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Considerations concerning the calculation of the reliability of deck systems

Paul Vasiliu[1]

[1]**Naval Academy "Mircea cel Batran" Constantza, Romania**

E-mail: p_vasiliu@yahoo.com

Abstract. As is well known, the calculation of the reliability of series and parallel systems is solved in the literature. To calculate the reliability of deck systems, methods based on the formula of Bayes are currently being used. In this paper we present a method of calculating the reliability of triangle-star systems and of transforming a triangle system into a star-type system that then leads to the calculation of the reliability of the deck systems.

Keywords: deck, reliability, parallel system, serial, star, triangle

1. Introduction

The calculation of a system's reliability is based mainly on the decomposition of the system into subsystems where the elements are connected in series or are connected in parallel.

The calculation of the reliability of the series structures and of the parallel structures is solved in the paper [1].

We consider the system $S = \{e_1, e_2, \cdots, e_n\}$ in Figure 1, where the elements $e_1, e_2, \cdots, e_n$ are connected in series and have the reliability respectively $R_1(t), R_2(t), \cdots, R_n(t)$. In the paper [1] it is demonstrated that the reliability of $R(t)$ of the serial system S is given by the expression:

$$R(t) = \prod_{i=1}^{n} R_i(t) \tag{1}$$

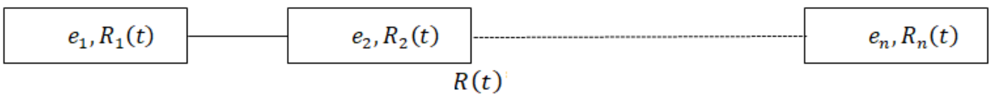

Figure 1. System with serial connection

We consider the system $S = \{e_1, e_2, \cdots, e_n\}$ in Figure 2, where the elements $e_1, e_2, \cdots, e_n$ are connected in parallel and have the reliability respectively $R_1(t), R_2(t), \cdots, R_n(t)$. In the paper [1] it is demonstrated that the reliability of $R(t)$ of the parallel system S is given by the expression:

$$R(t) = 1 - \prod_{i=1}^{n}\left(1 - R_i(t)\right) \tag{2}$$

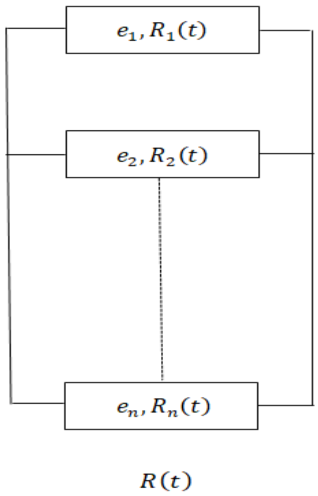

Figure 2. System with parallel connection

For calculating the reliability of systems where there are components that are not connected either in series or in parallel to the bridge structures, reliability can be calculated using probabilistic considerations such as the Bayes formula.

In this paper we present an algebraic method for calculating the reliability of systems in which the components are connected to the deck. The proposed method consists in transforming a system where the components are connected in a triangle in a system where the components are connected in the star. We will deduce the transformation formulas and then calculate the reliability of the system where the components are connected to the deck.

2. Transform triangle - star

We consider the system in Figure 3.

The system in Figure 3 is called a triangle connection system. Let us assume that the reliability of element e_1 is equal to $R_1(t)$, the reliability of element e_2 is equal to $R_2(t)$ and the reliability of element e_3 is equal to $R_3(t)$, $t \in [0, \infty)$.

To simplify exposure, enter the notations: R_1 for $R_1(t)$, R_2 for $R_2(t)$ and R_3 for $R_3(t)$.

