi_x}^-\left(x(\mu),z\right);$$
$$\left(\gamma_n^+\left(x(\mu),z\right)\cdot\left(\mu\bar{i}+\bar{k}\right)+\gamma_c^+\left(x(\mu),z\right)\cdot\bar{i}\right)\times\bar{\rho}_{\varphi_x}=\bar{\gamma}_{\varphi_x}^+\left(x(\mu),z\right);$$
$$\hat{\gamma}_c^-(z)\cdot\bar{i}\times\bar{\rho}_{\varphi_x}=\hat{\gamma}_{\varphi_x}^-(z);\ \hat{\gamma}_c^+(z)\cdot\bar{i}\times\bar{\rho}_{\varphi_x}=\hat{\gamma}_{\varphi_x}^+(z);\tag{13}$$
$$\left(\gamma_n^-\left(x(\mu),z\right)\cdot\left(\mu\bar{i}+\bar{k}\right)+\gamma_c^-\left(x(\mu),z\right)\cdot\bar{i}\right)\times\bar{\rho}_{\varphi_z}=\bar{\gamma}_{\varphi_z}^-\left(x(\mu),z\right);$$
$$\left(\gamma_n^+\left(x(\mu),z\right)\cdot\left(\mu\bar{i}+\bar{k}\right)+\gamma_c^+\left(x(\mu),z\right)\cdot\bar{i}\right)\times\bar{\rho}_{\varphi_z}=\bar{\gamma}_{\varphi_z}^+\left(x(\mu),z\right);$$
$$\hat{\gamma}_c^-(z)\cdot\bar{i}\times\bar{\rho}_{\varphi_z}=\hat{\gamma}_{\varphi_z}^-(z);\ \hat{\gamma}_c^+(z)\cdot\bar{i}\times\bar{\rho}_{\varphi_z}=\hat{\gamma}_{\varphi_z}^+(z);$$
$$\left(\gamma_n^-\left(x(\mu),z\right)\cdot\left(\mu\bar{i}+\bar{k}\right)+\gamma_c^-\left(x(\mu),z\right)\cdot\bar{i}\right)\times\bar{\rho}_h=\bar{\gamma}_h^-\left(x(\mu),z\right);$$
$$\left(\gamma_n^+\left(x(\mu),z\right)\cdot\left(\mu\bar{i}+\bar{k}\right)+\gamma_c^+\left(x(\mu),z\right)\cdot\bar{i}\right)\times\bar{\rho}_h=\bar{\gamma}_h^+\left(x(\mu),z\right);$$
$$\hat{\gamma}_c^-(z)\cdot\bar{i}\times\bar{\rho}_h=\hat{\gamma}_h^-(z);\ \hat{\gamma}_c^+(z)\cdot\bar{i}\times\bar{\rho}_h=\hat{\gamma}_h^+(z).$$

Then, taking into account (14), the induced velocity (11) will be linearly dependent on the parameters $\varphi_x, \varphi_y, \varphi_z, \Delta h$

