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Characterization of the network of a transport infrastructure

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Abstract. Networks (whatever their nature: telecommunications, freight transport, energy or people) are studied from several points of view, they have a fundamental role in the existence of the territory as well as in its development. The transport networks are of great diversity depending on the mode of transport to which they belong. The transport infrastructure network is characterized by arcs and nodes. Both arcs and nodes have characteristics that must be connected to the means of transport that use it. In order to carry out the transport in the relationship $(i, j) \in R$ for a given network (K, D), the sequence of arcs connecting to node $i \in K$ to the node $j \in K$ must be chosen (in the case of the network appropriate to the request). If the nodes i, j are neighbors the arc will constitute the shortest sequence of arcs.

Keywords: arcs, nodes, graph, didactic

1. Introduction

The transport transport networks are of great diversity depending on the mode of transport to which they belong. The transport infrastructure network is characterized by arcs and nodes. Both arcs and nodes have characteristics that must be connected to the means of transport that use it. For example, the uneven intersection of a highway with a local road is a network node only for those vehicles for which street access is allowed. For cyclists, agricultural tractors or animal-drawn vehicles, the intersection in question does not constitute a node of the network because these vehicles are not allowed to access the highway. Analogously, in the case of the railway network, the branching of lines from a railway station does not constitute a node for trains with electric traction if one of the branching lines is not electrified. The mathematical model is presented in [4]

2. Network parameters

Under the conditions of a transport network, if we write with $k = 1,2,\ldots,K,$ nodes of the transport network and with (k, l) the direct connections between nodes neighbors k and l, then the set $D = \{(k_1, l_1), (k_2, l_2), \ldots, (k_d, l_d), \ldots, (k_D, l_D)\}$ is called the set of paths and the set of nodes. The set of all nodes and the direct links between them constitutes the network $(K,\ D)$.

2.1. Example

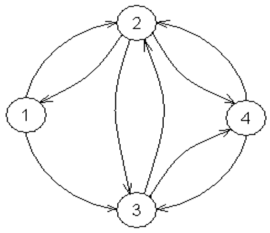

Fig.2.1 The graph of the network

For the network in the figure above we get

$$K = \{1, 2, 3, 4\} \text{ and}$$

$$D = \{(1, 2), (2, 1), (1, 3), (2, 3), (3, 2), (2, 4), (4, 2), (3, 4), (4, 3)\}.$$

For each arc of the network graph, parameters can be associated to characterize the movement of a certain type of means of transport. If the arc between two nodes is not homogeneous, then the homogeneous sections must be identified and characteristic parameters must be displayed for each one (length, maximum speed, minimum speed, with which means of transport of a certain type can move and correspondingly the minimum and maximum duration).

Modified network graph

For some nodes of the network, the transit time cannot be neglected (for example, transit through large cities in the case of road traffic between cities, or of intersections equested in the case of traffic within cities.

Definition: We call a complex node a node that has a significant transit time in relation to the duration of the arcs.

Each complex node with the number $n \in K$ is characterized by the set of values τ_{kl}^{n} that determine the duration of the passage from the road (k, n) to the road (n, l) through the node n . This duration depends on the transit capacity of the node (of its roads) and the intensity of the means of transport. At the same time, for each node, we can consider that the length of the path inside it is zero ($d_{kl}^{n} = 0$) given that the transit speed through the node is undetermined (it can change during the process of traveling the node).

We denote by $K_n^{'}$ the set of incident nodes for which node n is a neighbor (from which we can directly pass to node n) and by $K_n^{''}$ the set of emerging nodes, neighbors in relation to n (in which we can pass directly in relation to n). The value τ_{kl}^{n} must be determined for the pairs (k, l) belonging to the set $K_n^{'} \times K_n^{''}$. In particular when it is not possible to pass through the complex node from path (k, n) to path (n, l) the value τ_{kl}^{n} must be considered ∞.

For the graph associated with the network in Fig. 2.1. we admit that node 2 is complex node. This can be represented as in Fig 2.2. It is observed that the complex node is presented in the form of a graph in which the set of vertices is constituted by the set of ramifications of the complex node n on the output paths that connect the node with nodes whose number belongs to the set $K_n = K_n^{'} \cup K_n^{''}$. The set of arcs of the graph is determined for those arcs for which the value τ_{kl}^{n} is finite numbers.

