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A Brief Review on Detection of Cracks in Rotating Shafts

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Abstract. The occurrence and development of cracks in rotating shafts have effects on the reliability of machinery.

Using vibration monitoring, early cracks are detected and using multiple methods of analysis of recorded signals, these cracks can be evaluated and thus, it will extend the durability and reliability of these very expensive machines. Rotating shafts vibration monitoring is a very modern tool for determining crack initiation, because they change the dynamic rigidity of the shafts and therefore the dynamic behaviour, by changing the frequency spectrum and modal shapes.

The analysis of recorded vibrations has been used in the diagnosis of cracked rotating shafts since the 1980s.

This paper aims to briefly review the most significant research, showing the major progress in this field between 2000-2010, taking into account various key analysis tools, selecting parameters, intelligent methods for detecting the position and depth of their cracks and so on.

1 Introduction

Shafts are the main components of rotating machines which are used in many fields: food, energy, automotive, aerospace, naval, etc.

Although they are the most robust components of the rotating machines, while operate at different regimes and large number of cycles, rotation shafts undergo a process of fatigue and thus the possibility of cracking appears. Cracks appear after a certain number of operating cycles in areas of the shafts with material imperfections or areas with stress concentration, determined by the construction and the technological processes used. Cracks are classified according to their geometry: longitudinal crack, slant crack, transverse crack and breathing crack.

Vibration analysis is used in the process of monitoring and diagnosing cracks in the rotation shaft. This method is the most often used because it is made with easy-to-operate tools and vibrations accurately describe the state of the rotating shaft. Vibration analysis can be performed without knowing the constructive nature and operating regime of shaft. It determines a certain change in the operating mode, the appearance of a crack or other fault but without knowing the type of defect, its location or severity. From the beginning of signal recording different processing and analysis methods are used so that the data obtained can be used to make decisions in order to remove the installation from service. To obtain this information it is necessary to know the mechanical and operating characteristics of these shafts. This information is obtained (the structural properties of the shaft, the nature of the excitation forces) by processing the unregistered signals but they depend on the rotation regime. Recorded signal processing techniques are known such as: Fast Fourier Transform (FFT), Short Time Fourier Transform (STFT), Wigner-Ville Distribution (WVD), Wavelet Transform (WT), Hilbert-Haung Transform (HHT) and Artificial Intelligence (AI). These methods must be complemented by statistical processing techniques to validate the existence of cracks but they can also highlight an event or a specific area examined. To detect the different types of faults that can occur in the rotation shaft additional analyzes are performed such as nonlinear and transient analyzes.

This article presents the important scientific achievements in this field, from the period 2000-2010, without claiming that all the works are evaluated. The novelties regarding the model-based methods with analytical / numerical modeling of cracked shaft, the non-model methods in time or frequency domains and vibration-based signal processing techniques will be analyzed.

2. Literature Survey

Sekhar (2000) proposes a method based on the determination of Q factors for detecting the depth of cracked shafts in transient rotation mode. Sekhar suggested that the Q factor measured for component 2X is very sensitive to crack depth detection and the Q factor for component 1X can be used for crack monitoring. The author's conclusion is that a good indicator for crack detection are the sudden changes in Q factors during the coast-down [1].

Bachschmid et al. (2000a, 2000b), propose in mentioned papers a model-based method and a least-squares method for determining the position of the crack along the length of shaft. They used finite element analysis (FEA) to model the shaft in order to determine the effect of its static and dynamic behavior under the presence of cracks. The determination of crack depth is performed by comparing the equivalent periodic bending moment applied to the equivalent cracked shaft to the bending moment due by the weight of shaft. These equivalent moments are applied to each 1x, 2x and 3x subharmonic component corresponding to the vibrations of cracked shaft. The method was validated by experiment [2], [3].

He et al. (2001) have developed a model that formulates the detection of cracks in a rotor-bearing system as a finite element optimization problem and uses genetic algorithms to find the solution. The use of genetic algorithms solves a wide range of invers identification problems in a systematic and robust way. As the authors state, genetic algorithms avoid some of the weaknesses of traditional gradient based analytical search methods including the difficulty in constructing well-defined mathematical models directly from practical inverse problems [4].

Yang et al. (2001) contribute to the detection of shaft’s crack by analyzing the characteristics of transient vibrations, covering the subcritical, trans-critical and supercritical regions. The holo-spectral method used is a very sensitive technique and can be used successfully for diagnosis of shafts with incipient cracks and at high rotating speeds. The method proved to be efficient and was validated by experimental tests [5].

Varè and Andrieux (2001) develop a procedure based on the modeling and simulation of a cracked shaft, taking into account several factors: multicrack effect, local flexibilty of shaft in cracked area, the "switching" effect that introduces the breathing characteristic, temperature. They consider that three-dimensional finite element modeling (FEM) is the most accurate and they will implement it in the ASTER code developed by Electricité de France (EDF) [6].