$$\bar{V}_\kappa\left(\bar{r}_p\right)=\bar{V}_\kappa^0\left(\bar{r}_p\right)+\varphi_x\bar{V}_\kappa^{\varphi_x}\left(\bar{r}_p\right)+\varphi_z\bar{V}_\kappa^{\varphi_z}\left(\bar{r}_p\right)+\Delta\tilde{h}\bar{V}_\kappa^h\left(\bar{r}_p\right);$$
$$\bar{V}_\kappa^0\left(\bar{r}_p\right)=\int\limits_{-0.5l}^{0}\int\limits_{-2b/L}^{0}\bar{\gamma}_0^-\left(x(\mu),z\right)z\,d\mu dz+\int\limits_{0}^{0.5l}\int\limits_{0}^{2b/L}\bar{\gamma}_0^+\left(x(\mu),z\right)z\,d\mu\,dz+$$
$$+\int\limits_{-\infty}^{-0.5b}\int\limits_{-0.5l}^{0}\hat{\gamma}_0^-(z)\,dzdx+\int\limits_{-\infty}^{-0.5b}\int\limits_{0}^{0.5l}\hat{\gamma}_0^+(z)\,dz\,dx;$$
$$\bar{V}_\kappa^{\varphi_x}\left(\bar{r}_p\right)=\int\limits_{-0.5l}^{0}\int\limits_{-2b/L}^{0}\bar{\gamma}_{\varphi_x}^-\left(x(\mu),z\right)z\,d\mu dz+\int\limits_{0}^{0.5l}\int\limits_{0}^{2b/L}\bar{\gamma}_{\varphi_x}^+\left(x(\mu),z\right)z\,d\mu\,dz+$$
$$+\int\limits_{-\infty}^{-0.5b}\int\limits_{-0.5l}^{0}\hat{\gamma}_{\varphi_x}^-(z)\,dzdx+\int\limits_{-\infty}^{-0.5b}\int\limits_{0}^{0.5l}\hat{\gamma}_{\varphi_x}^+(z)\,dz\,dx;\tag{14}$$
$$\bar{V}_\kappa^{\varphi_z}\left(\bar{r}_p\right)=\int\limits_{-0.5l}^{0}\int\limits_{-2b/L}^{0}\bar{\gamma}_{\varphi_z}^-\left(x(\mu),z\right)z\,d\mu dz+\int\limits_{0}^{0.5l}\int\limits_{0}^{2b/L}\bar{\gamma}_{\varphi_z}^+\left(x(\mu),z\right)z\,d\mu\,dz+$$
$$+\int\limits_{-\infty}^{-0.5b}\int\limits_{-0.5l}^{0}\hat{\gamma}_{\varphi_z}^-(z)\,dzdx+\int\limits_{-\infty}^{-0.5b}\int\limits_{0}^{0.5l}\hat{\gamma}_{\varphi_z}^+(z)\,dzdx;$$
$$\bar{V}_\kappa^h\left(\bar{r}_p\right)=\int\limits_{-0.5l}^{0}\int\limits_{-2b/L}^{0}\bar{\gamma}_h^-\left(x(\mu),z\right)z\,d\mu dz+\int\limits_{0}^{0.5l}\int\limits_{0}^{2b/L}\bar{\gamma}_h^+\left(x(\mu),z\right)z\,d\mu\,dz+$$
$$+\int\limits_{-\infty}^{-0.5b}\int\limits_{-0.5l}^{0}\hat{\gamma}_h^-(z)\,dzdx+\int\limits_{-\infty}^{-0.5b}\int\limits_{0}^{0.5l}\hat{\gamma}_h^+(z)\,dz\,dx.$$

That is, the sought density of the vortex potential can be represented by a linear function of the parameters $\varphi_x, \varphi_y, \varphi_z, \Delta\tilde{h}$. This means that the potential $\Phi_\kappa(\bar{r})$ itself will be linear . Hence follows the linear dependence of the induced velocity $\bar{V}_\kappa$, and taking into account the smallness of these values, and the linearity $(\bar{V}_\kappa)^2$ and linearity of the pressure $p = p_0 + \varphi_x p_{\varphi_x} + \varphi_z p_{\varphi_z} + \Delta h p_h$, distributed over the wing surface. From the expression for the main force and moment, their linearity follows from $\varphi_x, \varphi_y, \varphi_z, \Delta\tilde{h}$:

$$\begin{aligned}
&\bar{R} = \bar{R}^0 + \varphi_x \bar{R}^{\varphi_x} + \varphi_z \bar{R}^{\varphi_z} + \Delta\tilde{h}\bar{R}^h;\\
&\bar{R}^0 = \iint_{S_\kappa} p_0 \bar{n} ds;\ \bar{R}^{\varphi_x} = \iint_{S_\kappa} p_{\varphi_x} \bar{n} ds;\ \bar{R}^{\varphi_z} = \iint_{S_\kappa} p_{\varphi_z} \bar{n} ds;\ \bar{R}^h = \iint_{S_\kappa} p_h \bar{n} ds;\\
&\bar{L} = \bar{L}^0 + \varphi_x \bar{L}^{\varphi_x} + \varphi_z \bar{L}^{\varphi_z} + \Delta\tilde{h}\bar{L}^h;\\
&\bar{L}^0 = \iint_{S_\kappa} r \times p_0 \bar{n} ds;\ \bar{L}^{\varphi_x} = \iint_{S_\kappa} r \times p_{\varphi_x} \bar{n} ds;\ \bar{L}^{\varphi_z} = \iint_{S_\kappa} r \times p_{\varphi_z} \bar{n} ds;\ \bar{L}^h = \iint_{S_\kappa} r \times p_h \bar{n} ds,
\end{aligned} \tag{15}$$