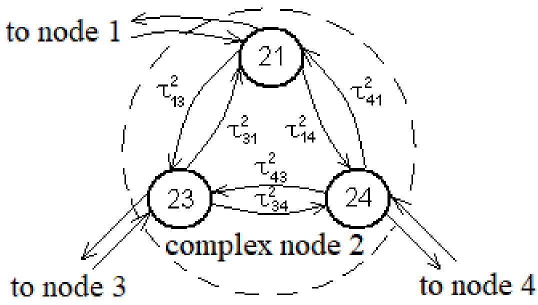

Fig.2.2 Complex node 2

We obtain a modified graph of the network in which all nodes are simple nodes with zero travel time if we plot each complex node for which we have the value $\tau_{kl}^{n} \neq 0$ and $\tau_{kl}^{n} \neq \infty$ with the help of the corresponding graph and we transpose the nodes into the network graph. In the modified graph (K, D) in which the durations of the (simple) nodes are negligible (equal to zero) the set of complex nodes is replaced by the set of branches, which have become simple nodes, and the set of arcs is supplemented with the set of links between the branches; *K* is the set of vertices of the modified graph of the network, and *D* is the set of arcs of the modified graph. Each arc $(k, l) \in D$ corresponds to certain quantities: $d_{kl} > 0$, $\underline{V}_{kl} \leq \overline{V}_{kl}$ and $\overline{\tau}_{kl} \geq \underline{\tau}_{kl}$, if the arc constitutes a link between the nodes in the initial network graph and $d_{kl} = 0$, $\underline{V}_{kl} = 0$, $\overline{V}_{kl} = \infty$, $\overline{\tau}_{kl} = \underline{\tau}_{kl}$, if the arc was introduced as a link between branches within the complex node reduction operation. The quantities d_{kl}, $\underline{V}_{kl}$, $\overline{V}_{kl}$, $\overline{\tau}_{kl}$ and $\underline{\tau}_{kl}$ are called arc parameters for all $(k, l) \in D$ arcs and their values are non-negative numbers.

3. The suitable network for request

The graph (I, R) describing transport requests should not be confused with the graph (K, L). The set *I* designates the starting or ending points of the transport (shipment) that partially coincide with the vertices of the network. The start (origin) and end (destination) points of a request can be found at any point on the arcs between the network nodes, where they are accessible to the means of transport moving on the network arcs. The network that corresponds to this condition and ensures the possibility of carrying out the transport in the relationship $(i, j) \in R$ is the network suitable for the transport request.

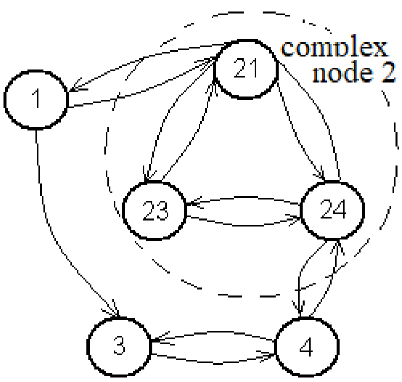

Fig.3.1 Modified network graph

All the nodes in the figure above are simple.

$$K = \{1, 21, 23, 24, 3, 4\}$$

$$D = \{(1, 21), (21, 1), (21, 23), (23, 21), (21, 24), (24, 21), (23, 24),$$
$$(24, 23), (23, 3), (3, 23), (3, 4), (1, 3), (4, 3), (24, 4), (4, 24)\}$$

For simplification, the set of network nodes is completed with the origin and destination points of the transports that did not coincide with the network nodes and the network graph nodes are renumbered with the symbols of the set $k = \{1, 2, \ldots, k, \ldots, K\}$ where $k = 1, 2, \ldots, K$ represents the graph nodes (network nodes, branches, dispatch or destination points). We admit that the nodes of the graph that were dispatch or destination points keep their previous numbering with $i = 1, 2, \ldots, I$, from which it follows that $I \subset K$.