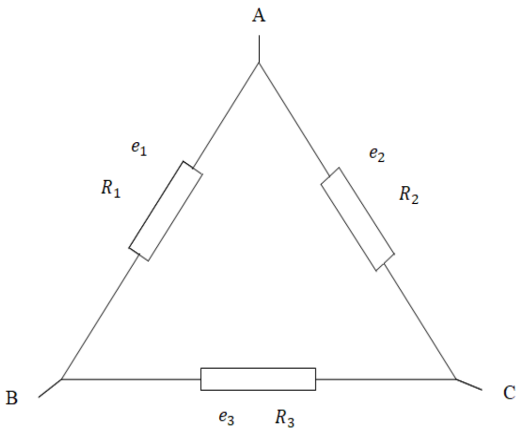

Figure 3. System with triangle connection

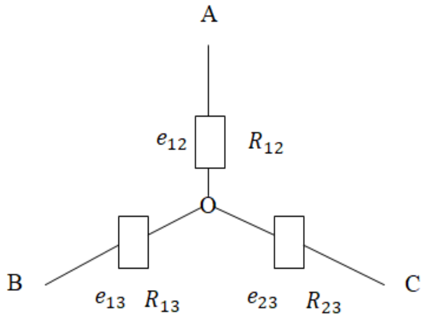

Figure 4. Starred system

The system in Figure 4 is called a star - connected system. Suppose that the reliability of element e_{12} is equal to $R_{12}(t)$, the reliability of element e_{13} is equal to $R_{13}(t)$ and the reliability of element e_{23} is equal to $R_{23}(t), t \in [0, \infty)$. To simplify exposure, enter the notations: R_{12} for $R_{12}(t)$, R_{13} for $R_{13}(t)$ and R_{23} for $R_{23}(t)$.

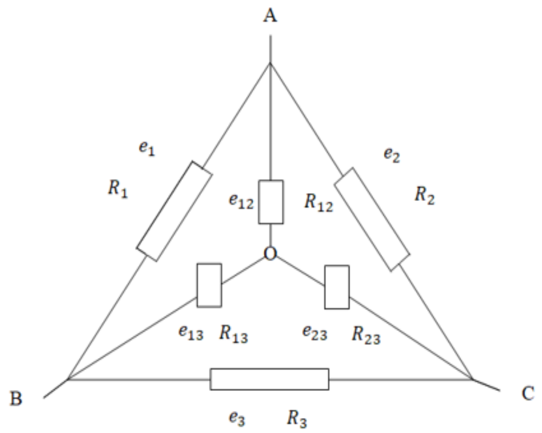

Figure 5. Triangle – star connection system

We will impose that the system with a triangle connection is equivalent to the star - connected system in the sense of the equivalence between points A and B of the triangle and between star points A, O and B, between points B and C in a triangle and between points B, O and C in the star and between points C and A in a triangle and between points A, O şi C in the star. Elements e_{12}, e_{13} şi e_{23} are the equivalents of elements e_1, e_2 şi e_3.

These assumptions ensure the equivalence of the system with triangle connection with the reliability of the star-connected system. Figure 5 shows both systems to highlight the triangle-to-star system transformation into a star-connected system. This transformation reduces the reliability of the deck system in calculating the reliability of serial and parallel systems.

Let $R_{BC}^{\triangle}$ the system reliability with a triangle connection between points B and C. Between these two points the elements e_1 and e_2 are connected in series and then in parallel with the element e_3. It follows that:

$$R_{BC}^{\triangle} = 1 - (1 - R_3) \cdot (1 - R_1 \cdot R_2) \tag{3}$$

Let $R_{BC}^{\curlywedge}$ the reliability of the star-to-point system between points B, O and C. Between these two points the elements e_{13} and e_{23} are connected in series and the element e_{12} is disconnected. It follows that:

$$R_{BC}^{\curlywedge} = R_{13} \cdot R_{23} \tag{4}$$

By condition $R_{BC}^{\curlywedge} = R_{BC}^{\triangle}$ the equation follows:

$$R_{13} \cdot R_{23} = 1 - (1 - R_3) \cdot (1 - R_1 \cdot R_2) \tag{5}$$

Let $R_{AB}^{\triangle}$ the system reliability with a triangle connection between points A and B. Between these two points the elements e_2 and e_3 are connected in series and then in parallel with the element e_1. It follows that:

$$R_{AB}^{\triangle} = 1 - (1 - R_1) \cdot (1 - R_2 \cdot R_3) \tag{6}$$

Let $R_{AB}^{\curlywedge}$ the reliability of the star-to-point system between points A, O and B. Between these two points the elements e_{12} and e_{13} are connected in series and the element e_{23} is disconnected. It follows that:

$$R_{AB}^{\curlywedge} = R_{12} \cdot R_{13} \tag{7}$$

By condition $R_{AB}^{\curlywedge} = R_{AB}^{\triangle}$ the equation follows:

$$R_{12} \cdot R_{13} = 1 - (1 - R_1) \cdot (1 - R_2 \cdot R_3) \tag{8}$$

Let $R_{AC}^{\triangle}$ the system reliability with a triangle connection between points A, O and C. Between these two points the elements e_1 şi e_3 are connected in series and then in parallel with the element e_2. It follows that:

$$R_{AC}^{\triangle} = 1 - (1 - R_2) \cdot (1 - R_1 \cdot R_3) \tag{9}$$

Let $R_{AC}^{\curlywedge}$ the reliability of the star-to-point system between points A, O and C. Between these two points the elements e_{12} and e_{23} are connected in series and the element e_{12} is disconnected. It follows that:

$$R_{AC}^{\curlywedge} = R_{12} \cdot R_{23} \tag{10}$$

By condition $R_{AC}^{\curlywedge} = R_{AC}^{\triangle}$ the equation follows:

$$R_{12} \cdot R_{23} = 1 - (1 - R_2) \cdot (1 - R_1 \cdot R_3) \tag{11}$$

The system of equations obtained from equations (5), (8) and (11) is solved in the unknowns R_{12}, R_{13} and R_{23}:

$$\begin{cases} R_{12} \cdot R_{13} = 1 - (1 - R_1) \cdot (1 - R_2 \cdot R_3) \\ R_{12} \cdot R_{23} = 1 - (1 - R_2) \cdot (1 - R_1 \cdot R_3) \\ R_{13} \cdot R_{23} = 1 - (1 - R_3) \cdot (1 - R_1 \cdot R_2) \end{cases} \tag{12}$$

To simplify exposure, we introduce substitutions:

$$a_1 = 1 - (1 - R_1) \cdot (1 - R_2 \cdot R_3) \tag{13}$$

$$a_2 = 1 - (1 - R_2) \cdot (1 - R_1 \cdot R_3) \tag{14}$$

$$a_3 = 1 - (1 - R_3) \cdot (1 - R_1 \cdot R_2) \tag{15}$$

With substitutions (13), (14) and (15) the system becomes:

$$\begin{cases} R_{12} \cdot R_{13} = a_1 \\ R_{12} \cdot R_{23} = a_2 \\ R_{13} \cdot R_{23} = a_3 \end{cases} \tag{16}$$

Calculate the member product with equations (16) and obtain the equation: $(R_{12} \cdot R_{13} \cdot R_{23})^2 = a_1 \cdot a_2 \cdot a_3$. Because $R_{12} \cdot R_{13} \cdot R_{23} > 0$ it follows that $R_{12} \cdot R_{13} \cdot R_{23} = \sqrt{a_1 \cdot a_2 \cdot a_3}$. Because $R_{12} \cdot R_{13} = a_1$ the equation $R_{12} \cdot R_{13} \cdot R_{23} = \sqrt{a_1 \cdot a_2 \cdot a_3}$ becomes:
$a_1 \cdot R_{23} = \sqrt{a_1 \cdot a_2 \cdot a_3}$ and hence it is obtained that:

$$R_{23} = \sqrt{\frac{a_2 \cdot a_3}{a_1}} = \sqrt{\frac{\left(1-(1-R_2)\cdot(1-R_1\cdot R_3)\right)\cdot\left(1-(1-R_3)\cdot(1-R_1\cdot R_2)\right)}{1-(1-R_1)\cdot(1-R_2\cdot R_3)}} \tag{17}$$