Consequently, the aerodynamic characteristics of the wing will have an idea:

$$\begin{aligned}
C_y &= C_y^0 + \varphi_x C_y^{\varphi_x} + \varphi_z C_y^{\varphi_z} + \Delta\tilde{h} C_y^h;\\
C_x &= C_x^0 + \varphi_x C_x^{\varphi_x} + \varphi_z C_x^{\varphi_z} + \Delta\tilde{h} C_x^h;\\
M_x &= M_x^0 + \varphi_x M_x^{\varphi_x} + \varphi_z M_x^{\varphi_z} + \Delta\tilde{h} M_x^h;\\
M_z &= M_z^0 + \varphi_x M_z^{\varphi_x} + \varphi_z M_z^{\varphi_z} + \Delta\tilde{h} M_z^h.
\end{aligned} \tag{16}$$

The coordinates of the point of application of the force $\bar{R}$ under the assumption that the change in its modulus is an order of magnitude lower than the change in the parameters $\varphi_x, \varphi_y, \varphi_z, \Delta\tilde{h}$, according to the formulas $x_c = \frac{L_x}{|\bar{R}|}; y_c = \frac{L_y}{|\bar{R}|}; z_c = \frac{L_z}{|\bar{R}|}$, whence from (16) follows the linear dependence of the coordinates of the center on these parameters.

4.1 Transfer functions of the system of equations of motion of the ekranoplan

Similarly to the one described in the previous section, it is possible to obtain a linear representation of the forces $\varphi_x, \varphi_y, \varphi_z, \Delta\tilde{h}$ on the horizontal and vertical rudders of the ekranoplan and the coordinates of the points of application of these forces:

$$
\begin{aligned}
&\bar{R}_1 = \bar{R}_1^0 + \varphi_x \bar{R}_1^{\varphi_x} + \varphi_z \bar{R}_1^{\varphi_z} + \Delta\tilde{h}\bar{R}_1^h;\\
&\bar{R}_1^0 = \iint\limits_{S_{гр}} p_0 \bar{n} ds;\ \bar{R}_1^{\varphi_x} = \iint\limits_{S_{гр}} p_{\varphi_x} \bar{n} ds;\ \bar{R}_1^{\varphi_z} = \iint\limits_{S_{гр}} p_{\varphi_z} \bar{n} ds;\ \bar{R}_1^h = \iint\limits_{S_{гр}} p_h \bar{n} ds;\\
&\bar{L}_1 = \bar{L}_1^0 + \varphi_x \bar{L}_1^{\varphi_x} + \varphi_z \bar{L}_1^{\varphi_z} + \Delta\tilde{h}\bar{L}_1^h;\\
&\bar{L}_1^0 = \iint\limits_{S_{гр}} r \times p_0 \bar{n} ds;\ \bar{L}_1^{\varphi_x} = \iint\limits_{S_{гр}} r \times p_{\varphi_x} \bar{n} ds;\ \bar{L}_1^{\varphi_z} = \iint\limits_{S_{гр}} r \times p_{\varphi_z} \bar{n} ds;\ \bar{L}_1^h = \iint\limits_{S_{гр}} r \times p_h \bar{n} ds;\\
&x_c^1 = \frac{L_x}{|\bar{R}|};\ y_c^1 = \frac{L_y}{|\bar{R}|};\ z_c^1 = \frac{L_z}{|\bar{R}|};
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
&\bar{R}_2 = \bar{R}_2^0 + \varphi_x \bar{R}_2^{\varphi_x} + \varphi_z \bar{R}_2^{\varphi_z} + \Delta\tilde{h}\bar{R}_2^h;\\
&\bar{R}_2^0 = \iint\limits_{S_{вр}} p_0 \bar{n} ds;\ \bar{R}_2^{\varphi_x} = \iint\limits_{S_{вр}} p_{\varphi_x} \bar{n} ds;\ \bar{R}_2^{\varphi_z} = \iint\limits_{S_{вр}} p_{\varphi_z} \bar{n} ds;\ \bar{R}_2^h = \iint\limits_{S_{вр}} p_h \bar{n} ds;\\
&\bar{L}_2 = \bar{L}_2^0 + \varphi_x \bar{L}_2^{\varphi_x} + \varphi_z \bar{L}_2^{\varphi_z} + \Delta\tilde{h}\bar{L}_2^h;\\
&\bar{L}_2^0 = \iint\limits_{S_{вр}} r \times p_0 \bar{n} ds;\ \bar{L}_2^{\varphi_x} = \iint\limits_{S_{вр}} r \times p_{\varphi_x} \bar{n} ds;\ \bar{L}_2^{\varphi_z} = \iint\limits_{S_{вр}} r \times p_{\varphi_z} \bar{n} ds;\ \bar{L}_2^h = \iint\limits_{S_{вр}} r \times p_h \bar{n} ds;\\
&x_c^2 = \frac{L_x}{|\bar{R}|};\ y_c^2 = \frac{L_y}{|\bar{R}|};\ z_c^2 = \frac{L_z}{|\bar{R}|}.
\end{aligned} \tag{18}
$$

