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# A parametric study for static deflections of symmetrically laminated rectangular thin plates

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**Abstract**. This study focuses on static deflection analysis of uniformly lateral loaded composite thin rectangular plate with uniformly lateral loads. Stacking sequence of symmetrically laminated quasi-isotropic plates has the sequence of the orientation angles -45°, 0°, +45°, 90°. Boundary conditions were selected as clamped and simply supported hinges at the edges of the plates. The Finite Difference Method (FDM) was used to determine the maximum deflection of the plates based on the governing differential equations of Classical Laminated Plate Theory (CLPT). Effect of the lamination type, boundary condition and aspect ratio (a/b, b/a) on maximum deflections was investigated parametrically, which are of importance in preliminary design phase of composite plates. Results obtained were compared with those of the Finite Element Method (FEM) received from literature. It was found out that the both groups of the results agree on.

#### 1. Introduction

Composite materials have been extensively preferred in marine engineering applications because of the high specific strength and high specific rigidity since few decades. Application of composites in commercial and military ships was given in Shenoi and Wellicome [1,2] and Mouritz et al. [3]. Ship structures consist of plates and shells supported with stiffener elements. Depending on the longitudinal or transversal positioning of stiffener elements, the construction type is named to either transversal or longitudinal supporting system.

It is not possible to solve the deflection problems in a plate by analytical methods except for some special cases. Due to any mathematical difficulties, some approximate numerical methods must be used to solve these problems. Although the existence of numerous finite element-based software packages, the Finite Difference Method (FDM) can still be regarded as a numerical method that has merits due to its straightforward approach and a minimum requirement on hardware [4].

The Finite Difference Method have been used in bending of the isotropic plates by many researchers. Ezeh et al. [5] studied pure bending analysis of thin rectangular flat isotropic plates using ordinary finite difference method. Ghods and Mir [6] examined maximum deflection of isotropic rectangular thin plates by using this method. Umeonyiagu et al. [7] considered flexural analysis of isotropic rectangular thin plates using an improved finite difference method. Deflection of laminated orthotropic plate subjected to uniformly distributed loaded was investigated based on the Classical Laminated Plate Theory (CLPT) using finite difference method by Saraçoğlu and Özçelikörs [8]. The researchers examined regular symmetric and regular antisymmetric composite square plates that were simply supported with four edges.

In symmetrically layered structures, the angles of the layers are symmetrical with respect to the central axis. This type of structures are preferred in production, since during the cooling process, they remain flat. Shear modulus of the quasi-isotropic plates with  $-45^\circ$ ,  $0^\circ$ ,  $45^\circ$  and  $90^\circ$  angles is greater than that of cross-ply. It is stated that, with a few exceptions, quasi-isotropic structure is commonly used for the air vehicles of NASA [9].

In this parametric study, maximum deflections of 24 different types of symmetrically laminated quasi-isotropic thin rectangular plates under uniformly lateral loads, were investigated with the use of the Finite Difference Method (FDM), with boundary conditions with clamped or simply supported edges of the plates. The obtained results were compared with the results of the Finite Element Method (FEM) received from another research of Altunsaray and Bayer [10]. It was found that the results of FDM agreed closely on those of the FEM.

# 2. Analysis

### 2.1. Material Geometry of plates, material properties and lamination types

The plate geometry is shown in Fig. 1. Material properties of T300-934 coded carbon/epoxy [12] are given in Table 1. Aspect ratios used in this parametrical study are given in Table 2. Twenty four different types of lamination of quasi-isotropic plates examined in this study are shown in Table 3. Quasi-isotropic plates have the combination of four different orientation angles ( $-45^\circ$ ,  $0^\circ$ ,  $45^\circ$  and  $90^\circ$ ). The thickness of each lamina of the plates (t) is 0.0002 meter and the thickness of the plate formed by 16 laminates (h) is equal to 0.0032 meter. Boundary conditions of simply supported and clamped cases were investigated. As the uniformly lateral load, the value of 10000 N/m<sup>2</sup> was used.



