



Volume XXIII 2020

ISSUE no.2

MBNA Publishing House Constanta 2020



## Scientific Bulletin of Naval Academy

SBNA PAPER • **OPEN ACCESS**

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To cite this article: Codruta Pricop, Mihail Pricop and George Novac, Scientific Bulletin of Naval Academy, Vol. XXIII 2020, pg.64-74.

Available online at [www.anmb.ro](http://www.anmb.ro)

ISSN: 2392-8956; ISSN-L: 1454-864X

doi: 10.21279/1454-864X-20-I2-009

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# Mother wavelet selection using signal energy for cracks detection in the rotation shafts

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**Abstract.** The mother wavelet greatly influences the wavelet analysis of a non-stationary and nonlinear recorded signal. Choosing mother wavelet must be done to determine cracks in rotating shafts so as to take into account the nature and type of information signals to be extracted from the signal. The difficulty in optimum selection of the mother wavelet is determined by their complex properties that determine different selection criteria. In the paper, several families of functions (Haar, Daubechies, Symlets, Coiflet, BiorSplines) were used for analysis and the proposed selection criterion is the energy dissipated on the frequency bands. Signal recordings were made on a stand to determine the presence of cracks in rotating shafts and their classification. For discrete decomposition of recorded signals (DWT) and the calculation of energy dissipated on the frequency bands the Matlab wavelet instrument was used.

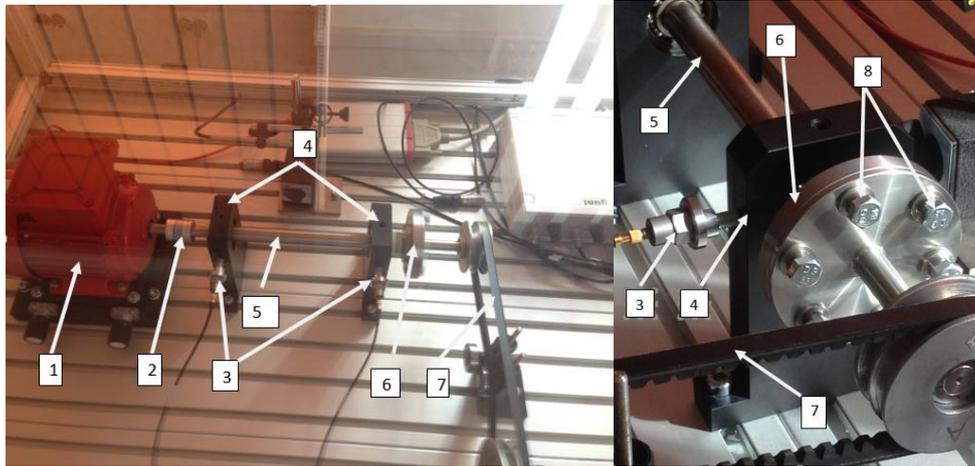
## 1. Introduction

Wavelets transform (WT) of the recorded signals, which are often nonlinear and non-stationary, is strongly influenced by the choice of the mother wavelet. The choice of mother wavelet for optimum determination of the cracks in the rotation shaft is made in such a way that it takes into account the nature and type of information that must be extracted from the signal. Selection of an optimal mother wavelet is a very difficult topic due to different selection criteria, which can indicate several types of mother wavelet. Mother wavelet functions have different properties in time and frequency, so it is difficult to choose the best wavelet function for extracting defects features from recorded signal.

In the paper to wavelet analysis were used five mother wavelet (Haar, Daubechies, Coiflets, Symlets, Biorthogonal) and their selection criterion is the energy distribution in the frequency bands of the original signal.

To detect faults in rotating machinery, bearings, gears and cracks in shafts are used widely wavelet transform. To detect cracks from rotating shafts, in work [1] Darpe used a “method based on the wavelet transform”. “Continuous Wavelet Transform” (CWT) was used in the paper [2] “describing the time and frequency characteristics of cracks in the rotation shaft”. The paper [3] presents a “review on the application of the wavelet transform in the diagnosis of rotary machine defects, which include: the time – frequency analysis of signals, the fault feature extraction, the singularity detection for signals, the denoising and extraction of the weak signals, the compression of vibration signals and the system identification”. To simulate cracks in rotating shafts and to make measurements, used "PT 500.11 - Crack Detection in the Rotating Shaft Kit" (figures. 1, 2), [4]. Signal recording and storage were

performed with Bruel & Kjaer equipment, LAN-XI Data Acquisition System, type DeltaTron 4506 accelerometer, PULSE software. Accelerometers mounted on the two bearing blocks, in vertical and horizontal direction, were used to record the signals. The shaft, with three operating states (healthy shaft, depths of the crack 16%, 33% and 66%) it is rotated by means of an electric motor at four rotational speeds 600 rpm, 1200 rpm and 2400 rpm [5].