After concretizing the forces and moments acting on the ekranoplan, it is possible to concretize the condition of static stability (4) and the system of equations of motion (3).

Let us write down condition (4) ($\frac{M_z}{\varphi_z} < 0$)in an explicit form, taking into account that $\varphi_x, \varphi_y, \varphi_z, \Delta\tilde{h}$ the quantities are small:

$$
\begin{aligned}
\frac{M_z}{\varphi_z} = \frac{1}{\varphi_z}&\left\{\left(\bar{r}_c^0 \times \bar{R}^0\right) + \left(\bar{r}_{1c}^0 \times \bar{R}_1^0\right) + \left(\bar{r}_{2c}^0 \times \bar{R}_2^0\right)\right\}\bar{k} +\\
&+ \frac{\varphi_x}{\varphi_z}\left\{\left(\bar{r}_c^{\varphi_x} \times \bar{R}^{\varphi_x}\right) + \left(\bar{r}_{1c}^{\varphi_x} \times \bar{R}_1^{\varphi_x}\right) + \left(\bar{r}_{2c}^{\varphi_x} \times \bar{R}_2^{\varphi_x}\right)\right\}\bar{k} +\\
&+ \left\{\left(\bar{r}_c^{\varphi_z} \times \bar{R}^{\varphi_z}\right) + \left(\bar{r}_{1c}^{\varphi_z} \times \bar{R}_1^{\varphi_z}\right) + \left(\bar{r}_{2c}^{\varphi_z} \times \bar{R}_2^{\varphi_z}\right)\right\}\bar{k} +\\
&+ \frac{\Delta\tilde{h}}{\varphi_z}\left\{\left(\bar{r}_c^h \times \bar{R}^h\right) + \left(\bar{r}_{1c}^h \times \bar{R}_1^h\right) + \left(\bar{r}_{2c}^h \times \bar{R}_2^h\right)\right\}\bar{\mathrm{k}} < 0.
\end{aligned} \tag{19}
$$

This check can be performed as a verification calculation of the statistical stability of the designed body of an ekranoplan with known aerodynamic characteristics.