Figure 1. Uniformly lateral loaded (q) with rectangular plate

<b>Table 1.</b> Mechanical properties of carbon/epoxy (1500-954) [11	Ta	able	<b>1.</b> N	Mecl	nanical	pro	perties	of	carbon/e	poxy	(T3	600-	934)	) [	11	[]
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Longitudinal Young Modulus (E <sub>11</sub> )	$148 \times 10^9  (\text{N/m}^2)$
Transversal Young Modulus (E <sub>22</sub> )	$9.65 \times 10^9  (\text{N/m}^2)$
Longitudinal Shear Modulus (G <sub>12</sub> )	$4.55 \times 10^9  (\text{N/m}^2)$
Longitudinal Poisson ratio (v <sub>12</sub> )	0.3
Laminate thickness (t)	$0.185 \times 10^{-3} - 0.213 \times 10^{-3}$ (m)
Density $(\rho_0)$	$1.5 \text{ x} 10^3 \text{ (kg/m}^3)$

Short half side (a or b) is selected 0.2 m. Six different aspect ratios (a/b or b/a) are given in Table 2.

**Table 2.** Aspect ratios of the plates studied

a/b	1	1.2	1.4	1.6	1.8	2
b/a	1	1.2	1.4	1.6	1.8	2

		ie prace cypes	
LT1	$[-45_2/0_2/45_2/90_2]_s$	LT13	$[45_2/-45_2/0_2/90_2]_s$
LT2	$[-45_2/0_2/90_2/45_2]_s$	LT14	$[45_2/-45_2/90_2/0_2]_s$
LT3	$[-45_2/45_2/0_2/90_2]_s$	LT15	$[45_2/0_2/-45_2/90_2]_s$
LT4	$[-45_2/45_2/90_2/0_2]_s$	LT16	[45 <sub>2</sub> /0 <sub>2</sub> /90 <sub>2</sub> /-45 <sub>2</sub> ] <sub>s</sub>
LT5	$[-45_2/90_2/0_2/45_2]_s$	LT17	$[45_2/9_{02}/-45_2/0_2]_s$
LT6	$[-45_2/90_2/45_2/0_2]_s$	LT18	$[45_2/90_2/0_2/-45_2]_s$
LT7	$[0_2/-45_2/45_2/90_2]_s$	LT19	$[90_2/-45_2/0_2/45_2]_s$
LT8	$[0_2/-45_2/90_2/45_2]_s$	LT20	$[90_2/-45_2/45_2/0_2]_s$
LT9	$[0_2/45_2/-45_2/90_2]_s$	LT21	$[90_2/0_2/-45_2/45_2]_s$
LT10	$[0_2/45_2/90_2/-45_2]_s$	LT22	$[90_2/0_2/45_2/-45_2]_s$
LT11	$[0_2/90_2/-45_2/45_2]_s$	LT23	$[90_2/45_2/-45_2/0_2]_s$
LT12	$[0_2/90_2/45_2/-45_2]_s$	LT24	$[90_2/45_2/0_2/-45_2]_s$

Table 3. Symmetrically laminated quasi-isotropic plate types

# 2.2 Classical Laminated Plate Theory (CLPT)

In the Classical Laminated Plate Theory, the bending-strain coupling matrix of symmetrically laminated plates  $B_{ij}$  is zero, the governing differential equation of the symmetrically laminated composite plates under the effect of uniform laterally load is given in Eq. (1) [12].

$$\left(D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4}\right) - q(\mathbf{x}, \mathbf{y}) = 0$$
(Equation 1)

where w and q indicate deflection function and load, respectively. The bending stiffness matrix elements  $D_{11}$ ,  $D_{12}$ ,  $D_{16}$ ,  $D_{22}$ ,  $D_{26}$  and  $D_{66}$  are calculated as in the Eq.(2).

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} \overline{Q}_{ij}^{(k)} \left( z_{k+1}^3 - z_k^3 \right)$$
(Equation 2)

The elements of the transformed-reduced stiffness matrix  $Q_{ij}$  are calculated separately for each lamina and given in below Eq.(3).