**Figure 1.** Experimental setup for “shaft with crack” **Figure 2.** Crack simulation in simulated with elastic rotor [5]  
rotating shaft kit

Discrete Wavelet Transform (DWT) and Parseval's Theorem is used in this work to study the phenomenon of cracking in the shaft rotation. DWT breaks down the recorded signal into approximations and details by successively passing through low pass filters (LPF) and high pass filters (HPF). By decomposing the signal the information does not change over time, it is distributed at the level of each sub-band. A feature in the optimal evaluation of cracks in the rotation shaft is the energy of the coefficients, calculated according to Parseval's Theorem. The energy used as a feature can be all from the same level of signal decomposition (called the single-level basis solution), or on each level of decomposition, a selection called multiple level basis selection.

## 2. Discrete Wavelet Transform and Parseval's Theorem

The energy distribution on the component levels of the recorded signal is determined using DWT and Parseval's Theorem. DWT is a multi-resolution method of non-stationary signals recorded on rotary machines. In work [6] Kim used DWT to perform a comparative analysis for detect defects in the vibration signal recorded on rotating mechanical systems. Of the methods of analysis in the domain time - frequency, DWT is the most efficient method for detecting cracks in the rotation shaft during acceleration and deceleration processes. By using the DWT and CWT in the domain time - frequency, Ohue et al. in [7] found that a gear element damaged can be identified by changing the intensity of the wavelet coefficients. For detect and locate gear tooth defects Omar and Gaoanda proposed the use of a dynamic windowing process [8]. DWT was used by Djebala et al. for the detection of defects in rolling bearing [9]. Kumar and Singh used the Symlet wavelet as the wavelet function to perform DWT on the bearing vibration signal for measuring its outer race defect width [10]. For the multi-fault diagnosis of a gear Li et al. they used an integrated method based on DWT, autoregressive (AR) model and principal component analysis (PCA). In the analysis, DWT it was used for denoise the vibration signals [11, 12]. DWT was used by Kwak in the paper [13] for detecting defects in the cutting tool and machine tool failure by analyzing the wavelet coefficients. Extracting a good set of fault-related features from wavelet coefficients helps to identify machine defects in a much effective way. For classification of bearings defects and gearbox with high accuracy and stability, Li et al. used the slope characteristics extracted from slope logarithmic variances calculated from the DWT

coefficients [14]. Yu et al. have extracted a cluster-based feature from DWT coefficients and probabilistic neural networks for bearing fault detection [15]. Using other techniques, the DWT's ability to diagnose mechanics errors has been improved. For example, Castejon et al. developed a method for diagnosing errors in two stages, in the first step used a DWT-based multi-resolution analysis to extract interesting features from the signals, and in the second stage, to classify the defects from bearings in the incipient phase, used the neural network method [16]. Because the translation of the wavelet function depends directly on the scale, the translation and scaling parameters  $s$  and  $\tau$  in dyadic meshing, are expressed by the relations  $s=2^i, \tau=k2^j$ . DWT is a mathematical function:

$$DWT(j, k) = \langle x(t), \psi_{j,k}(t) \rangle = \frac{1}{\sqrt{2^j}} \int x(t) \cdot \psi^* \left( \frac{t-k2^j}{2^j} \right) dt, \quad (1)$$

where symbol  $\langle . \rangle$  is inner product operation.  $\psi_{j,k}(t)$  is a mother wavelet, expressed by:

$$\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}} \psi \left( \frac{t-k2^j}{2^j} \right).$$

Through DWT the recorded signal is decomposed by LPF and HPF filters constructed from the selected wavelet function  $\psi(t)$  and the corresponding scaling function  $\phi(t)$ , given by the relationships [17]:

$$\begin{cases} \phi(t) = \sqrt{2} \sum_k h(k) \phi(2t - k) \\ \psi(t) = \sqrt{2} \sum_k g(k) \psi(2t - k) \end{cases}, \quad (2)$$

with  $\sum_k h(k) = \sqrt{2}$  and  $\sum_k g(k) = 0$ .

The decomposition results, up to a certain level, are the components of the low and high frequency signal:

$$\begin{cases} a_{j,k} = \langle x(t), \phi_{j,k}(t) \rangle \\ d_{j,k} = \langle x(t), \psi_{j,k}(t) \rangle \end{cases} \quad (3)$$

$a_{j,k}$  are the approximation coefficients for the low frequency components of the signal, and

$d_{j,k}$  are the detail coefficients for the high frequency components of the signal.

