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Optimal design for a cylindrical gear with inclined teeth

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Abstract. Designing is an act of creation, which is a technical activity carried out for productive purpose, and which aims to provide all the data necessary for the implementation of correlated material and financial means, a theme or an idea in practice.
Depending on the operating conditions of the machines and their technological destination, optimum reliability, maintainability and ergonomics must be ensured from the design stage.
For some areas where toothed gears are used, such as the aerospace industry, the automotive construction etc., reduced gear mass (volume) can be an important parameter to become an optimization criterion.
This paper aims to study the gearing mass and proposes to minimize this function.

$$M_{gearing} = V_{gearing} \cdot \rho_{material} \rightarrow min$$

1. Introduction

The gear mechanism is made up of two gears, which are sequentially transmitted through the teeth and continuously contacting (engaging) - rotating movement and torque between the two shafts.

Gears are used to transmit the rotation movement from the drive shaft to the other driven, achieving a constant transmission ratio between speeds. Transmission of movement is always accompanied by the transmission of torque, which is a mechanical work, so a power.

A gear is made up of a pair of gears, one driving and the other driven. Relative sliding of the surfaces in contact is excluded because the movement is not transmitted by the frictional force but by a pushing force between the teeth.

Gears have a wide use in mechanical transmissions, due to their advantages: constant transmission ratio; safety in operation; high durability; high efficiency; reduced size; the possibility of using for a wide range of powers, speeds and transmission reports. As drawbacks, there can be mentioned: high execution and mounting precision; complicated technology; noise and vibration in operation.

In the modern construction of machines and appliances, gearing is the most important and most used mechanism. The construction of a car like that of a lathe contains dozens of gears.

Properly groomed and properly assembled can guarantee safe operation at speeds and reduced power up to thousands of kilowatts of power and at high speeds up to 100-150 m / s. At present, toothed wheels with dimensions between fractions of millimeters up to 10 m in diameter can be built.

When selecting the material, a number of factors must be taken into account: the load that loads the gear; the required service life; the mechanical characteristics of the materials; how to obtain the semi-finished product; execution technology; economic efficiency; operating conditions.

2. The optimisation problem

This paper is a study on the mass of a gear speed reducer single stage and the objective function is to minimize this value.

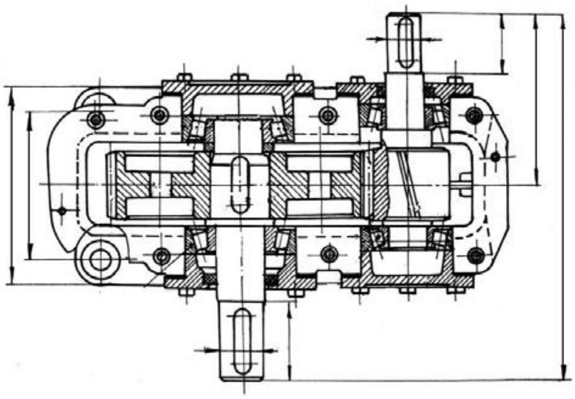

Figure 1 - Speed reducer single stage

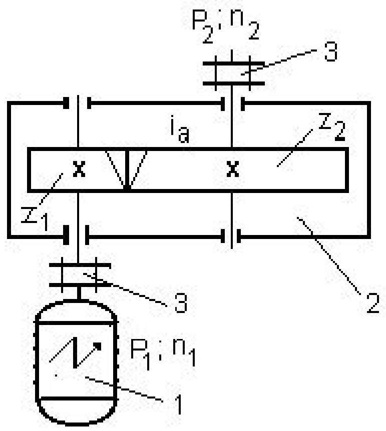

Figure 2 – Kinematic scheme of the reducer

It is proposed to design a cylindrical gear with inclined teeth having the following input data:

- Power of the second shaft: $P_2 = 24\ kW$
- Total gear ratio: $i_R = 2$
- Speed of the second shaft: $n_2 = 1000\ rot/min$
- The mean time in service between two successive repairs: $L_h = 9000\ hours$
- Number of wheels in contact with the wheel gear: $\chi_{1,2} = 1$
- Difference in wheel width: $\Delta_b = 5\ mm$
- Toothed wheel materials:
  - Pinion: 41MoCr11 - improved steel: $HB_1 = 2800\ MPa$
  - Gear: 40Cr10 – improved steel: $HB_2 = 2500\ MPa$
  - Density materials: $\rho_{max} = 7{,}85 \cdot 10^{-6}\ kg/mm^3$