Let us write explicitly the system (3). From the above, it follows that aerodynamic forces and moments are represented by a linear combination of parameters $\varphi_x, \varphi_y, \varphi_z, \Delta\tilde{h}$:

$$
\begin{aligned}
&\bar{F}_a = \bar{F}_a^0 + \varphi_x \bar{F}_a^{\varphi_x} + \varphi_z \bar{F}_a^{\varphi_z} + \Delta\tilde{h}\bar{F}_a^{\ h};\\
&\bar{F}_a^{\ 0} = \bar{R}^0 + \bar{R}_1^0 + \bar{R}_2^0;\ \bar{F}_a^{\ \varphi_x} = \bar{R}^{\varphi_x} + \bar{R}_1^{\varphi_x} + \bar{R}_2^{\varphi_x};\\
&\bar{F}_a^{\ \varphi_z} = \bar{R}^{\varphi_z} + \bar{R}_1^{\varphi_z} + \bar{R}_2^{\varphi_z};\ \bar{F}_a^{\ h} = \bar{R}^h + \bar{R}_1^h + \bar{R}_2^h;\\
&\bar{M}_a = \bar{M}_a^{\ 0} + \varphi_x \bar{M}_a^{\ \varphi_x} + \varphi_z \bar{M}_a^{\ \varphi_z} + \Delta\tilde{h}\bar{M}_a^{\ h};\\
&\bar{M}_a^{\ 0} = \left(\bar{r}_c^{\,0} \times \bar{R}^0\right) + \left(\bar{r}_{1c}^{\,0} \times \bar{R}_1^0\right) + \left(\bar{r}_{2c}^{\,0} \times \bar{R}_2^0\right);\\
&\bar{M}_a^{\ \varphi_x} = \left(\bar{r}_c^{\,\varphi_x} \times \bar{R}^{\varphi_x}\right) + \left(\bar{r}_{1c}^{\,\varphi_x} \times \bar{R}_1^{\varphi_x}\right) + \left(\bar{r}_{2c}^{\,\varphi_x} \times \bar{R}_2^{\varphi_x}\right);\\
&\bar{M}_a^{\ \varphi_z} = \left(\bar{r}_c^{\,\varphi_z} \times \bar{R}^{\varphi_z}\right) + \left(\bar{r}_{1c}^{\,\varphi_z} \times \bar{R}_1^{\varphi_z}\right) + \left(\bar{r}_{2c}^{\,\varphi_z} \times \bar{R}_2^{\varphi_z}\right);\\
&\bar{M}_a^{\ h} = \left(\bar{r}_c^{\,h} \times \bar{R}^h\right) + \left(\bar{r}_{1c}^{\,h} \times \bar{R}_1^h\right) + \left(\bar{r}_{2c}^{\,h} \times \bar{R}_2^h\right).
\end{aligned}
\tag{20}
$$

The coordinates of the center C of the application of the resulting force are determined by the ratio:

$$
\begin{aligned}
x_C &= \frac{M_{ax}}{\left|\bar{F}_a\right|} = x_C^0 + \varphi_x x_C^{\varphi_x} + \varphi_z x_C^{\varphi_x} + \Delta\tilde{h} x_C^h;\\
y_C &= \frac{M_{ay}}{\left|\bar{F}_a\right|} = y_C^0 + \varphi_x y_C^{\varphi_x} + \varphi_z y_C^{\varphi_x} + \Delta\tilde{h} y_C^h;\\
z_C &= \frac{M_{az}}{\left|\bar{F}_a\right|} = z_C^0 + \varphi_x z_C^{\varphi_x} + \varphi_z z_C^{\varphi_x} + \Delta\tilde{h} z_C^h.
\end{aligned}
\tag{21}
$$

Then the equations of motion (3) have the form:

$$
\begin{cases}
(m+\lambda_{11})\dot{v}_x + \lambda_{12}\dot{v}_y + \lambda_{13}\dot{v}_z + \lambda_{14}\ddot{\varphi}_x + \lambda_{15}\ddot{\varphi}_y + \lambda_{16}\ddot{\varphi}_z = F_{ax}^0 + \varphi_x\left(F_{ax}^{\varphi_x} - mg\right) + \varphi_z F_{ax}^{\varphi_z} + \Delta\tilde{h}F_{ax}^h + P\\
\lambda_{21}\dot{v}_x + (m+\lambda_{22})\dot{v}_y + \lambda_{23}\dot{v}_z + \lambda_{24}\ddot{\varphi}_x + \lambda_{25}\ddot{\varphi}_y + \lambda_{26}\ddot{\varphi}_z = F_{ay}^0 + \varphi_x F_{ay}^{\varphi_x} + \varphi_z F_{ay}^{\varphi_z} + \Delta\tilde{h}F_{ay}^h - mg\\
\lambda_{31}\dot{v}_x + \lambda_{32}\dot{v}_y + (m+\lambda_{33})\dot{v}_z + \lambda_{34}\ddot{\varphi}_x + \lambda_{35}\ddot{\varphi}_y + \lambda_{36}\ddot{\varphi}_z = F_{az}^0 + \varphi_x F_{az}^{\varphi_x} + \varphi_z\left(F_{az}^{\varphi_z} - mg\right) + \Delta\tilde{h}F_{az}^h\\
\lambda_{41}\dot{v}_x + \lambda_{42}\dot{v}_y + \lambda_{43}\dot{v}_z + (J_{xx}+\lambda_{44})\ddot{\varphi}_x + (J_{xy}+\lambda_{45})\ddot{\varphi}_y + (J_{xz}+\lambda_{46})\ddot{\varphi}_z = M_{ax}\\
\lambda_{51}\dot{v}_x + \lambda_{52}\dot{v}_y + \lambda_{53}\dot{v}_z + (J_{yx}+\lambda_{54})\ddot{\varphi}_x + (J_{yy}+\lambda_{55})\ddot{\varphi}_y + (J_{yz}+\lambda_{56})\ddot{\varphi}_z = M_{ay};\\
\lambda_{61}\dot{v}_x + \lambda_{62}\dot{v}_y + \lambda_{63}\dot{v}_z + (J_{zx}+\lambda_{64})\ddot{\varphi}_x + (J_{zy}+\lambda_{65})\ddot{\varphi}_y + (J_{zz}+\lambda_{66})\ddot{\varphi}_z = M_{az},
\end{cases}
\tag{22}
$$

$$
\begin{aligned}
&M_{ax}=\left\{\left(y_C^0-y_G\right)F_{az}^0-\left(z_C^0-z_G\right)F_{ay}^0\right\}+\varphi_x\left\{\left(\left(y_C^0-y_G\right)F_{az}^{\varphi_x}+y_C^{\varphi_x}F_{az}^0\right)-\left(\left(z_C^0-z_G\right)F_{ay}^{\varphi_x}+z_C^{\varphi_x}F_{ay}^0\right)\right\}+\\
&+\varphi_z\left\{\left(\left(y_C^0-y_G\right)F_{az}^{\varphi_z}+y_C^{\varphi_z}F_{az}^0\right)-\left(\left(z_C^0-z_G\right)F_{ay}^{\varphi_z}+z_C^{\varphi_z}F_{ay}^0\right)\right\}+\Delta\tilde{h}\left\{\left(\left(y_C^0-y_G\right)F_{az}^h+y_C^hF_{az}^0\right)-\left(\left(z_C^0-z_G\right)F_{ay}^h+z_C^hF_{ay}^0\right)\right\}\\
&M_{ay}=\left\{\left(z_C^0-z_G\right)F_{ax}^0-\left(x_C^0-x_G\right)F_{az}^0\right\}+\varphi_x\left\{\left(\left(z_C^0-z_G\right)F_{ax}^{\varphi_x}+z_C^{\varphi_x}F_{ax}^0\right)-\left(\left(x_C^0-x_G\right)F_{az}^{\varphi_x}+x_C^{\varphi_x}F_{az}^0\right)\right\}+\\
&+\varphi_z\left\{\left(\left(z_C^0-z_G\right)F_{ax}^{\varphi_z}+z_C^{\varphi_z}F_{ax}^0\right)-\left(\left(x_C^0-x_G\right)F_{az}^{\varphi_z}+x_C^{\varphi_z}F_{az}^0\right)\right\}+\Delta\tilde{h}\left\{\left(\left(z_C^0-z_G\right)F_{ax}^h+z_C^hF_{ax}^0\right)-\left(\left(x_C^0-x_G\right)F_{az}^h+x_C^hF_{az}^0\right)\right\}\\
&M_{az}=\left\{\left(x_C^0-x_G\right)F_{ay}^0-\left(y_C^0-y_G\right)F_{ax}^0\right\}+\varphi_x\left\{\left(\left(x_C^0-x_G\right)F_{ay}^{\varphi_x}+x_C^{\varphi_x}F_{ay}^0\right)-\left(\left(y_C^0-y_G\right)F_{ax}^{\varphi_x}+y_C^{\varphi_x}F_{ax}^0\right)\right\}+\\
&+\varphi_z\left\{\left(\left(x_C^0-x_G\right)F_{ay}^{\varphi_z}+x_C^{\varphi_z}F_{ay}^0\right)-\left(\left(y_C^0-y_G\right)F_{ax}^{\varphi_z}+y_C^{\varphi_z}F_{ax}^0\right)\right\}+\Delta\tilde{h}\left\{\left(\left(x_C^0-x_G\right)F_{ay}^h+x_C^hF_{ay}^0\right)-\left(\left(y_C^0-y_G\right)F_{ax}^h+y_C^hF_{ax}^0\right)\right\}
\end{aligned}
\qquad (23)
$$