Figure 4 shows an example of DWT decomposition of a signal recorded for the intact shaft at 2400 rpm. It is observed from the detail components that the activity is greatly reduced once the level of decomposition or the scale increases.

Parseval's Theorem establishes an energy distribution on the different levels of decomposition signal (frequency bands).

By integrating the square of the recording signal, its total energy is obtained, according relationship:

$$E = \int_{-\infty}^{+\infty} x^2 dt. \quad (4)$$

According to the multiresolution analysis, the recorded signals are decomposed, using DWT, into components with different frequency bands on each level. After the decomposition, the recorded signal can be obtained as the sum of the low-frequency and high-frequency components

$$x(t) = x_j^a(t) + \sum_{j=1}^l x_j^d(t), \quad (5)$$

where  $x_j^a(t)$  is the approximate information and  $x_j^d(t)$  is the detailed information at scale  $j$  after the discrete wavelet transform at the  $j$  times. Taking into account the property of orthogonality of wavelet and scaling functions, the signal energy is calculated as the sum of the energies of the detail components and the energy of the approximate component at the large-scale

$$E = \int_{-\infty}^{+\infty} [x_j^a(t)]^2 + \int_{-\infty}^{+\infty} [x_j^d(t)]^2 = E_j^a + \sum_{j=1}^l E_j^d. \quad (6)$$

The calculated signal energies at the  $j$  frequency bands form a vector  $[E_1, E_2, \dots, E_j]$ , which can be used to analyze the energy characteristics of recorded signal. This is because that the induced damage may suppress or enhanced the recorded signal at some frequency bands when the recorded signal is input into a system as excitation, so at some frequency bands may be suppressed while that at others may be enhanced. Correspondingly, there is a large difference between the energies calculated before and after the occurrence of the damage, the energy being undergoing redistribution on the frequency bands of the signal components.

### 3. Mother Wavelet Selection Methods

In WT, mother wavelets are very important in the analysis, they have different shapes depending on the type of application.

Taking into account the diversity of mother wavelets, it is the problem of selecting them, in order to obtain the most accurate results of the various analyzes, which means a good correlation between the signal and the mother wavelet.

From the category of wavelet families, the work uses the following:

- Haar, Daubechies, Coiflet and Symlets orthogonal wavelets with FIR filters that are defined by a scaling coefficient.

- Biorthogonal wavelets with FIR filters defined by the two scaling coefficients, for reconstruction and decomposition respectively.

Haar wavelet is orthogonal, symmetrical and compact.

Daubechies wavelet is orthogonal, asymmetrical, introduces a phase distortion, without an explicit expression.

Coiflet wavelet is orthogonal, near symmetrical, compact support, regularity.

Symlet wavelet is orthogonal, near symmetrical, compact support and regularity.

Biorthogonal wavelet is orthogonal, symmetrical, compact support and regularity.

As an application for detecting cracks in rotating shafts, the accuracy of the classification, for different operating conditions, depends on the decomposition level of the signal and the type of mother wavelet used for the analysis.

About thirteen families of mother wavelets are known, and their use will lead to different results for the same wavelet analysis. The use of a particular mother wavelet depends on their properties and the type of analysis performed.

A first selection of mother wavelet is based on a qualitative approach to the properties. The following properties of mother wavelet are known: regularity, compact support, symmetry, vanishing moment, orthogonality, explicit expression.

Because the methods in the first category are difficult to use in the analysis of non-stationary signals, a quantitative approach to the methods of classification of the mother wavelet is recently used. There are several methods used of which we emphasize the most used: Shannon entropy [18], maximum energy to entropy ration [19], maximum cross correlation coefficient [20], distribution error [21], minimum description length [22], variances [23], genetic algorithm [24], energy of the wavelet coefficients [25], vanishing moment, shift variance and regularity [26].

The methods of selection of mother wavelet based on qualitative and quantitative approaches have been proposed and used in different fields, such as: analyzing power system transients, biomedical, engineering acoustic emission, denoise vibration signals, machines rotation fault detection, compression of power distribution data, partial discharge signal detection and extraction, denoising of ECG signal, image denoising, automatic ultrasonic non-destructive foreign body detection and classification, etc.

In principle, the methods presented for selecting mother wavelet consider that the wavelet coefficients reflect the similarity between the signal and the corresponding mother wavelet. The conclusion of the study is that certain mother wavelets lead to some performance results in the analysis of cracks in the rotating shafts.