- Tensions limit for material gears:
  - $\sigma_{H\,\lim 1} = 720\ MPa$
  - $\sigma_{H\,\lim 2} = 675\ MPa$
  - $\sigma_{F\,\lim 1} = 460\ MPa$
  - $\sigma_{F\,\lim 2} = 445\ MPa$
- Safety factor for contact stress:
  - $S_H = 1{,}25$
  - $S_F = 1{,}5$
- Flank hardness factor: $Z_w = 1$
- Elasticity factor of the wheel material: $Z_E = 189{,}8\ MPa^{1/2}$
- Material factor: $Z_M = 271\ N/mm^2$
- Precision class (tolerance grade): 7 toothing milling with the snail cutter and grinding
- Rack reference: ISO 53;
- Profile of generating rack:
  - Pressure angle normal reference plane: $\alpha_{n=}20^{\circ}$
  - Tooth head height factor: $h_{0a}^{*} = 1$
  - Match factor at the head of the reference tooth: $c^{*} = 0{,}25$
- Factor of operating mode: $K_A = 1$
- Lubricant type: TIN 125 EP with kinematic viscosity 125÷140 mm^2/s at $50^{\circ}C$
- Overloading coefficient $c_s = 1$

3. Optimisation problem genes

In the broad sense, optimization means the action of determining, on the basis of a predetermined criterion, the best decision in a given situation where more than one decision is possible, as well as the action to implement the established decision as well as its outcome.

Narrow optimization simply means action establishing the right choice (solution) called Decision optimal (optimal solution).

Five variables are considered to optimize the problem.

- Variable 1 - $\boldsymbol{a_w}$ – the center distance, axial distance values are those standardized in the field 40÷315 mm
- Variable 2 - $\boldsymbol{\psi_a}$ – the coefficient ratio between the width and the axial distance (variable real continuous), taking values in the range of 0,1 ÷0,6
- Variable 3 - $\boldsymbol{\beta}$ – the inclination helix angle on the pitch cylinder (variable real continuous), with value in the field 7,25° ÷ 15°
- Variable 4 - $\boldsymbol{z_1}$ – the number of pinion teeth (full variable),with values in range 24 ÷ 50
- Variable 5 - $\boldsymbol{x_s}$ – the profile displacement coefficient sum for both gears (real continuos variable), having values in the range -0,5 ÷ +1,1

4. Calculate the amount necessary for describing the problem of optimization

Taking into account the inputs and variables mentioned above, it is necessary to go through a series of steps to determine the essential dimensions for describing the objective function and the optimization problem constraints.

4.1. Determining the power of the electric motor

$$P_c = \frac{P_2}{\eta_{tot}} = \frac{24}{0{,}949} = 25{,}29\ kW$$

$\eta_{tot} = \eta_{12} \cdot \eta_b^2 \cdot \eta_l = 0{,}97 \cdot 0{,}994^2 \cdot 0{,}99 = 0{,}949$

Where:
η_{tot} – total efficiency of the drive mechanism
$\eta_{12} = 0{,}97$ - efficiency of the gears
$\eta_b = 0{,}994$ - a pair of bearings efficiency:
$\eta_l = 0{,}99$ - the lubrification efficiency:

4.2. Choosing electric motor
From STAS we choose the EM type 225M-60-6 with :

- → The nominal power: $P_{EM} = 30kW > 25{,}29kW$
- → The loaded speed: $n_1 = 2870 rot/min > n_{12} = n_2 \cdot i_R = 1000 \cdot 2 = 2000\, rot/min$

4.3. Determination of the torque moments of the shafts
The moments developed for the two shafts of the reducer are:

$$M_{t1} = \frac{30 \cdot P_1}{\pi \cdot n_1} \cdot 10^6 = 99821 Nmm$$
$$M_{t2} = \frac{30 \cdot P_2}{\pi \cdot n_2} \cdot 10^6 = 229299 Nmm$$

4.4. Calculating the normal module, the distance between the axes and the number of teeth

- ⇨ The distance between axes $\boldsymbol{a}$

$$a \geq (1+u)\sqrt[3]{\frac{K_A \cdot K_V \cdot K_{H\beta} \cdot M_{t2}}{2 \cdot u \cdot \psi_a} \cdot \left(\frac{Z_M \cdot Z_H \cdot Z_\varepsilon}{\frac{\sigma_{H\,lim}}{S_H} \cdot K_{HN} \cdot Z_R \cdot Z_W}\right)^2} = 122{,}761\, mm$$

Where:
$u = i_R = 2$
The center distance is standardizedand is given in table 1 and we'll choose the next superior value but whether the calculated value is les than 5% than the lower one, we may choose the lower one.