Prabhakar et al. (2001) analyzed the position and depth of transverse cracks in the rotor-bearing system, using mechanical impedance, which was calculated for both open cracks and breathing cracks. For the modeling of the two types of cracks, the flexibility matrices of cracked sections were used for lateral vibrations in finite elements analysis. The impulse forces applied in different positions of the shaft and induced speeds were measured, through which the impedance was calculated for natural and running frequencies of shaft. It has been observed that impedance decreases as the depth of crack increases in the rotor-bearing system and this tendency is more pronounced in breathing cracks than in the open ones [7].

Zakhezin and Malysheva (2001) have calculated eigenvalues and eigenvectors for a single span shaft up to a frequency of 1100 Hz with and without transverse cracks at different locations and depths. The shafts, in the presented situations, were numerically modeled with finite elements taking into account the system damping. The method was validated by the results obtained in the experimental tests [8].

Maynard et al. (2001) demonstrated the feasibility of using the methodology for detecting changes in natural torsional frequencies of cracked shafts in two electric power generation plants in case of a cracked shaft of a turbine and another of an induced draft fan. Non-intrusive sensors were used and by excitation of natural torsional frequencies with random forces, changes of the first natural torsional frequencies were observed. The results were confirmed by finite element simulations for the two cracked shafts [9].

Kim, J. et al (2002) developed the Directional Harmonic Wavelet Transform method for more efficient monitoring of the cracked rotor, applied to the measured transient vibration response.

Directional Harmonic Wavelet Transform is an extension of Harmonic Wavelet Transform used to determine more accurately the local characteristics of non-stationary signals from rotating structures by using further-decomposed wavelets or directional-wavelets (FW Wavelet & BW Wavelet) [10].

Saavedra and Cuitino (2002) apply the finite element method to model a cracked cylindrical shaft from rotor-bearing system. The breathing crack is modeled by a local flexibility estimated by fracture mechanics linear theory. For the nonlinear rotor-bearing system the equations of stationary vibrations are integrated by the Hilbert, Hughes and Taylor method using the MATLAB program. It is concluded that the 2x harmonic component of the rotor-bearing system and typical orbit can be indicators for diagnosing cracked shafts. The method was experimentally validated [11].

Mohiuddin and Khulief (2002) develop a computational scheme for diagnosis of cracked rotating multi-stepped shaft using a simplified finite element modeling. The crack in the shaft causes additional flexibility in the finite element and is modeled using the crack flexibility influence coefficient. The order of the equations of motion is reduced by complex modal transformations. A computer program is developed to integrate the full-order as well as the reduced order equations of motion. For different types of excitation in different positions of the shaft, the amplitudes of response are determined and compared. The obtained results validate the use of the reduced model for diagnosis of cracks in rotor-bearing systems [12].

Subbiah et al. (2002) analyzed surface cracks and subsurface cracks determining the effect of bending and torsional vibrations on them. A x-harmonic with eight-node elements was used for shaft modeling and for transverse cracks modeling was used the finite element analysis. The authors determined the effect of torsion and bending on the initiation of cracks and then on propagation of the phenomena. By measuring displacements and calculating the strain energy for cracked shafts with different crack lengths, the authors concluded that transverse cracks are decisive for lateral vibrations and less decisive for torsional vibrations and also the effect is greater for surface cracks. The influence of torsional vibrations depends on the position of the cracks [13].

Adewusi and Al-Bedoor (2002) used the principles of neural networks to detect cracks in shafts. The authors analyzed vibration signals from 5 experimental tests of shaft with and without cracks at different positions to train the Multi-layer Feed-forward Neural Networks using back-propagation algorithm. The data obtained show that two-layer network does not always detect a crack that propagates; this is mainly due to crack's position. Three-layer neural networks have performed much better in detecting the propagation but they cannot always accurately determine the severity of the crack. In conclusion, the accuracy of the network depends on the number of neurons in each layer. The method was validated by authors who applied FFT to the measured vibration signals [14].

Zou et al. (2003) propose the Wigner-Ville distribution for the determination of time-frequency characteristics of shafts with and without cracks. The Jefcott rotor was used with a disk with mass on a massless elastic shaft, with and without crack. It is modeled by the equation of motion whose numerical solutions are obtained by Runge-Kutta method. The time-frequency characteristics are analyzed and it is concluded that Wigner-Ville distribution is very efficient in the analysis of non-stationary and nonlinear shafts systems. In this paper the authors demonstrate that the proposed method is very sensitive to small variations of shaft stiffness, haft unbalances do not influence the quality of the distribution and inhabiting angle influences the distribution Wigner-Ville [15].

M.S. Lebold et al. (2003) propose improvement techniques of non-intrusive torsional vibration method for monitoring cracked shafts. This effort is being made for the online diagnosis of Westinghouse 93A reactor cooling pumps. For this study, experiments were performed on scale shafts in laboratory controlled conditions. Shafts with real cracks and their propagation through bending fatigue loads were used. The characteristics of the cracks (position, depth) were determined periodically by ultrasonic techniques, following the evolution of frequency spectrum characteristics of torsional vibrations. The torsional vibration method is able to determine the first natural frequency of the cracked shaft in the frequency range 0.1 - 0.2 Hz [16].