### 4. Results

In order to evaluate the presence of cracks in the rotation shaft, the following steps are gone in this paper, using the stand presented in paragraph 1, as follows:

1. Shaft preparation in four operating situations: healthy shaft, crack simulated in the incipient phase of 16%, intermediate phase of 33% and an advanced phase of 66%;

2. Measurement of signals in two bearings, in vertical and horizontal direction at each bearing, at three operating speeds 600 rpm, 1200 rpm and 2400 rpm of the shaft, using Bruel & Kjaer equipment;

3. The processing of the signals recorded with DWT, using five mother wavelets from MATLAB, for decomposition and calculation of the energy associated with each level of decomposition, used to accurately diagnose cracks in the rotation shaft.

In general, it is heavy to find recommendations in the literature as to which function is the best for such analyses, but the change of the wavelet functions will certainly have an impact on the energy distribution of the decomposed signal energy. The signal is characterized by energy, which represents a quantitative measure of it. The signal energy can be calculated by the coefficients of the signal wavelet transform for each frequency band (6). The energetic content of each frequency band of the signal is a characteristic of the state of the rotation shaft, because it is directly related to the presence of the crack or not.

In this paper, the energy characteristics associated with the different levels of the frequency components of the signals, realized with DWT, accept modifications on the respective frequency bands, where the characteristics of the signal are concentrated. If there are increases in a frequency component of the signal corresponding to a certain scale, the corresponding wave coefficients extracted from the signal will have larger values and the energy content will be higher. This is evident from table 1.

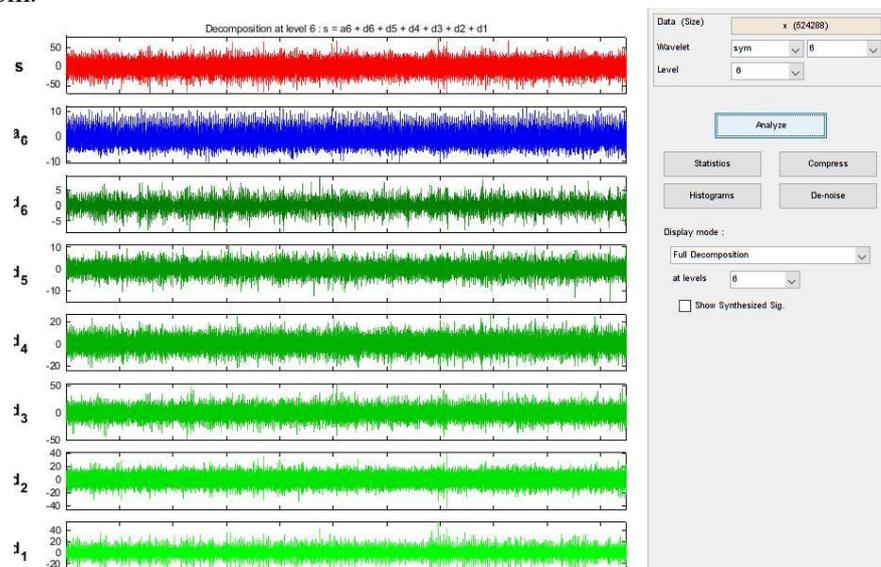
**Table 1.** Energy distribution by decomposition levels, using Haar, Db, Sym, Coif, Bior wavelets, bearing 1, vertical measurement direction, 2400 rpm

Wavelet	Healthy	Crack 66%
Sym2 (level 6)	Ea = 2.7079 Ed = 18.8891 28.0772 30.2439 13.6161 4.7488 1.7170	Ea = 11.7145 Ed = 19.0446 19.2770 30.1308 13.9261 3.5093 2.3977
Sym3 (level 6)	Ea = 2.6487 Ed = 17.6535 28.5610 31.0429 13.9831 4.5086 1.6022	Ea = 11.9811 Ed = 18.5386 18.5100 31.5497 13.9310 3.2818 2.2079
Sym4 (level 6)	Ea = 2.6163 Ed = 16.9198 29.0814 31.3643 14.1335 4.2879 1.5969	Ea = 12.0171 Ed = 18.4562 18.0231 32.4325 13.7546 3.1743 2.1421
Sym5 (level 6)	Ea = 2.6182 Ed = 16.5586 29.3849 31.4356 14.2898 4.0781 1.6349	Ea = 12.1331 Ed = 18.3644 17.7294 33.1277 13.4184 3.1362 2.0908
Sym6 (level 6)	Ea = 2.6041 Ed = 16.3454 29.4733 31.6273 14.3110 4.0257 1.6132	Ea = 12.1909 Ed = 18.3421 17.4739 33.4797 13.3995 3.0536 2.0603
Sym7 (level 6)	Ea = 2.6000 Ed = 16.2413 29.6162 31.6643 14.3178 3.8756 1.6848	Ea = 12.1704 Ed = 18.2755 17.5320 33.9135 13.0047 3.0315 2.0724
Sym8 (level 6)	Ea = 2.5954 Ed = 16.1006 29.8728 31.5526 14.3849 3.8634 1.6304	Ea = 12.2538 Ed = 18.3013 17.4918 33.9989 12.9569 2.9858 2.0115
Haar (level 6)	Ea = 2.8763 Ed = 22.4270 27.9252 27.7888 12.0087 5.0934 1.8805	Ea = 10.8458 Ed = 21.2853 21.2244 26.7027 12.8018 4.0479 3.0921
Db2 (level 6)	Ea = 2.7079 Ed = 18.8891 28.0772 30.2439 13.6161 4.7488 1.7170	Ea = 11.7145 Ed = 19.0446 19.2770 30.1308 13.9261 3.5093 2.3977
Db3 (level 6)	Ea = 2.6487 Ed = 17.6535 28.5610 31.0429 13.9831 4.5086 1.6022	Ea = 11.981 Ed = 18.5386 18.5100 31.5497 13.9310 3.2818 2.2079
Db4 (level 6)	Ea = 2.6312 Ed = 17.0050 29.0207 31.1952 14.2044 4.2723 1.6713	Ea = 12.0753 Ed = 18.3832 17.9981 32.4051 13.8200 3.1473 2.1710