To analyze the dynamic stability of the ground effect vehicle near the horizontal reference motion from system (23), one can obtain the transfer functions of the system for each of the quantities $\varphi_x, \varphi_y, \varphi_z, \Delta\tilde{h}$ and determine their roots

In the operator form, the system is:

$$
\begin{cases}
p\left(m+\lambda_{11}\right)V_x+\left(p\lambda_{12}-\dfrac{F_{ax}^h}{p}\right)V_y+p\lambda_{13}V_z+\left(p^2\lambda_{14}+mg-F_{ax}^{\varphi_x}\right)\Phi_x+p^2\lambda_{15}\Phi_y+\left(p^2\lambda_{16}-F_{ax}^{\varphi_z}\right)\Phi_z=F_{ax}^0+P\\
p\lambda_{21}V_x+\left(p\left(m+\lambda_{22}\right)-\dfrac{F_{ay}^h}{p}\right)V_y+p\lambda_{23}V_z+\left(p^2\lambda_{24}-F_{ax}^{\varphi_x}\right)\Phi_x+p^2\lambda_{25}\Phi_y+\left(p^2\lambda_{26}-F_{ay}^{\varphi_z}\right)\Phi_z=F_{ay}^0-mg\\
p\lambda_{31}V_x+\left(p\lambda_{32}-\dfrac{F_{az}^h}{p}\right)V_y+p\left(m+\lambda_{33}\right)V_z+\left(p^2\lambda_{34}-F_{az}^{\varphi_x}\right)\Phi_x+p^2\lambda_{35}\Phi_y+\left(m+p^2\lambda_{36}-F_{az}^{\varphi_z}\right)\Phi_z=F_{az}^0\\
p\lambda_{41}V_x+\left(p\lambda_{32}-\dfrac{\left(\left(y_C^0-y_G\right)F_{az}^h+y_C^hF_{az}^0\right)-\left(\left(z_C^0-z_G\right)F_{ay}^h+z_C^hF_{ay}^0\right)}{p}\right)V_y+p\lambda_{43}V_z+\\
\quad+\left(p^2\left(J_{xx}+\lambda_{44}\right)-\left(\left(\left(y_C^0-y_G\right)F_{az}^{\varphi_x}+y_C^{\varphi_x}F_{az}^0\right)-\left(\left(z_C^0-z_G\right)F_{ay}^{\varphi_x}+z_C^{\varphi_x}F_{ay}^0\right)\right)\right)\Phi_x+p^2\left(J_{xy}+\lambda_{45}\right)\Phi_y+\\
+\left(p^2\left(J_{xz}+\lambda_{46}\right)-\left(\left(\left(y_C^0-y_G\right)F_{az}^{\varphi_z}+y_C^{\varphi_z}F_{az}^0\right)-\left(\left(z_C^0-z_G\right)F_{ay}^{\varphi_z}+z_C^{\varphi_z}F_{ay}^0\right)\right)\right)\Phi_z=\left(z_C^0-z_G\right)F_{ax}^0-\left(x_C^0-x_G\right)F_{az}^0\\
p\lambda_{51}V_x+\left(p\lambda_{52}-\dfrac{\left(\left(z_C^0-z_G\right)F_{ax}^h+z_C^hF_{ax}^0\right)-\left(\left(x_C^0-x_G\right)F_{az}^h+x_C^hF_{az}^0\right)}{p}\right)V_y+p\lambda_{53}V_z+\\
\quad+\left(p^2\left(J_{yx}+\lambda_{54}\right)-\left(\left(\left(z_C^0-z_G\right)F_{ax}^{\varphi_x}+z_C^{\varphi_x}F_{ax}^0\right)-\left(\left(x_C^0-x_G\right)F_{az}^{\varphi_x}+x_C^{\varphi_x}F_{az}^0\right)\right)\right)\Phi_x+p^2\left(J_{yy}+\lambda_{55}\right)\Phi_y+\\