J. Sawicki et al. (2003) proposed the study of vibrations on Jeffcott rotor, unbalanced, with different crack depths, passing through fundamental resonant velocities at different constant accelerations. Simple hinge model for shallow cracks and Meyes modified steering function for deep cracks were used in the equations of motion in inertial coordinates. The equations of motion were solved by the Runge-Kutta method. Nonlinear and transient analysis was used for this model. The presence of cracks leads to a reduction in shaft’s rigidity with an increasing effect of the amplitude of fundamental frequency and appearance of subcritical harmonics. From the presented graphs it was observed that the fundamental resonance speed expands and amplitudes of vibrations increase with depth. The authors conclude that the angle of unbalance eccentricity orientation has an influence only on the amplitude of fundamental resonance and less on subharmonics [17].

Guo et al. (2003) study the diagnosis of shafts with transverse cracks, numerically modeled with finite elements, determining the modal characteristics (frequencies and eigenmodes) of axial, lateral and torsional vibrations. To consider the three modes of vibration, the complete stiffness matrix (12x12) is used to model open cracks or breathing crack. The responses of the cracked shaft to eccentric and gravitational excitations are analyzed by numerical integer method and by spectral technique. The authors conclude that 1x axial vibrations are a good indicator for the presence of cracks, that 2x and 3x subharmonic vibrations of lateral vibrations also highlight the presence of cracks and that torsional vibrations are not a good indicator for small cracks [18].

Sekhar (2004) extracted the subharmonics from the transient vibration signal of the shaft supported on fluid film bearings, using the continuous wavelet transform (CWT). The shaft was numerically modeled with finite elements and the crack was shaped by changing the flexibility in the cracked section of the shaft. The dynamic analysis was performed by the motion equations of cracked shaft transient response, taking into account the phenomenon of fluid dissipation and decelerations for each critical speed [19].

W. Qin, G. Chen, and X. Ren (2004) study the influence of grazing bifurcation on the nonlinear response of cracked shaft. The nonlinearity of cracked Jeffcott shaft is modeled in linear pieces due to the presence of the breath of crack (PWL model). The cracked shaft has two degrees of freedom, determining the differential equations that are integrated by the fifth order Runge-Kutta method. The Gasch method is used to model the breathing of crack. From the analysis the response of cracked shaft shows a grazing bifurcation that causes jumps from one periodic motion of the response to another and the transformation of periodic motion into quasi-periodic motion and from the periodic motion to go directly into chaos [20].

N. Bachschmid and E. Tanzi (2004) use the 3D model of the shaft with nonlinear contact forces in the cracked area to analyze the response to bending, shear, torsion and axial stresses. Additional deviations are determined for the 6 degrees of freedom of cracked shaft, due to loads for different angles to the crack (0-360^0), by three methods: a 3D finite element model (3D model), a fracture mechanics-based model (SERR), a simplified 1D model (FLEX). The calculation time for the 3D model is much longer than for the 1D model and the results obtained with the two methods are comparable. The results calculated by the simulations performed with the 1D model proposed by the authors were compared and validated by experimental tests [21].

Sekhar, A.S. (2005) propose a study for detection two types of defects that are found simultaneously in a shaft: the unbalance of shaft and the presence of crack. The study is based on a finite element model and the way to identify the defects was done online. Defects are mathematically modeled by a system of virtual loads (forces and moments) that act on the undamaged shaft and on the damage shaft, to compare the dynamic response in the two situations. The position and depth of crack and the position and size of the unbalance were determined for faulted shaft [22].

J.J. Sinou and A.W. Lees (2005) proposed a method for detecting cracks in shafts by analyzing frequencies, harmonic components and the evolution of orbits. The shaft is modeled through finite elements of Timoshenko bars with four degrees of freedom in node and the breathing crack is modeled by truncated Fourier series. The authors applied the Alternate Frequency / Time Domain method (AFT method) and determined with a good approximation the nonlinear response of a shaft with breathing cracks. The analysis of harmonic responses is completed with the evolution of shaft's orbits. As the speed of the shaft increases, the orbit distortion increases, it transforms into a double loop and at half the resonant speed, it changes again into the inside loop. Changes in the orbit cause changes in the phase and amplitudes of subharmonic components, being a convenient indicator for monitoring cracks in trees, determining the characteristics of cracks through the amplitude of orbits. The method is validated by comparing the results obtained by the fourth order Runge-Kutta integration method [23].

T. Zhou et al. (2005) performed experiments to confirm methods based on nonlinear models of cracked shafts. The real crack was obtained by applying fatigue test. The frequency spectra and the evolution of shaft's orbit were analyzed and the authors concluded that the influence of crack on the shaft dynamics is obvious when it is deep enough. The experiments validate the numerical analyzes [24].