Db5 (level 6)	Ea = 2.6182 Ed = 16.5992 29.2442 31.5817 14.2252 4.0673 1.6643	Ea = 12.1062 Ed = 18.3537 17.6408 33.1678 13.4723 3.1347 2.1245
Db 6 (level 6)	Ea = 2.6125 Ed = 16.3415 29.4923 31.5795 14.3451 4.0271 1.6020	Ea = 12.1262 Ed = 18.3552 17.5744 33.4501 13.3592 3.0736 2.0613
Db7 (level 6)	Ea = 2.6103 Ed = 16.1893 29.7503 31.6009 14.2805 3.9221 1.6467	Ea = 12.0975 Ed = 18.3373 17.5561 33.8881 13.0827 3.0026 2.0357
Db8 (level 6)	Ea = 2.5974 Ed = 16.1059 29.8280 31.5970 14.4199 3.7676 1.6843	Ea = 12.0701 Ed = 18.3055 17.4079 34.1135 12.9922 3.0624 2.0483
Db9 (level 6)	Ea = 2.5865 Ed = 16.0512 29.8600 31.7221 14.3762 3.7726 1.6313	Ea = 12.1195 Ed = 18.2782 17.4535 34.4691 12.6659 2.9861 2.0278
Db10 (level 6)	Ea = 2.5830 Ed = 16.0002 30.0043 31.6229 14.4052 3.7567 1.6278	Ea = 12.2073 Ed = 18.2660 17.4830 34.3695 12.7473 2.9526 1.9742
Coif1 (level 6)	Ea = 2.6951 Ed = 18.7093 28.2940 30.3274 13.5747 4.7645 1.6351	Ea = 11.7781 Ed = 19.0324 19.2910 30.2226 13.8274 3.4790 2.3696
Coif2 (level 6)	Ea = 2.6265 Ed = 16.8355 29.1516 31.2626 14.2299 4.2806 1.6131	Ea = 12.1645 Ed = 18.4091 17.9152 32.4844 13.7639 3.1387 2.1243
Coif3 (level 6)	Ea = 2.6155 Ed = 16.2854 29.6480 31.5778 14.2526 3.9729 1.6478	Ea = 12.1091 Ed = 18.3413 17.5825 33.6942 13.1909 3.0289 2.0531
Coif4 (level 6)	Ea = 2.5793 Ed = 16.0601 29.9294 31.5725 14.4383 3.7441 1.6762	Ea = 12.1405 Ed = 18.3073 17.4734 34.1941 12.8409 3.0359 2.0080
Coif5 (level 6)	Ea = 2.5846 Ed = 15.9497 30.0840 31.6566 14.4274 3.6123 1.6854	Ea = 12.2063 Ed = 18.2766 17.4182 34.6974 12.4292 2.9918 1.9805
Bior1.1 (level 6)	Ea = 2.8763 Ed = 22.4270 27.9252 27.7888 12.0087 5.0934 1.8805	Ea = 10.8458 Ed = 21.2853 21.2244 26.7027 12.8018 4.0479 3.0921
Bior1.3 (level 6)	Ea = 2.4676 Ed = 20.3098 28.8525 29.6146 12.8597 4.2943 1.6014	Ea = 11.4090 Ed = 19.0369 20.7422 29.1584 13.2245 3.5579 2.8710
Bior1.5 (level 6)	Ea = 2.3487 Ed = 19.3415 29.5974 30.0767 13.0584 3.9933 1.5841	Ea = 11.4935 Ed = 18.1963 20.7581 30.4035 12.7909 3.4979 2.8597
Bior2.2 (level 6)	Ea = 3.5254 Ed = 9.2578 17.8310 38.2466 17.0815 10.1610 3.8968	Ea = 12.9553 Ed = 9.6586 11.6838 32.7780 22.5184 6.8170 3.5888
Bior2.4 (level 6)	Ea = 2.7325 Ed = 9.8973 19.9655 40.4889 16.8104 7.6781 2.4272	Ea = 13.4341 Ed = 10.0476 11.9521 34.6989 22.1663 5.2766 2.4245
Bior2.6 (level 6)	Ea = 2.6057 Ed = 9.9947 21.0594 40.7155 16.5171 6.8683 2.2394	Ea = 13.7706 Ed = 10.1495 12.3380 35.4544 21.0431 4.9447 2.2997
Bior2.8 (level 6)	Ea = 2.5545 Ed = 9.9722 21.5181 41.0182 16.2563 6.4467 2.2341	Ea = 13.9037 Ed = 10.2072 12.5424 36.4274 19.8399 4.7986 2.2809
Bior3.1 (level 6)	Ea = 35.8338	Ea = 37.0660