3.1. Example

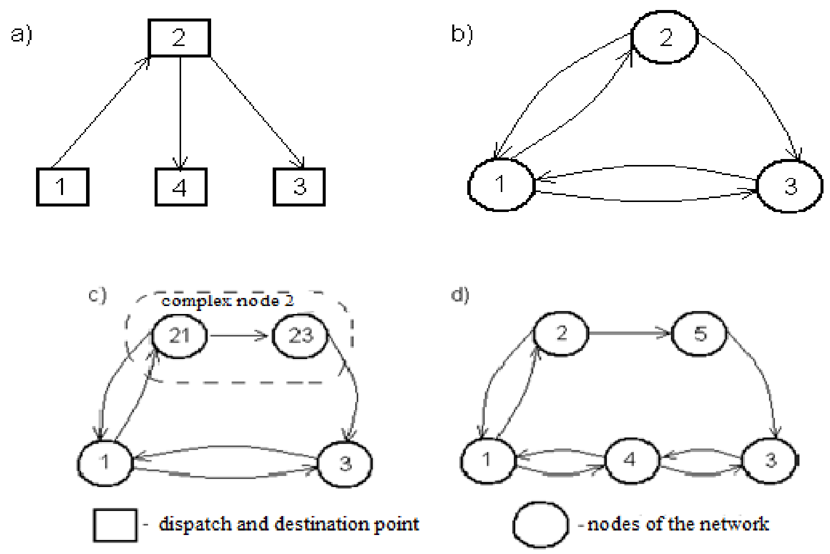

Fig.3.2 The transformation of the network into the network suitable for the demand

In figure a) shows the demand graph (I, R) where $I = \{1,2,3,4\}$; $R = \{(1, 2), (2, 3), (2, 4)\}$.
In figure b) shows the initial graph of the network (K, D) with the observation that node 4 is found on the arcs connecting nodes 1 and 3 and node 2 of the network is a complex node, $K = \{1, 2, 3\}$; $D = \{(1, 2), (2, 1), (2, 3), (1, 3), (3, 1)\}$.
In figure c) shows the modified graph of the network, $K = \{1, 21, 23, \cdots 3\}$; $D = \{(1, 2), (2, 1), (1, 3), (3, 1), (21, 23), (23, 3)\}$.
In figure d) shows the network graph appropriate to the request, $K = \{1, 2, 3, 4, 5\}$; $D = \{(1, 2), (2, 1), (1, 4), (4, 1), (3, 4), (4, 3), (2, 5), (5, 3)\}$.

4. Itineraries in the network

To carry out the transport in relation $(i, j) \in R$ for the given network (K, D), the sequence of arcs connecting node $i \in K$ with node $j \in K$ must be chosen (in the case of the network appropriate to the request). If nodes $i,\ j$ are neighbors, the arc $(i,\ j)$ will constitute the shortest sequence of arcs see [2], [4]. The succession of arcs that constitute the S_{ij} course is of the form

$$S_{ij} = \{(k_1 = i,\ k_2),\ (k_2,\ k_3),\ldots,\ (k_r,\ k_{r+1}),\ldots,\ (k_s,\ k_{s+1} = j)\}$$

or a more simplified writing in which only the set of nodes between *i* and *j* of the form are retained

$$S_{ij}^{'} = \{(i,\ k_1, k_2,\ldots, k_r,\ldots, k_s,\ j)\}$$

If the path between *i* and *j* includes a single arc, then $S = 1$ and

$$S_{ij} = \{(i,\ j)\} \text{ and } S_{ij}^{'} = \{i,\ j\}.$$

For paths S_{ij} connecting nodes *i* and *j*, the length can be set

$$d_{ij} = \sum_{(k,l)\in S_{jl}} d_{kl}$$

sau durata minimă necesară pentru parcurgerea itinerarului

$$\underline{\tau}_{ij} = \sum_{(k,l)\in S_{jl}} \underline{\tau}_{kl},$$

where d_{ij} is the length of the arc between nodes *k* and *l*, and $\underline{\tau}_{kl}$ is the minimum duration required to traverse the arc.

If the durations of stops in different nodes of the route are also taken into account, then the duration of travel between nodes *i* and *j* is:

$$t_{ij}^{T} = \sum_{(k,l)\in S_{ij}} \tau_{k,l} + \sum_{k\in S_{ij}^{'}} \Delta_k,$$

where Δ_k is the normal length of stay in the nodes along the route $(\Delta_k = 0)$, and $\underline{\tau}_{kl} \le \tau_{kl} \le \overline{\tau}_{kl}$ is the estimated duration of the arc $(k,\ l)$. The minimum value $t_{i,j}^{T}$ is obtained for $\tau_{kl} = \underline{\tau}_{kl}$ and $\Delta_k = 0$

For larger values of Δ_k, the transport duration can be arbitrarily increased and the travel speed reduced accordingly, $V_{ij}^{T} = d_{ij} / t_{ij}^{T}$ see [1].