Green I. and Casey C.(2005) theoretically analyzed the qualitative response of a system with a shaft with a gaping transverse crack using two models: global asymmetry crack model and local asymmetry crack model. The system dynamics was analyzed in the two approximations for the 2X harmonic component using the transfer matrix method thus coupling the system components, including the presence of crack. In the first approximation the shaft is modeled with an asymmetrical cross section along its entire length, model similar to many dynamic systems, containing designed asymmetric components. The analysis of forced response of model showed that the critical speeds of the 2X harmonic components decrease with the increase of crack’s depth, the damping being omit in this case. In the second approximation the gaping transverse crack in the shaft is modeled by introducing an additional flexibility to be embedded in the transfer matrix of the system through a field matrix. For this model it was observed that amplitudes of the response for 2X harmonic components increase with the depth of crack and therefore with asymmetry. The corresponding critical speeds decrease due to reduced rigidity and natural frequencies change. The authors concluded that these changes in analyzed response were due to the presence of transverse cracks in trees and that analyzed methods could be used in monitoring and diagnosing cracks [25].

Wu X. et al. (2005) studied the coupled effect of lateral and torsional vibrations on shaft with two breathing cracks using finite element analysis. The presence of cracks is modeled by the mechanics of breaking. The cracked shaft is modeled using stiffness matrix of finite elements with six degrees of freedom in nodes. The authors studied the natural frequencies and dynamic response of the shaft with two cracks for different geometries of shafts, the position of cracks and the relative phase of cracks. For close cracks the frequencies also depend on relative orientation and geometric ratios of shaft. For the in-phase cracks, the answer is identical as in case of a single crack but with different amplitudes and for phase-shifter cracks the vibration spectrum has different signatures, with diminished 1X, 3X and 5X harmonics. The authors analyze the evolution of orbit for quasi-periodic vibration which rotates in the opposite direction compared to shaft [26].

Paolo Pennacchi et. al. (2006) used frequency analysis to determine the position and depth of a crack in the shaft by experimental tests. The authors used a three-part compound shaft at scale modeled with finite elements. The frequency spectrum analysis was performed for three cases of cracks, made in the middle part of the shaft. For the first case the crack was obtained by electro-erosion with a depth of

34%. The second crack was obtained at the same position as the first, by fatigue solicitation, starting from a crack made by electro-erosion and the third crack with an average depth of 47% at another position on shaft axis. The central position of cracked shaft is non-linear with the rest of the shaft and has a bow due to fatigue stress which can distort certain results. By analyzing the response measured on bearings for the 1X, 2X and 3X harmonic components, the position was identified and the crack depth in the shaft was determined [27].

Mani et al. (2006) presented a theoretical method for detecting cracks in shafts by analyzing the combination resonance of the critical shaft speed, shaft speed and excitation frequency of active magnetic bearing (AMB). The authors used a dynamic system consisting of a Jeffcott shaft with a breathing fissure, two conventional bearings and an active magnetic bearing. The diagnostic forces in the active magnetic bearing are chosen using the multiple scale method to create spectral responses to frequencies that stimulate combination resonance. The value of corresponding vibration amplitude identifies the variation of shaft stiffness due to the presence of crack and determines the depth of crack [28].

Darpe et al. (2006) studied the qualitative and quantitative effect of residual bow on the rigidity of rotating cracked shaft and the dynamics of the system. The external excitations of a shaft, modeled like a Timoshenko beam with a breathing crack, caused changes and additions frequencies around the excitation frequency. From the study, the authors concluded that shaft bow does not significantly affect its rigidity and the nonlinear nature of cracked shaft response,but the bow affects the orbital response of cracked shaft which shows that this clue cannot be used for diagnosis [29].

Ishida and Inoue (2006) analyzed nonlinear vibrations using harmonic excitations on a cracked shaft. The breathing crack is modeled by a power series function and the equations of motion of the cracked shaft have linear and nonlinear parameters. Harmonic combinations between excitation frequencies and cracked rotor frequencies are investigated. For the detection of cracks the types of nonlinear resonances due to the crack, the resonance points and dominant frequency component were analyzed numerically and experimentally. The amplitudes of additional frequencies increase with the depth of crack. Harmonic combinations detect cracks in the shaft and certain resonances can detect cracks both in a horizontal shaft and in a vertical shaft [30].

Guo and Peng (2007) applied the Hilbert Huang transform for the diagnosis of a cracked shaft, studying the transient response and the effectiveness of the method on the propagation of transverse breathing cracks. The shaft is modeled with finite elements (FEM) and Dimarogonas method and breathing crack is modeled by a switching function. The motion of loaded shaft with three axial forces and three torques is modeled with equations of motion and instantaneous response is analyzed when passing through the critical speed and the resonant subharmonics 1x, 2x and 3x. The authors conclude that analysis of transient response is efficient at a low rotational speed and that the propagation of the crack decreases the efficiency of the transient response [31].