	Ed = 1.0048 2.6320 14.0626 8.0576 20.5449 17.8643	Ed = 1.2551 2.2293 13.1754 12.0532 18.0481 16.1730
Bior3.3 (level 6)	Ea = 6.0304 Ed = 3.5497 8.8406 38.9334 17.3025 16.8710 8.4725	Ea = 11.9876 Ed = 3.9817 5.7563 32.0715 28.8763 11.1152 6.2113
Bior3.5 (level 6)	Ea = 2.8396 Ed = 4.6610 11.7934 46.4766 18.3662 11.8726 3.9905	Ea = 12.0382 Ed = 4.8219 6.5766 35.7598 30.0214 7.8267 2.9553
Bior3.7 (level 6)	Ea = 2.4452 Ed = 5.0521 13.0203 48.5583 17.7560 10.1086 3.0595	Ea = 12.5551 Ed = 5.0973 6.8491 37.2344 28.8436 7.0911 2.3294
Bior3.9 (level 6)	Ea = 2.3902 Ed = 5.2336 13.7792 48.9849 17.4526 9.3121 2.8473	Ea = 12.9511 Ed = 5.2637 7.1145 37.6616 27.8921 6.8874 2.2295
Bior4.4 (level 6)	Ea = 2.6486 Ed = 17.4232 25.7044 33.5425 14.1132 4.8931 1.6749	Ea = 11.4648 Ed = 19.6150 15.6994 32.0212 15.8176 3.4687 1.9134
Bior5.5 (level 6)	Ea = 2.5251 Ed = 27.4818 29.3294 25.4712 11.0393 3.0088 1.1444	Ea = 9.1910 Ed = 32.1011 18.1207 26.7523 10.2022 2.1549 1.4777
Bior6.8 (level 6)	Ea = 2.6429 Ed = 15.6140 26.6300 34.0241 14.7497 4.5592 1.7802	Ea = 12.6835 Ed = 17.6980 15.3398 33.9881 14.8620 3.4332 1.9955

Table 1 shows the percentage distribution of energy on different frequency bands, using 37 mother wavelets (haar, db2-db10, coif1-coif5, sym2-sym8 and bior1.1-bior6.8). Of all the analyzed cases, in the paper, the following operating conditions were considered: the accelerometer is positioned in the vertical direction in bearing one (figure 1), state the healthy shaft and with 66% crack, the shaft speed being 2400 rpm.

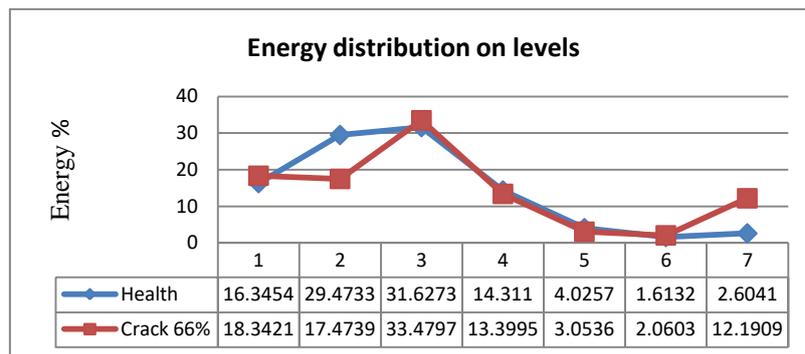


**Figure 3.** DWT for the discrete signal with 524288 samples, using the *sym6* wavelet function (levels 6)

The recorded signals were decomposed on six levels using DWT, for which the approximation coefficients  $a_6$  and the detail coefficients  $d_1, d_2, d_3, d_4, d_5$  and  $d_6$  were obtained. Using the *wenergy* function from MATLAB to determine the energy distribution on each level (frequency band) of the

signal, the presence of cracks in the rotation shaft was identified. The distribution of energy levels using *sym6* wavelet function is represented in figure 3.