Table 1 – Standard center distances [mm]

I	II	I	II	I	II	I	II	I	II	I	II
40	40 45	63	63 71	100	100 112	160	160 180	250	250 280	400	400 450
50	50 56	80	80 90	125	125 140	200	200 225	315	315 355	500	500 560

Table 1 shows the standard values for axle spacing between cylindrical and worm gears. The values of the I string are preferential.

We'll take $a_w = 125\, mm$

- ⇨ The normal module $\boldsymbol{m_n}$

$$m_n \geq \frac{M_{t2} \cdot (1+u) \cdot K_A \cdot K_V \cdot K_\alpha \cdot K_{F\beta} \cdot Y_F \cdot Y_\beta}{\Psi_a \cdot a^2 \frac{\sigma_{F\,lim}}{S_F} \cdot K_{FN} \cdot Y_Z \cdot Y_{Fx}} = 0{,}927\, mm$$

If the value is under 1 mm then m_n is selected 1 mm. Thus $m_n = 1\, mm$

⇨ Establishing the number of teeth for the driving gear

The maximum number of teeth is calculated taking into account the center distance and the normal module

$$z_{1\,max} = \frac{2 \cdot a_w \cdot cos\beta}{m_n \cdot (1+u)} = \frac{2 \cdot 125 \cdot \cos(15°)}{1 \cdot (1+2)} = 80{,}49\ mm$$

$z_1 < z_{1\,max}$ => $z_1 = 30 \div 35$ if $z_{1\,max} = 45 \div 80$ and more

We'll choose $z_1 = 35$

4.5. The final selection of the normal module, the distance between the axes and the number of teeth

$$m_n = \frac{2 \cdot a_w \cdot cos\beta}{z_1 \cdot (1+u)} = \frac{2 \cdot 125 \cdot \cos(15°)}{35 \cdot (1+2)} = 2{,}30\ mm$$

The new module now is $m_n = 2{,}25$

With this new value one may recalculate the number of teeth:

$$z_1 = \frac{2 \cdot a_w \cdot cos\beta}{m_n \cdot (1+u)} = \frac{2 \cdot 125 \cdot \cos(15°)}{2{,}25 \cdot (1+2)} = 35{,}77$$

The new selected number is identical with the preliminary selected one $z_1 = 35$

With z_1 final we may calculate the number of teeth for the driven gear

$$z_2 = u \cdot z_1 = 2 \cdot 35 = 70$$

The rational for selection of z_2 is recommended that the number of teeth of both gears not to divide exactly (relative prime number).

We'll choose $z_2 = 71$

The reference center distance is recalculated

$$a_0 = \frac{m_n \cdot (z_1 + z_2)}{2 \cdot cos\beta} = \frac{2{,}25 \cdot (35+71)}{2 \cdot \cos(15°)} = 123{,}46\ mm$$

4.6. Geometrical elements calculation

► The frontal pitch pressure angle $\boldsymbol{\alpha_t}$

$$\alpha_t = arctg\left(\frac{tg\alpha_n}{cos\beta}\right) = arctg\left(\frac{tg(20°)}{\cos(15°)}\right) = 20{,}65°$$

► The pitch gearing normal angle $\boldsymbol{a_{wt}}$

$$\alpha_{wt} = arccos\left(\frac{a_0 \cdot cos\alpha_t}{a_w}\right) = 22{,}45°$$

► The profile displacement coefficient sum for both gears $\boldsymbol{x_s}$

$$x_s = x_1 + x_2 = (z_1 + z_2) \cdot \frac{inv(\alpha_{wt}) - inv(\alpha_t)}{2 \cdot tg(\alpha_{n)}} = 0{,}6931$$

Where:

$$inv(\alpha_{wt}) = tg(\alpha_{wt}) - \frac{\pi}{180} \cdot \alpha_{wt} = 0{,}0212$$

$$inv(\alpha_t) = tg(\alpha_t) - \frac{\pi}{180} \cdot \alpha_t = 0{,}0164$$

In order to distribute this sum on both gears we'll use the chart (Figure 3).