Babu and Sekhar (2007) used two methods for the qualitative and quantitative diagnosis of cracked shaft: artificial neural networks (ANN) and continuous wavelet transform (CWT). The shaft-bearing system was modeled using the finite element method (FEM) and the breathing crack was modeled with an additional local flexibility matrix. The excitation matrix consists of excitation forces due to unbalance of mass disk m. The transient bending vibrations were analyzed. The crack is qualitatively detected by CWT determining the variation diagrams of the CWT coefficients for the critical peaks 1/3 and 1/2. Quantitative identification of theposition and depth of crack is performed with ANN having as input parameters the amplitudes of the subcritical peaks 1/3 and 1/2. The method can be used to real time monitoring the operation of steam turbines, aero engines, etc. [32].

Xiang et al. (2007) presented a model-based method for diagnosing a cracked shaft. B-spline wavelet on the interval (FEM BSWI) Rayleigh beam is modeled with finite elements taking into account the influences of rotational inertia on lateral vibrations, based on Rayleigh beam theory. The shaft is modeled with BSWI Rayleigh-Euler beam element and the disc with BSWI Rayleigh-Timoshenko beam element. The crack is modeled by changing the stiffness in cracked section. The authors applied the forward problem and inverse problem to detect and estimate the position and depth of shaft's crack. The method has been experimentally validated and can be used to monitor and diagnose cracked shaft [33].

Samanta and Nataraj (2007) proposed a study for detection of cracks in shafts using the adaptive neuro-fuzzy inference system (ANFIS), artificial neural networks (ANN) and genetic algorithms (GA). The authors used a Jeffcott shaft with a breathing crack. The time-frequency characteristics of the transient signal are extracted using the continuous wavelet transform (CWT) for the healthy and cracked shaft. The wavelet energy distributions and gray moment vectors (GMV) of the CWT scalograms, classifiers parameters, ANFIS and ANN are selected using GA to optimize crack level diagnosis. Test data with known crack levels are used to train the classified parameters. To validate the method the diagnostic performance is compared with classified parameters [34].

A.K. Darpe (2007a) proposed a method for detecting nonlinear breathing cracks, taking into account the coupled bending-torsion vibrations. The cracked shaft was excited by a short torque and the lateral vibration responses were analyzed. The transient characteristics for lateral resonant vibrations are determined using wavelet transforms. The values of the wavelet coefficients for the transient lateral vibration were evaluated in accordance with the torsion excitation angle and the correlation with the breathing crack for a horizontal shaft. Other faults do not cause susceptible responses under similar excitation conditions so the proposed method was unambiguous. The method can be used without rotary system being turned off [35].

Darpe (2007b) proposed a method of analyzing the vibration response for a shaft with slant crack. He used the Jeffcott shaft model with a stiffness matrix for equations of motion with three degrees of freedom. The responses to axial, lateral, and torsional vibrations for the shaft slant crack were analyzed by comparing the responses to transverse cracks with subharmonic resonances. The axial and lateral rigidity of shaft with slant crack is higher but the elasticity is better at torsion. The responses for axial and longitudinal vibrations differ for shafts with slant and transverse cracks. At the same time the author studied the influence of the angle of slant crack with longitudinal axis of shaft on the rigidity of shaft and the response to vibrations [36].

Darpe (2007c) analyzed the characteristics of the coupled vibration response of a shaft with the slant crack and the effect of stiffness coefficients. The results were compared with those of the transverse cracks. The slant crack was modeled with a flexibility matrix, taking into account the additional stress intensity factors, using the theory of fracture mechanics. The transverse coupling of lateral-axial-torsional vibrations is stronger in the case of slant cracks than in the case of transverse cracks, although there is the same torsional rigidity. The influence of the angle of the oblique crack with the shaft axis on the rigidity is also investigated [37].

Chang et al. (2007) proposed a numerical method of nonlinear analysis of a cracked shaft with asymmetrical viscoelastic supports. The masses of shaft and disc, the geometric nonlinearity, the presence of additional displacements given by cracks and the asymmetrical viscoelastic supports are considered, for the generation of nonlinear equations of motion. The effect of the position and depth of crack, the location on disc and the speed of shaft on the response of cracked shaft on asymmetrical viscoelastic supports are analyzed. An important role for the shaft asymmetrical viscoelastic supports system is the dynamic stability influenced by the presence of the crack [38].

Sinou (2007) analyzed the periodic response of nonlinear model and the dynamic stability of nonlinear periodic solutions for a rotating cracked shaft. Using the finite element method (FEM), the Timoshenko beam is modeled, with four degrees of freedom in the node. The breathing crack modeled with the additional flexibility matrix is used, due to the concentration of the deformation energy in the vicinity of the crack. To approximate the nonlinear response, the author used the harmonic equilibrium method, which uses truncated Fourier series. The dynamic stability was analyzed by applying nonlinear perturbations to the periodic solution of the cracked shaft. The author analyzed the influence of some parameters of the rotor system (depth and location of the crack, the position of the disk on the shaft axis and the support rigidity) regarding the stability of the periodic nonlinear response [39].