By analogy or using circular permutations the equations are obtained:

$$R_{13} = \sqrt{\frac{a_1 \cdot a_3}{a_2}} = \sqrt{\frac{\left(1-(1-R_1)\cdot(1-R_2\cdot R_3)\right)\cdot\left(1-(1-R_3)\cdot(1-R_1\cdot R_2)\right)}{1-(1-R_2)\cdot(1-R_1\cdot R_3)}} \tag{18}$$

$$R_{12} = \sqrt{\frac{a_1 \cdot a_2}{a_3}} = \sqrt{\frac{\left(1-(1-R_1)\cdot(1-R_2\cdot R_3)\right)\cdot\left(1-(1-R_2)\cdot(1-R_1\cdot R_3)\right)}{1-(1-R_3)\cdot(1-R_1\cdot R_2)}} \tag{19}$$

Relationships (17), (18) and (19) are relations of transformation of the system with triangle connection in the star connection system.

3. Reliability calculation of deck system

We consider the deck system of Figure 6 where the elements e_1, e_2, e_3, e_4 and e_5 have the reliability $R_1(t)$, $R_2(t)$, $R_3(t)$, $R_4(t)$ and respectively $R_5(t)$, $t \in [0, \infty)$.

To simplify exposure, we introduce substitutions: R_1 for $R_1(t)$, R_2 for $R_2(t)$, R_3 for $R_3(t)$, R_4 for $R_4(t)$ and R_5 for $R_5(t)$.

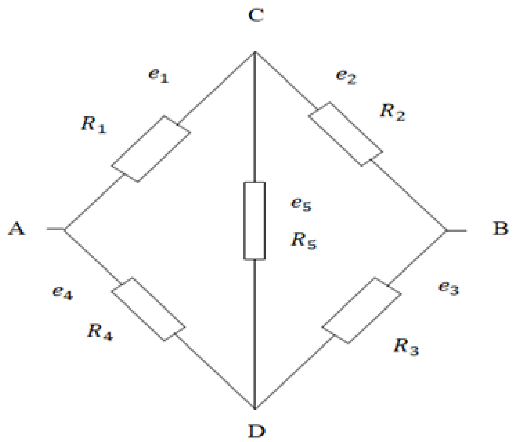

Figure 6. System with deck connection

We will calculate the reliability of the system in Figure 6 between points A and B. For this purpose, the ACD triangle connection is transformed into the $AOCD$ star connection.

The transformed system is shown in Figure 7.

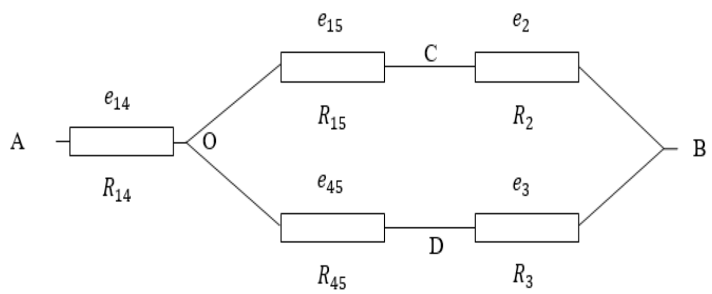

Figure 7. Transformed deck system

To simplify exposure, using relationships (13), (14) and (15), we introduce the substitutions:

$$a_1 = 1 - (1 - R_1) \cdot (1 - R_4 \cdot R_5) \quad (20)$$

$$a_4 = 1 - (1 - R_4) \cdot (1 - R_1 \cdot R_5) \quad (21)$$

$$a_5 = 1 - (1 - R_5) \cdot (1 - R_1 \cdot R_4) \quad (22)$$

Between points A and O the equivalent element is e_{14} with the reliability:

$$R_{14} = \sqrt{\frac{a_1 \cdot a_4}{a_5}} = \sqrt{\frac{(1-(1-R_1)\cdot(1-R_4\cdot R_5))\cdot(1-(1-R_4)\cdot(1-R_1\cdot R_5))}{1-(1-R_5)\cdot(1-R_1\cdot R_4)}} \quad (23)$$