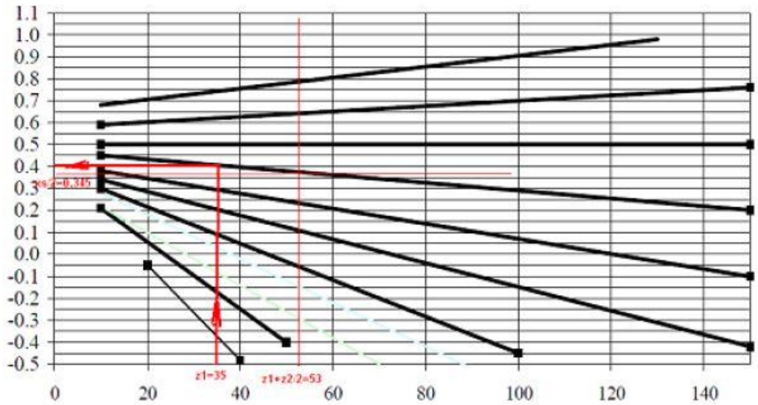

Figure 3 – Profile displacement distribution

From the chart we have $x_1 = 0{,}4$ so that $x_2 = 0{,}293$

- ► Pitch diameter $\boldsymbol{d_1};\ \boldsymbol{d_2}$

$$d_1 = \frac{m_n \cdot z_1}{cos\beta} = 81{,}515\ mm$$

$$d_2 = \frac{m_n \cdot z_2}{cos\beta} = 165{,}359\ mm$$

- ► Top land diameter $\boldsymbol{d_{a1}};\ \boldsymbol{d_{a2}}$

$$d_{a1} = d_1 + 2 \cdot h_{a1} = 87{,}815\ mm$$
$$d_{a2} = d_2 + 2 \cdot h_{a2} = 171{,}177\ mm$$

Where addendum is:

$h_{a1} = m_n \cdot (h_{0a}^* + x_1) = 3{,}15\ mm$

$h_{a2} = m_n \cdot (h_{0a}^* + x_2) = 2{,}909\ mm$

$h_{0a}^* = 1$

- ► Root diameter $\boldsymbol{d_{f1}};\boldsymbol{d_{f2}}$

$$d_{f1} = d_1 - 2 \cdot h_{f1} = 77{,}695\ mm$$
$$d_{f2} = d_2 - 2 \cdot h_{f2} = 161{,}053\ mm$$

Where dedendum is:

$h_{f1} = m_n \cdot \left(h_{0f}^* - x_1\right) = 1{,}91\ mm$

$h_{f2} = m_n \cdot \left(h_{0f}^* - x_2\right) = 2{,}153\ mm$

$h_{0f}^* = 1{,}25$

► Base circle diameter d_{b1} ; d_{b2}

$$d_{b1} = d_1 \cdot \cos(\alpha_t) = 76{,}277\ mm$$
$$d_{b2} = d_2 \cdot \cos(\alpha_t) = 154{,}735\ mm$$

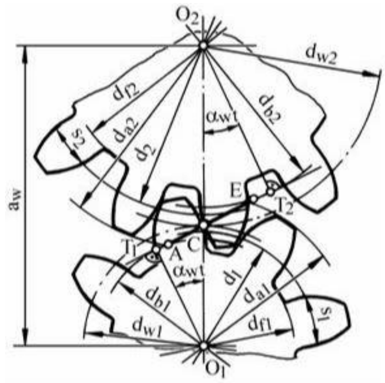

Figure 4 – Geometrical elements for gears

► Rolling diameter d_{w1}; d_{w2}

$$d_{w1} = d_1 \cdot \frac{cos(\alpha_t)}{cos(\alpha_{wt})} = 82{,}543\ mm$$
$$d_{w2} = d_2 \cdot \frac{cos(\alpha_t)}{cos(\alpha_{wt})} = 167{,}425\ mm$$

► Gear width b_1; b_2

$$b_2 = a_w \cdot \psi_a = 58\ mm$$
$$b_1 = b_2 + m_n = 63\ mm$$

4.7. Echivalent gear elements

- The echivalent gear teeth number z_{n1}; z_{n2}

$$z_{n1} = \frac{z_1}{cos\beta^3} = 38{,}83 \;=>\; z_{n1} = 39$$
$$z_{n2} = \frac{z_2}{cos\beta^3} = 78{,}78 \;=>\; z_{n2} = 79$$

- The equivalent gear pitch diameter d_{n1}; d_{n2}

$$d_{n1} = \frac{d_1}{cos^2\beta} = 87{,}367\ mm$$
$$d_{n2} = \frac{d_2}{cos^2\beta} = 177{,}231\ mm$$