Jiao et al. (2008) determined the presence of crack in the shaft with a nonlinear and non-stationary analysis using the Hilbert-Huang transform (HHT), which is based on empirical mode decomposition

(EMD). Data has many simple modes, coexisting with very different frequencies that overlap, test and verify empirically. The data set is divided into small and finite intrinsic mode functions (IMF). Through experimental tests, the authors demonstrated the applicability of HHT to the detection of cracks in the shaft by representing the phenomenon of phase modulation excited by torsional vibration. The method is efficient, can be adapted and applied to time-frequency-energy processes [40].

Babu et al. (2008) used the Hilbert-Huang transform (HHT) for analyzing a transient signal to monitor a cracked shaft. The HHT spectrum shows subharmonics that characterize the presence of crack in the shaft. Through the empirical mode decomposition (EMD) method and the Hilbert transform (HT), complex data sets have been decomposed into a finite number of intrinsic mode functions (IMF). The authors concluded that for transient vibration analysis of a cracked shaft, HHT gives better results compared to FFT and CWT methods [41].

Xiang et al. (2008) proposed a method for diagnosing the location and depth of a shaft crack by combining wavelet-based elements and the genetic algorithm. For the B-spline wavelet on the interval (BSWI) the finite element method was used with Raleigh-Euler beam elements for the slender shaft and with Rayleigh-Timoshenko beam for the stiffness disk. The crack was modeled by adding a stiffness matrix, represented by a concentrated parameter element, according to the theory of fracture mechanics. The direct problem was analyzed, using experimental modal analysis, to detect cracks accurately with the first three natural frequencies. The inverse problem was applied using the genetic algorithm to optimize the differences between the numerically and experimentally obtained frequencies. Although the method is used with high performance in monitoring cracked shafts, still operational modal analysis (OMA) techniques must be developed for determining natural frequencies [42].

Saridakis et al. (2008) proposed a model based on the deformation energy and the theory of fracture mechanics, applied for the diagnosis of the shaft with multiple cracks and the determination of three crack parameters: the location, depth and angle crack in relation to the shaft axis. The two-crack shaft is modeled using the Euler – Bernoulli beam, introducing the coupling effect between horizontal and vertical bending. The authors calculated the stress intensity factors for the crack modeled with the local compliance matrix for two degrees of freedom, at all angles of rotation of the shaft depending on the other two parameters (depth and position). The natural frequencies and the responses of the cracked shaft on the two degrees of freedom are calculated, excited by a bending moment in the vertical plane. Several objective functions are defined using the differences between the values of the results obtained by numerical methods with those experimental. Two of the objective functions are based on fuzzy-logic. The authors use the inverse problem method to identify the three parameters for the two cracks: location, depth, and orientation relative to the longitudinal axis of the shaft. Due to the fact that many parameters must be identified, the artificial neural network is used to solve the inverse problem and the genetic algorithm to optimize the process, which produces the parameter values as input data in the artificial neural network and compares the outputs with experimental ones. The methodology proposed by the authors can be used for real-time diagnosis of shafts with multiple cracks, significantly reducing the computation time, without losing information [43].

Patel and Darpe (2008) examined the effects of dynamic parameters (unbalance eccentricity level and its phase, crack depth, shaft speed and damping) on the nonlinear behavior at subcritical velocities of a shaft with two crack types: switching cracks and breathing cracks. The bifurcation, amplitude, orbit and Poincaré section properties were analyzed according to the dynamic parameters for the two cracked shaft models. Jeffcott shaft mounted on rigid bearings support was used. Following the simulations performed on shafts with switching crack, the phenomenon of chaos and bifurcation was observed and in the shafts with breathing crack, their absence was found. Also, the response of the shaft with the breathing crack does not include quasi-periodic and subharmonic vibrations. It should be noted, however, that there are some differences between numerical and experimental analysis, determined by the mathematical models used [44].

Bachschmid et al. (2008) studied the nonlinear behavior of a helically cracked shaft, subjected to coupled excitations at bending and torsion. Previously, the authors studied the mechanism of the nonlinear 3D model of the breathing crack and a proposed simplified model. Comparing the results on the two methods, it was concluded that both accurately determine the variation of the stiffness of the cracked shaft during a complete rotation and simulate the behavior of the breathing crack. Simplified modeling becomes an iterative nonlinear procedure, the mechanism being affected by temperature, pre-stresses and other nonlinearities of the shaft-bearing system. For the crack with the helicoidal surface, torsional deflections were determined, in the absence of torsional loads, generated by the bending moments. This is due to the coupling effect and the appearance of torsional vibrations, which show the presence of helicoidal cracks in the shaft [45].