In all cases of use of orthogonal mother wavelet functions, for the intact tree, the signal energy is concentrated on the high frequency decomposition levels  $E_{d1}$ ,  $E_{d2}$  and  $E_{d3}$  (approximately 75%). It is also observed that the energy, for cracked shafts, is redistributed on lower levels of signal decomposition ( $E_a$  reaches from 2% to 12%). From the use of orthogonal mother wavelets it can be seen that some variations between the energy levels are same (Ed2 reaches from 29% to 17% for cracked shaft), which means that they are recommended for diagnosis of cracks in the rotation shafts (except Haar wavelet) (figure 4).



**Figure 4.** Energy distribution diagram of healthy shaft and crack shaft using *sym6* wavelet

Of the biorthogonal wavelet functions only *bior4.4* and *bior6.8* have redistributions similar to the orthogonal functions, in addition the others have decreasing variations on the 3rd frequency band ( $E_{d3}$ ), and all have an increase on the low frequency band  $E_a$ .

In tables 2 and 3 are presented examples looking influence on the accuracy of the results, of the measuring point position (bearing 1 or bearing 2), measurement direction (vertically or horizontal), the condition of the shaft with the intermediate cracks (16%, 33% and 66 %) and different speeds for the shaft (600 rpm, 1200 rpm and 2400rpm).

**Table 2.**

Vertically direction			
Sym6 (level 6)	Bearing 1 (channel 2)	Bearing 2 (channel 3)	RPM
Healthy	Ea = 0.5243 Ed = 12.4836 28.3266 21.0977 19.8910 15.5373 2.1395	Ea = 0.0667 Ed = 30.3147 53.6964 5.0422 4.8233 5.5123 0.5444	600rpm
	Ea = 1.1715 Ed = 14.9350 32.2342 30.7813 14.2274 5.2754 1.3752	Ea = 0.1782 Ed = 42.7567 48.4268 4.8395 2.2909 1.1316 0.3764	1200rpm
	Ea = 2.6041 Ed = 16.3454 29.4733 31.6273 14.3110 4.0257 1.6132	Ea = 0.1256 Ed = 45.9964 47.3796 4.5115 1.1488 0.5277 0.3104	2400rpm
Crack 16%	Ea = 0.3424 Ed = 10.5937 23.6031 33.2985 15.8923 14.2437 2.0263	Ea = 0.0550 Ed = 37.8038 50.0591 6.8680 2.3798 2.5304 0.3039	600rpm
	Ea = 0.8996 Ed = 11.4858 29.6356 33.6638 15.7774 7.7186 0.8192	Ea = 0.1186 Ed = 38.4436 50.6974 5.9324 2.8681 1.7530 0.1869	1200rpm
	Ea = 3.6087 Ed = 13.9961 27.3231 33.2146 15.5599 4.5317 1.7659	Ea = 0.1448 Ed = 43.0245 50.2342 4.8481 1.0464 0.4955 0.2065	2400rpm
Crack 33%	Ea = 0.3794	Ea = 0.0434	600rpm

	Ed = 13.4981 31.2572 30.6373 13.0282 9.7596 1.4403	Ed = 33.2259 54.8752 6.6820 2.3716 2.5217 0.2803	
	Ea = 1.9432 Ed = 12.3036 32.6857 32.8658 13.1559 6.1981 0.8477	Ea = 0.1969 Ed = 37.3893 51.4728 6.3834 2.7296 1.6199 0.2081	1200rpm
	Ea = 6.8933 Ed = 12.4246 29.6137 32.9345 12.6907 3.6927 1.7505	Ea = 0.2392 Ed = 38.4749 54.0475 5.5387 1.0763 0.4508 0.1726	2400rpm
<b>Crack 66%</b>	Ea = 0.8781 Ed = 16.4028 24.0940 31.8196 12.8958 11.9284 1.9815	Ea = 0.0740 Ed = 30.4103 55.0416 6.1347 3.7235 4.2223 0.3936	600rpm
	Ea = 3.6406 Ed = 16.7156 29.2970 34.2937 11.0638 4.0814 0.9079	Ea = 0.3049 Ed = 37.2893 52.4191 5.8979 2.5030 1.3808 0.2051	1200rpm
	Ea = 12.1909 Ed = 18.3421 17.4739 33.4797 13.3995 3.0536 2.0603	Ea = 1.0137 Ed = 36.0831 55.7215 5.2467 1.0208 0.5540 0.3602	2400rpm