- The equivalent top land diameter d_{an1}; d_{an2}

$$d_{an1} = d_{n1} + d_{a1} - d_1 = 93{,}667\ mm$$
$$d_{an2} = d_{n2} + d_{a2} - d_2 = 183{,}049\ mm$$

- The equivalent base circle diameter d_{bn1}; d_{bn2}

$$d_{bn1=}d_{n1} \cdot cos\alpha_n = 82{,}098\ mm$$
$$d_{bn2=}d_{n2} \cdot cos\alpha_n = 166{,}543\ mm$$

- The echivalent distance between axes, [mm]a_{wn}

$$a_{wn} \geq \frac{a_w}{cos\beta_b} \cdot \frac{cos\alpha_n}{cos\alpha_{wn}} = 110{,}045\ mm$$

Where:

$\beta_b = arctg\left(\frac{d_{b1}}{d1}tg\beta\right) = 14{,}07°$

$\beta_w = arctg\left(\frac{d_{w1}}{d1}tg\beta\right) = 15{,}18°$

$\alpha_{wn} = arccos\left(\frac{cos\alpha_{wt} \cdot cos\beta_b}{cos\beta_w}\right) = 21{,}78°$

From Table 1 we'll take $a_{wn} = 112\ mm$

- The equivalente normale module $\boldsymbol{m_{nn}}$

$$m_{nn} = \frac{2 \cdot a_{wn} \cdot cos\beta_w}{z_{n1} + z_{n2}} = 1{,}83\ mm$$

The new module now is $m_{nn} = 1{,}75$

5. Calculating the gear mass (volume)

The gearing weight is:

$$M_{gearing} = V_{gearing} \cdot \rho_{material}$$

Where:

$V_{gearing}$ – gear unit volume

$\rho_{material}$ – density of the gear wheel material: $\rho_{max} = 7{,}85 \cdot 10^{-6}\ kg/mm^3$

The gear unit volume is:

$$V_{gearing} = V_{z1} + V_{z2}$$

Where:

V_{z1} – pinion volume, $[mm^3]$

V_{z2} – gear volume, $[mm^3]$

We write in generalized form the relationships for volume determination using the index ***"i"*** *(i-1 for the pinion, i-2 for the gear)*. Based on this notation the volume of the cylindrical inclined teeth is:

$$V_{z\,i} = A_i \cdot b_i$$

Where:

A_i – area of the cylindrical front surface with inclined teeth, $[mm^2]$

b_i – width of cylindrical wheels with inclined teeth, [mm]

Area of the cylindrical front surface with inclined teeth is:

$$A_i = A_{disc\,i} + A_{z\,i} \cdot z_i$$

Where:

$A_{disc\,i}$ – front surface area of the disc

$A_{z\,i}$ – area of the cylindrical tooth front surface

$$A_{disc\,i} = \frac{\pi \cdot d_{f\,i}^2}{4}$$

Where:

$d_{f\,i}$ – root diameter of the cylindrical wheel

For calculating the area of the front surface of the cylindrical toothed wheel tooth, its surface has been divided into several circular sectors of known rays and angles (Figure 5).

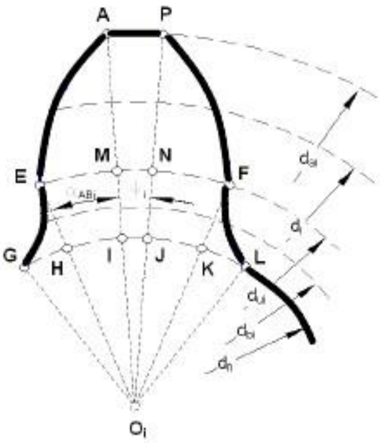

Figure 5 – Dividing the front surface of the gear wheel

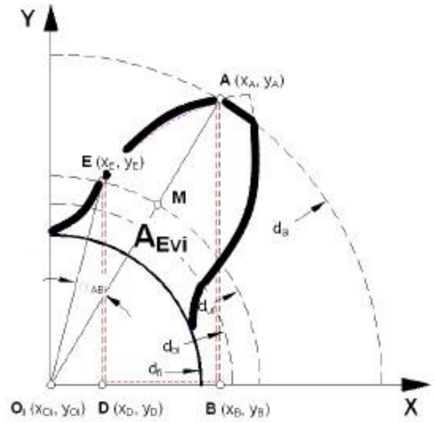

Figure 6 – Involute chart area

With notation in Figure 5, the front surface area can be written as follows:

$$A_{z\,i} = 2 \cdot A_{AEM\,i} + A_{AMNP\,i} + A_{EHKF\,i} + 2 \cdot A_{R\,i}$$

Where:
$A_{AEM\,i}$ – contour area defined by points A, E, M, $[mm^2]$
$A_{AMNP\,i}$ – contour area defined by points A, M, N, P, $[mm^2]$
$A_{EHKF\,i}$ – contour area defined by points E, H, K, F, $[mm^2]$
$A_{R\,i}$ – zone conection area (outline bounded by points E, G, H, $[mm^2]$

Contour area defined by points A, E and M is:

$$A_{AEM\,i} = A_{Oi\,EABD\,i} - A_{\Delta Oi\,BA} - A_{sect\,OiEM} = 3428{,}824\,mm^2$$

Contour area defined by points A, M, N and P is:

$$A_{AMNP\,i} = A_{sect\,OiAP} - A_{sect\,OiMN} = 2241{,}68\,mm^2$$

Contour area defined by points E, H, K and F is:

$$A_{EHKF\,i} = A_{sect\,OiEF} - A_{sect\,OiHK} = 3725{,}512\,mm^2$$

Zone conection area is:

$$A_{R\,i} = A_{\Delta OOiE} - A_{sect\,OiGH} - A_{sect\,OGE} = 2231{,}278\,mm^2$$

Area of the cylindrical front surface with inclined teeth is:

$$A_i = A_{disc\,i} + A_{z\,i} \cdot z_{i=} = 17287{,}396\,mm^2$$

The volume of the cylindrical inclined teeth is:

$$V_{z\,i} = A_i \cdot b_i = 1099910{,}57\,mm^3$$

The gearing weight is:

$$M_{gearing} = V_{gearing} \cdot \rho_{material} = 8{,}634\,kg$$

6. The objective of the optimization problem

For some areas where toothed gears are used, such as the aerospace industry, the automotive construction etc., reduced gear mass (volume) can be an important parameter to become an optimization criterion.

For this reazon it was considered as a function gear unit mass. We want to minimize this function.

$$M_{gearing} = V_{gearing} \cdot \rho_{material} \rightarrow min$$

The Mathcad 15 software was used to solve the optimal design problem. The values of the minimum mass variable are shown in Table 2.

Tabel 2 – Values of solution variables with minimum mass

Nr.	Variable	Symbol	Valoare
1	Distance between axes, [mm]	a_w	112
2	Coefficient of displacement in the normal plane for the pinion	x_s	1
3	Coefficient ratio between the width and the axial distance	ψ_a	0,45
4	Inclination helix angle on the pitch cylinder	β	14,07°
5	Gear teeth number	z_1	39

Table 3 presents a comparison of the main geometric elements of the gear in the two variants (classical and optimal respectively).

Tabel 3 - Comparison between the two variants of gears

Nr.	Characteristic	Classic version		Optimal version	
		pinion	wheel	pinion	wheel
1	Normal module, [mm]	2,25		1,75	
2	Distance between axes, [mm]	125		112	
3	Number of teeth of gears	35	71	39	79
4	Width gears, [mm]	63	58	67	62
5	Root diameter, [mm]	77,695	161,053	69,925	145,558
6	Pitch diameter, [mm]	81,515	165,359	73,363	148,823
7	Rolling diameter, [mm]	82,543	167,425	74,288	150,682
8	Top land diameter, [mm]	87,815	171,177	79,033	154,059
9	Base circle diameter, [mm]	76,277	154,735	67,124	140,808
10	Gearing weith, [kg]	8,634		7,403	

7. Conclusions

For over 20 years, Mathcad is the standard recognized performing, documenting and working collaboratively with engineering calculations, methods and algorithms in design.

Unlike spreadsheet programs, where equations are expressed cryptic and conversion between systems of different units is impossible, or programming languages, accessible mainly programmers, Mathcad is a much better perform and manage engineering calculations, they being easy to achieve, understood, verified, communicated and followed logically.

Based on the values in Table 3, it can be noticed that in the optimal variant the wheels of gearing are wider (coefficient of ratio between the width and the axial distance increased from 0,46 – 0,59) but with smaller diameters. The gearing weight decreased from 8,634 to 7,403 kg (which which represents a 14,25% decrease in mass) on the idea that the same conditions function for both variants.

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