Sinou (2008) analyzes the effects of the depth and location of the breathing fissure on the nonlinear response of the 2x and 3x superharmonic components of the cracked rotor and the crack – unbalance interactions effect. The Timoshenko beam element was used to model the shaft with finite elements, with four degrees of freedom in each node and a rigid disk positioned in the middle of the shaft. The authors used the Mayes and Davies model for respiratory fissure. The truncated Fourier series were used to obtain the nonlinear dynamic response of the cracked shaft. The obtained results show that the two parameters of the crack produce variations of the nonlinear response and the appearance of new harmonic components. The crack-unbalance interaction has an influence on the evolution of the 2x, 3x superharmonic components and the associated resonance peaks. The proposed method has been validated for numerous numerical examples of shafts with crack positions and locations, unbalance magnitude and crack size orientation [46].

El Arem Saber and Maitournam Habibou (2008) presented a method of constructing a cracked beam with finite element through a stiffness matrix, which is assembled with the rest of the stiffness matrices of the elements without cracks. It is considered that the elastic energy determined by the crack is distributed over the entire length of the cracked beam finite element. Because the construction of the cracked beam finite element is done by three-dimensional calculations, considering the conditions of unilateral contact between the faces of the crack, the precision of modeling the crack breathing mechanism increases. The advantages of the method proposed by the authors are the increase in modeling accuracy and the reduction of the calculation time. Using the Floquet theory, the stability of the cracked shaft is analyzed for a range of viscous damping coefficients [47].

Jun and Gadala (2008) developed a method for the dynamic analysis of the cracked shaft using the additional slope in the crack breathing area. The additional slope in the crack breathing area is calculated by integrating on the open crack surface and depends on the bending moments as the first response, which acts at the ends of the cracked beam element. In turn, the bending moments are calculated according to the additional slope in the crack breathing area, considered a input data in the reverse iterative process. The authors used the transfer matrix for the analytical derivation of the influence coefficients and calculated the cracked and unbalanced shaft response and the quasi-static response determined by its own weight. The presence of the crack in the shaft produces nonlinear excitations in the system, so that for the transient regime the additional slope in the crack breathing area becomes a concept of "moving" Fourier series expansion. The distribution of the intensity factor changes along the front line of the crack, changes with the depth of the crack and the speed of the shaft. Due to the nonlinear stiffness asymmetry, in the area of the critical and subcritical speeds of the shaft, in the whirl orbits a temporary whirl direction reversal and phase shift exist. The authors compared the results of the proposed method with experimental tests in the literature for crack propagation models in shafts [48].

H. B. Dong et al. (2009) proposed a way to determine the depth and position of a transverse crack in a shaft by combining two methods: the modal analysis method and the wavelet finite element method. The shaft was modeled using finite element B-spline wavelet on the interval (FEM BSWI) and the crack was modeled with a weightless rotating spring. The authors proposed a method based on empirical mode decomposition (EMD) and Laplace wavelet to make it easier to identify cracks in shafts. The analysis of the first three natural frequencies and the corresponding modal forms give information about the position and depth of the cracks. The method was experimentally validated and its operability and functionality were verified [49].

Sawicki et al. (2009) used multi-resolution analysis of the discrete wavelet transform (DWT) to identify transverse cracks in shafts. The signals recorded on healthy and cracked shaft, with or without external excitation were decomposed into eight levels of frequency bands, analyzing the first three low frequency bands. The test stand consists of a shaft supported on ball bearings and an active magnetic bearing (AMB), which produces the excitation forces of the shaft. The authors analyzed the influence of the presence of the crack on the RMS amplitudes of vibrations for the healthy shaft and cracked with or without excitation forces [50].

Nagaraju et al. (2009) analyzed transient bending vibrations with 3D Continuous Wavelet Transform (CWT 3D), using sub-critical harmonics to detect breathing cracks in shafts. Using Cross Wavelet Transform (XWT), the authors analyzed the phase angles of the sub-critical frequency components between the un-cracked and the cracked signals. The advantages of using XWT are better resolution and combined representation of sub-critical amplitudes and phase angles. Detection of crack parameters (position and depth) was determined by the inverse problem, using the artificial neural network (ANN) [51].

Andrés Bejarano et al. (2009) proposed an online method for diagnosing open cracks in shafts, using a wireless sensor, which records vibrational signals in different positions of the shaft. The authors used the Jeffcott shaft, modeled with finite elements, placed on rigids bearings. Due to the eccentricity of the rigid disk, the system is excited by harmonic loads, coming from the weight of the rotating disk. For different positions and depths of the crack, are determined both numerically with the finite element method and experimentally, the natural frequencies and the corresponding modal forms. The inverse problem for determining crack parameters (position and depth) used the method of neural networks artificial, with the Backpropagation Levenberg Marquardt (LMBP) algorithm. This method, efficient and fast, predicts a relationship between the input parameters and the target values [52].