+\left(p^2\left(J_{yz}+\lambda_{56}\right)-\left(\left(\left(z_C^0-z_G\right)F_{ax}^{\varphi_z}+z_C^{\varphi_z}F_{ax}^0\right)-\left(\left(x_C^0-x_G\right)F_{az}^{\varphi_z}+x_C^{\varphi_z}F_{az}^0\right)\right)\right)\Phi_z=\left(z_C^0-z_G\right)F_{ax}^0-\left(x_C^0-x_G\right)F_{az}^0\\
p\lambda_{61}V_x+\left(p\lambda_{62}-\dfrac{\left(\left(x_C^0-x_G\right)F_{ay}^h+x_C^hF_{ay}^0\right)-\left(\left(y_C^0-y_G\right)F_{ax}^h+y_C^hF_{ax}^0\right)}{p}\right)V_y+p\lambda_{63}V_z+\\
\quad+\left(p^2\left(J_{zx}+\lambda_{54}\right)-\left(\left(\left(x_C^0-x_G\right)F_{ay}^{\varphi_x}+x_C^{\varphi_x}F_{ay}^0\right)-\left(\left(y_C^0-y_G\right)F_{ax}^{\varphi_x}+y_C^{\varphi_x}F_{ax}^0\right)\right)\right)\Phi_x+p^2\left(J_{zy}+\lambda_{65}\right)\Phi_y+\\
+\left(p^2\left(J_{zz}+\lambda_{66}\right)-\left(\left(\left(x_C^0-x_G\right)F_{ay}^{\varphi_z}+x_C^{\varphi_z}F_{ay}^0\right)-\left(\left(y_C^0-y_G\right)F_{ax}^{\varphi_z}+y_C^{\varphi_z}F_{ax}^0\right)\right)\right)\Phi_z=\left(x_C^0-x_G\right)F_{ay}^0-\left(y_C^0-y_G\right)F_{ax}^0
\end{cases}
\qquad (24)
$$

Conclusion

Ekranoplans can be viewed as such an unconventional type of superfast water transport, utilizing the

influence of the underlying surface (ground) upon its motion stability, lift-to-drag ratio and, consequently, on its economic efficiency, expressed in terms of fuel consumption and direct operating costs.
To reach one of the most important technical problems proposed mathematical model of WIG vehicle longitudinal motion was proposed considering both hydrodynamics and aerodynamics.
For solving problem of steering of the ekranoplan on the horizontal movement under small perturbations from the side of the water surface was considered to within the framework of the linear theory of the vehicle's motion and the linear theory of medium perturbations, what in the general case, these are two conjugate problems with mutually influencing parameters and the most complete solution can be given only as a result of imitation modeling or a numerical experiment of the dynamic system "Ekranoplan – air - moving boundary".
As result of research of such modeling will allow for each specific WIG to evaluate its behavior on time intervals of calculation and to obtain a qualitative analysis of the operation of the system with a given geometry and the nature of perturbations to assess the trends in the behavior of the ekranoplan.

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