Between points C and O the equivalent element is e_{15} with the reliability:

$$R_{15} = \sqrt{\frac{a_1 \cdot a_5}{a_4}} = \sqrt{\frac{(1-(1-R_1)\cdot(1-R_4\cdot R_5))\cdot(1-(1-R_5)\cdot(1-R_1\cdot R_4))}{1-(1-R_4)\cdot(1-R_1\cdot R_5)}} \quad (24)$$

Between points D and O the equivalent element is e_{45} with the reliability:

$$R_{45} = \sqrt{\frac{a_4 \cdot a_5}{a_1}} = \sqrt{\frac{(1-(1-R_4)\cdot(1-R_1\cdot R_5))\cdot(1-(1-R_5)\cdot(1-R_1\cdot R_4))}{1-(1-R_1)\cdot(1-R_4\cdot R_5)}} \quad (25)$$

Between points O, C and B the element e_{15} is connected in series with the element e_2. The reliability of the OCB side is equal to:

$$R_{OCB} = R_2 \cdot R_{15} \tag{26}$$

Between points O, D and B the element e_{45} is connected in series with the element e_3. The reliability of the ODB side is equal to:

$$R_{ODB} = R_3 \cdot R_{45} \tag{27}$$

The OCB and ODB sides are connected in parallel. The reliability of this parallel connection is equal to:

$$R_{OB} = 1 - (1 - R_{OCB}) \cdot (1 - R_{ODB}) \tag{28}$$

By replacing (26) and (27) in relation (28) the following is obtained:

$$R_{OB} = 1 - (1 - R_2 \cdot R_{15}) \cdot (1 - R_3 \cdot R_{45}) \tag{29}$$

Element e_{14} is connected in series with the equivalent element of the OB side. It follows that the reliability of the deck system between points A and B is equal to:

$$R_{AB} = R_{14} \cdot R_{OB} = R_{14} \cdot \left(1 - (1 - R_2 \cdot R_{15}) \cdot (1 - R_3 \cdot R_{45})\right) \tag{30}$$

By replacing (23), (24) and (25) in (30) we obtain the expression of the reliability of the system with a deck connection between points A and B:

$$R_{AB} = \sqrt{\frac{a_1 \cdot a_4}{a_5}} \cdot \left(1 - \left(1 - R_2 \cdot \sqrt{\frac{a_1 \cdot a_5}{a_4}}\right) \cdot \left(1 - R_3 \cdot \sqrt{\frac{a_4 \cdot a_5}{a_1}}\right)\right) =$$

$$\sqrt{\frac{\left(1-(1-R_1)\cdot(1-R_4\cdot R_5)\right)\cdot\left(1-(1-R_4)\cdot(1-R_1\cdot R_5)\right)}{1-(1-R_5)\cdot(1-R_1\cdot R_4)}} \cdot \left(1 - \left(1 - R_2 \cdot \sqrt{\frac{\left(1-(1-R_1)\cdot(1-R_4\cdot R_5)\right)\cdot\left(1-(1-R_5)\cdot(1-R_1\cdot R_4)\right)}{1-(1-R_4)\cdot(1-R_1\cdot R_5)}}\right) \cdot\right.$$

$$\left.\left(1 - R_3 \cdot \sqrt{\frac{\left(1-(1-R_4)\cdot(1-R_1\cdot R_5)\right)\cdot\left(1-(1-R_5)\cdot(1-R_1\cdot R_4)\right)}{1-(1-R_1)\cdot(1-R_4\cdot R_5)}}\right)\right) \tag{31}$$

4. Conclusions

In this paper we deduced the reliability calculating relationships of the equivalent elements obtained by transforming a system in which the components are connected in a triangle in an equivalent system in which the components are connected in the star (17), (18), (19).

We have then shown how the reliability of a bridge system can be reduced in calculating the reliability of serial and parallel systems by transforming the equivalent of a system where the components are connected in a triangle in a system where the components are connected in the star (20) ÷ (31).

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5. References

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