It is possible that between two nodes *i* and *j* there are several routes $m = 1,\ 2,\ldots, M$, so for each one the sequence of arcs must be specified.

4.1. Example

In this example, three itineraries between nodes 1 and 4 are presented and the variants corresponding to the shortest, respectively fast route are mentioned. The numbers on the arcs of the graph signify the distance (those from the numerator), respectively the duration (those from the denominator).

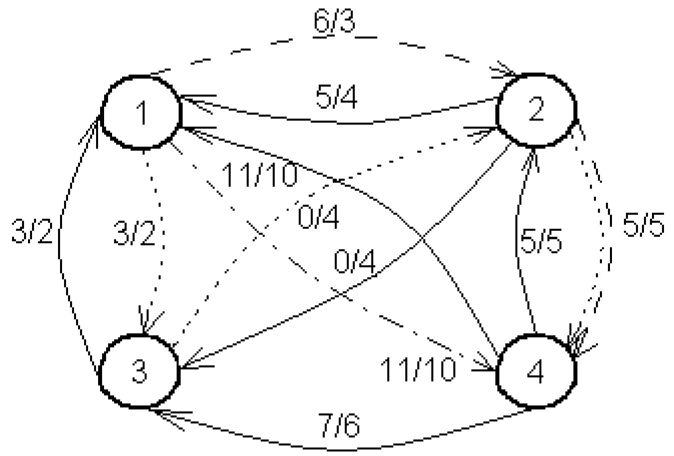

Fig. 4.1

$$S_{14}^{(1)} = \{(1,3),(3,2),(2,4)\} \text{ the shortest route}$$
$$d_{14}^{(1)} = 3+0+5 = 8 = d_{14}^{*}$$
$$\underline{\tau}_{14}^{(1)} = \underline{\tau}_{13} + \underline{\tau}_{32} + \underline{\tau}_{24} = 2+4+5 = 11$$
$$S_{14}^{(2)} = \{(1,2),(2,4)\} \text{ the fastest route}$$
$$d_{14}^{(2)} = 6+5 = 11$$
$$\underline{\tau}_{14}^{(2)} = \underline{\tau}_{12} + \underline{\tau}_{24} = 3+5 = 8 = \underline{\tau}_{14}^{*}$$
$$S_{14}^{(3)} = \{(1,4)\}$$
$$d_{14}^{(3)} = 11$$
$$\underline{\tau}_{14}^{(3)} = \underline{\tau}_{14} = 10$$

For each route $S_{ij}^{(m)}$ can be set:

- the length $d_{ij}^{(m)} = \sum_{(k,l)\in S_{ij}^{(m)}} d_{kl}$,
- minimum travel time $\underline{\tau}_{ij}^{(m)} = \sum_{(k,l)\in S_{ij}^{(m)}} \tau_{kl}$,
- maximum travel speed $V_{ij}^{(m)} = \dfrac{d_{ij}^{(m)}}{\underline{\tau}_{ij}^{(m)}}$.

One can choose the shortest route, among all M routes, marked S_{ij}^{d} with the property $\min_m d_{ij}^{(m)} = d_{ij}^{*}$ consisting of that as its length

$$d_{ij}^{*} = \sum_{(k,l)\in S_{ij}^{(d)}} d_{kl} ,$$

representing the minimum of the lengths

$$d_{ij}^{(m)} = \sum_{(k,l)\in S_{ij}^{(m)}} d_{kl} ,$$

for all M routes $S_{ij}^{(m)}$, $(m = 1, 2, \ldots, M)$.

Independently of all the *M* routes, one can choose the route with the minimum travel time, denoted $S_{ij}^{(\tau)}$ with the property

$$\min_m \underline{\tau}_{ij}^{(m)} = \underline{\tau}_{ij}^*$$

consisting in the fact that the duration of travel

$$\underline{\tau}_{ij}^* = \sum_{(k,l)\in S_{ij}^{(\tau)}} \underline{\tau}_{kl} ,$$

represents the minimum travel times

$$\underline{\tau}_{ij}^{(m)} = \sum_{(k,l)\in S_{ij}^{(m)}} \underline{\tau}_{kl} ,$$

for all M routes $S_{ij}^{(m)}$, $(m = 1, 2, \ldots, M)$.