**Table 3.**

Horizontal direction			
Sym6 (level 6)	Bearing 1 (channel 2)	Bearing 2 (channel 3)	RPM
Healthy	Ea = 8.7733 Ed = 13.0208 26.5098 22.8175 16.8328 9.9481 2.0977	Ea = 0.4359 Ed = 27.8674 36.9514 20.2941 8.4622 5.3505 0.6386	600rpm
	Ea = 11.8496 Ed = 13.7266 32.7663 23.6637 12.7319 3.5608 1.7012	Ea = 1.4286 Ed = 29.5587 36.2749 20.5808 8.2299 2.9412 0.9859	1200rpm
	Ea = 12.1909 Ed = 18.3421 17.4739 33.4797 13.3995 3.0536 2.0603	Ea = 0.5063 Ed = 33.1121 44.7731 17.9015 2.6264 0.6095 0.4710	2400rpm
<b>Crack 16%</b>	Ea = 7.2992 Ed = 12.1625 29.7926 23.6622 16.3115 8.5724 2.1995	Ea = 0.3670 Ed = 28.1670 37.0997 21.2007 8.1167 4.3811 0.6679	600rpm
	Ea = 12.8710 Ed = 12.8900 32.6105 23.9067 12.0031 3.8011 1.9177	Ea = 1.9613 Ed = 28.7822 36.4418 20.3066 8.0582 3.1456 1.3044	1200rpm
	Ea = 25.6363 Ed = 11.6085 28.6883 20.7118 9.8345 1.6398 1.8808	Ea = 0.4944 Ed = 33.5584 44.2654 17.4796 2.9462 0.7496 0.5066	2400rpm
<b>Crack 33%</b>	Ea = 9.2147 Ed = 11.2746 26.1303 22.4714 15.8464 12.4215 2.6410	Ea = 0.4454 Ed = 29.5364 37.3886 17.2337 8.4573 6.1647 0.7738	600rpm
	Ea = 12.6210 Ed = 12.1086 33.0525 24.2551 11.6056 4.5956 1.7616	Ea = 1.6281 Ed = 31.3377 36.0611 19.3172 7.3527 3.0807 1.2225	1200rpm
	Ea = 21.0275 Ed = 11.1743 29.4365 23.6531 10.1344 2.5192 2.0550	Ea = 0.5290 Ed = 31.6633 45.2728 18.3519 2.9206 0.7400 0.5223	2400rpm
<b>Crack 66%</b>	Ea = 8.4105 Ed = 12.7789 28.2805 22.1689 16.4680 9.5656 2.3276	Ea = 0.4073 Ed = 26.7722 39.2487 21.0220 7.2478 4.6577 0.6444	600rpm
	Ea = 14.8772 Ed = 11.9306 32.7154 21.7701 11.5244 4.7340 2.4484	Ea = 2.4103 Ed = 27.3860 36.8452 19.1663 8.3580 3.9476 1.8865	1200rpm

	Ea = 28.6831 Ed = 15.5365 20.0448 19.6481 9.4085 2.7243 3.9547	Ea = 1.6446 Ed = 30.8186 43.2380 18.6814 3.2102 0.9662 1.4411	2400rpm
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With the increase of the speed from 600 rpm to 2400 rpm, the energy of the band corresponding to the defect in the shaft (crack) increases, due to the increase of the coefficients of the signal transformation at the frequency bands specific to the defect. Because the energy content in the frequency bands is a rough characteristic of the signal, it is not sensitive in the early stages of the crack in the shaft (see the 16% stage).

The energy characteristic of the crack frequency is influenced by factors such as shaft speed, location of measurement sensors, shaft status, which in turn will influence the quality of the analysis.

## 5. Discussions and conclusions

In the literature there are few recommendations for the selection of mother wavelet necessary for a more accurate analysis of phenomena. Signals with sharp changes are better analyzed using irregular wavelet functions, which are responsible for local analyse (Haar, Daubechies, Coiflet, Symlet, Biorthogonal). Of these, the optimal wavelet mother for signal analysis is the one with the highest energy at the highest decomposition level. The energy distributed on each frequency band of the signal, used as a criterion for choosing mother wavelet for the classification of cracks in rotating shafts also depends on the level of decomposition. Due to the largest energy variations in the detection of cracks in shaft, Daubechies, Coiflet, Symlet, Biorthogonal4.4 and Biorthogonal6.8 mother wavelet are the most efficient.

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