Sinou (2009) experimentally analyzed the nonlinear dynamic response of a shaft with transverse cracks, of different depths. The presence of cracks and their propagation, from certain depths, sensitizes the 2x and 3x vibration harmonics at 1/2 and 1/3 subcritical resonant speeds, causing amplitudes increase. At the same time, the presence of cracks determines the decrease of the subcritical resonance speeds. The experimental results validate the theories presented by many researchers [53].

Yanli and Chuhe (2009) analyzed the response of a shaft with a slant crack at 45^0, showing the influence of the couplings of some stiffnesses. Using a Jeffcott shaft, modeled with finite elements, with four degrees in node, the authors observed, in the case of the slant-cracked shaft, the coupling of some rigidities, bending – torsion, bending – tensile and torsion – tensile, and for the transverse-cracked shaft a single bending– tensile coupling. The eccentricity of the disc also introduces an additional bending-torsion coupling. The authors also studied the response to two types of slant cracks: open crack and breathing crack. In both cases the frequency spectrum is the same, which recommends using the open crack for computational economy. The combined frequencies of rotating speed and torsional excitation speeds and torsional excitation frequencies are useful for diagnosing shafts with the slant crack [54].

Zakhezin and Malysheva (2010) investigated the presence of cracks in shafts using cepstral analysis. For the analysis of the autocorrelation function of a filtrated cepstrum, measurements of the vibrational signals on the bearing housing were performed. The authors determined the depth of the cracks using the parameters of the autocorrelation function, the number and peaks of the harmonics and the relative distance between them [55].

Tao Yu et al. (2010) proposed a method of diagnosing a shaft with a crack or multiple cracks, using a combination of modal analysis and artificial neural networks (ANN). The cracked shaft is modeled with finite elements based on the theory of fracture mechanics and the Paris energy principle. Different modal shapes will be obtained for open cracks with known locations and depths. These will be the input for the inverse problem, nonlinear analysis with high accuracy and efficiency by ANN, pre-designed to identify the location and depth of the cracks [56].

Bachschmid et al. (2010) studied the dynamic response of a shaft with different longitudinal positions and depths of the crack at uniform or transitional velocities. The numerical model of the horizontal shaft with simplified transverse breathing crack was analyzed in the frequency range. The simplified rotating system, which is modeled with FEM comprising a 1D shaft, an equivalent cracked bar, support bearings, has been extended in a Fourier series. The dependence between the cracking forces, the crack depth, the static bending moment was analyzed. The curvature of the axial line and its variation in the sections of the cracked shaft were analyzed. The dynamic response to the critical speeds of the crack shaft was analyzed.

The aim of these numerical studies is to quickly and efficiently assess the sensitivity and response in real turbo generator units, caused by the presence of cracks, through standard monitoring and measurements, performed on the support bearings [57].

Singh and Tiwari (2010) proposed a method for identifying cracks in shafts, estimating the number of cracks, their location and depth. The cracked shaft, fixed at the ends, is modeled with finite elements, using the Timoshenko beam element. In the first stage, using the forced transverse responses in a shaft, excited by harmonic loads with different frequencies, in two orthogonal planes, the curvature of shaft elastic line is obtained, which presents discontinuities at the cracks. The shaft excitation mode reduces the signal noise. In the second stage, using the objective functions in the genetic algorithms (GA), defined by the shaft's response to several frequencies, the location and depth of the cracks in the shafts are accurately determined [58].

3. Conclusion

In this paper were highlighted some of the works from the years 2000-2010 comprising the main achievements on the diagnosis of cracked shafts, obtained by analyzing the vibrations of shafts in power plants, in laboratories, through studies on models with numerical analysis. From a vast literature, researchers in this field have proposed various tools and methodologies for cracks identification in shafts, while showing the specific advantages of the method and validation of results by already established methods.

From the 58 works studied and presented in the form of a summaries, some general conclusions can be presented, useful for method understanding and for the correct use of methods in future research by other specialists:

- understanding the behavior of a cracked shaft, through the classical approach, stiffness, energy functions and analysis of signals recorded in real rotating machines;
- the correct learning of the numerical techniques and methods used by the researchers for the detection of cracks in the shafts;
- determining the behavior of rotating shafts in the presence of different cracks: transverse, slant, helicoidally or multiple cracks;
- understanding the mechanism of breathing cracks, modeling it, variation of the corresponding rigidity and analysis of the dynamic response;
- the used methods determine the sensitivity of the shaft response to the position of the crack along the shaft, the shape and depth of the crack;
- the influence of: axial and torsion excitations, unbalance, nonlinearities, rotor-bearing system, etc.;
- the use of the finite element method, the wavelet transform and artificial intelligence methods, as a inverse technique, for modal analysis and cracks monitoring;
- efficient use of the genetic algorithm, as a tool for inverse diagnosis, for monitoring the cracked shaft.

In conclusion, the papers presented in the summary investigate the effects of the presence of cracks on the dynamic properties of a shaft including natural frequencies, stiffness, transient response, whirl orbits, changes of center orbit at critical speeds, damping ratios and others.

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