It is possible to have more than one route with the same minimum d_{ij}^* or $\underline{\tau}_{ij}^*$ value. The choice of one of these is made in relation to additional criteria. For any relationship $(i, j) \in R$, an unequivocal choice of the shortest, fastest route or in relation to another criterion (cost, safety, etc.) is possible.

In figure 4.1, the fastest route does not coincide with the shortest route. Such situations are encountered in large urban agglomerations, when on the shortest route there are a large number of required intersections, the crossing of which takes a lot of time. If the traffic intensities on the intersecting paths are insignificant, then we have $S_{ij}^d = S_{ij}^\tau = S_{ij}^*$, that is, the shortest route is also the fastest, being considered the optimal choice.

To simplify the calculations, we admit that for each pair $(i, j) \in R$ the optimal route S_{ij}^* has been determined, of length d_{ij}^* and duration $\underline{\tau}_{ij}^*$ (the travel speed being maximum, $\overline{V}_{ij} = d_{ij}^* / \overline{\tau}_{ij}^*$).
The set of all optimal routes is written in the form of a matrix

$$[S_{ij}] = \begin{bmatrix} 0 & S_{12} & S_{13} & \cdots & S_{1j} & \cdots & S_{1I} \\ S_{21} & 0 & S_{23} & \cdots & S_{2j} & \cdots & S_{2I} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ S_{i1} & S_{i2} & S_{i3} & \cdots & S_{ij} & \cdots & S_{iI} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ S_{I1} & S_{I2} & S_{I3} & \cdots & S_{Ii} & \cdots & S_{II} \end{bmatrix}$$

where S_{ij} corresponds to the shortest and fastest route, which we called the optimal route

$$S_{ij} = \{(i, k_2), (k_2, k_3), \ldots, (k_r, k_{r+1}), \ldots, (k_s, j)\}.$$

Each step is characterized by three values d_{ij}, $\underline{\tau}_{ij}$ and $\overline{V}_{ij}$ so that the matrix $[S_{ij}]$ must correspond to three matrices: $[d_{ij}]$, $[\underline{\tau}_{ij}]$ and $[\overline{V}_{ij}]$.

For the network graph proposed in example 4.1, we have figure 4.2 on the arcs of which the distances between the nodes are written, the matrix $[S_{ij}]$ from table 1 and the matrix $[d_{ij}]$ from table 2 correspond.

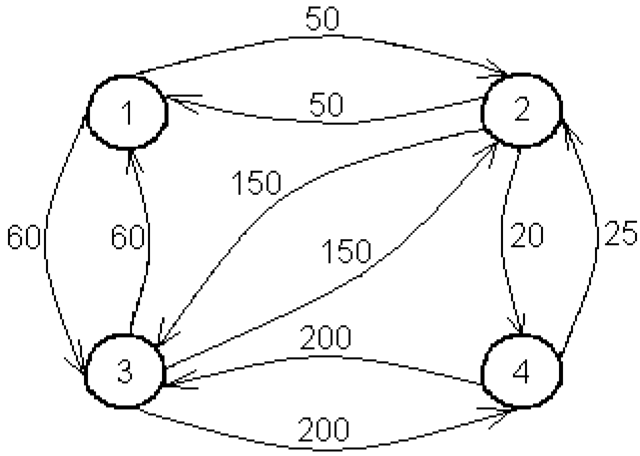

Fig. 4.2

j / i	1	2	3	4
1	0	{(1,2)}	{(1,3)}	{(1,2),(2,4)}
2	{(2,1)}	0	{(2,1),(1,3)}	{(2,4)}
3	{(3,1)}	{(3,1),(1,2)}	0	{(3,1),(1,2),(2,4)}
4	{(4,2),(2,1)}	{(4,2)}	{(4,2),(2,1),(1,3)}	0

Table 1 Matrix $[S_{ij}]$

j / i	1	2	3	4
1	0	50	60	70
2	50	0	110	20
3	60	110	0	130
4	75	25	135	0

Table 2 Matrix $[d_{ij}]$

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