$$\overline{Q}_{11} = Q_{11}\cos^4(\theta) + 2(Q_{12} + 2Q_{66})\sin^2(\theta)\cos^2(\theta) + Q_{22}\sin^4(\theta)$$
(Equation 3-1)

$$\overline{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})\sin^2(\theta)\cos^2(\theta) + Q_{12}(\sin^4(\theta) + \cos^4(\theta))$$
(Equation 3-2)

$$\overline{Q}_{22} = Q_{11}\sin^4(\theta) + 2(Q_{12} + 2Q_{66})\sin^2(\theta)\cos^2(\theta) + Q_{22}\cos^4(\theta)$$
(Equation 3-3)

$$\overline{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})\sin(\theta)\cos^3(\theta) + (Q_{12} - Q_{22} + 2Q_{66})\sin^3(\theta)\cos(\theta) \quad \text{(Equation 3-4)}$$

$$\overline{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})\sin^3(\theta)\cos(\theta) + (Q_{12} - Q_{22} + 2Q_{66})\sin(\theta)\cos^3(\theta)$$
(Equation 3-5)  
$$\overline{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})\sin^3(\theta)\cos(\theta) + (Q_{12} - Q_{22} + 2Q_{66})\sin(\theta)\cos^3(\theta)$$
(Equation 3-5)

$$Q_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2(\theta)\cos^2(\theta) + Q_{66}(\sin^4(\theta) + \cos^4(\theta))$$
(Equation 3-6)

The elements of the reduced stiffness matrix  $Q_{ij}$  is given in terms of material constants in Eq.(4).

$$Q_{11} = E_{11}/(1 - v_{12}v_{21}),$$
 (Equation 4-1)  

$$Q_{12} = v_{12}E_{22}/(1 - v_{12}v_{21}),$$
 (Equation 4-2)  

$$Q_{22} = E_{22}/(1 - v_{12}v_{21}),$$
 (Equation 4-3)  

$$Q_{66} = G_{12}$$
 (Equation 4-4)

By substituting the material constants (*E*, *G* and *v*), the angles of the laminas ( $\theta$ ) and the distance of each lamina from the reference plane into the Eqs. (2)-(3)-(4), the bending stiffness matrix elements  $D_{ij}$ 

# in Eq. (2) were determined.

In the case of clamped edges, the deflection and the slope along the edges of the plate are zero.

$$w = \frac{\partial w}{\partial x} = 0 \text{ at } x = 0 \text{ and } x = a$$
(Equation 5-1)
$$w = \frac{\partial w}{\partial y} = 0 \text{ at } y = 0 \text{ and } y = b$$
(Equation 5-2)

In the case of simply supported edges, the deflection and the bending moment along the edges of the plate are zero.

$$w = M_x = \frac{\partial w^2}{\partial x^2} = 0 \text{ at } x = 0 \text{ and } x = a$$
(Equation 6-1)  

$$w = M_y = \frac{\partial w^2}{\partial y^2} = 0 \text{ at } y = 0 \text{ and } y = b$$
(Equation 6-2)

### 2.3 Finite differences method (FDM)

Finite Difference Method is a numerical method used for obtaining approximate solutions of engineering problems. The derivatives in the governing differential equations are replaced by difference equations at some selected points of the plate. These points can be located at the joints of a square, or some other reference network, called a finite difference mesh. If the static problems of a plate can be described by a differential equation, it can be replaced at each mesh point by an equivalent finite difference equation. The boundary conditions are applied using a similar approach, setting then into the given values. Therefore, the domain in question becomes a system of linear equations, from which the values at the nodes of the mesh can be solved by conventional methods of linear algebra [4]. In this study, intervals between pivotal points in x and y directions ( $\Delta x$  and  $\Delta y$ ) are taken to be a/20 and b/20 for a square plate and increasing mesh size 24 to 40 for rectangular plates shown in Table 4. The software MATLAB [13] was used for preparing a computer code to carry out these parametric analyses.

a/b	FDM mesh size	b/a	FDM mesh size
1,0	20x20	1,0	20x20
1,2	20x24	1,2	24x20
1,4	20x28	1,4	28x20
1,6	20x32	1,6	32x20
1,8	20x36	1,8	36x20
2,0	20x40	2,0	40x20

Table 4. Finite differences mesh grid of the square to rectangular plates studied

The following central differences operators in Eq.(7-1–7-12) are used to obtain a finite difference scheme for the governing differential equation [9].

$\frac{\partial^2 w}{\partial x^2} \approx \frac{w_{i+\Delta x,j} - 2w_{i,j} + w_{i-\Delta x,j}}{(\Delta x)^2}$	(Equation 7-1)
$\frac{\partial^2 w}{\partial x \partial y} \approx \frac{w_{i+\Delta x, j+\Delta y} - w_{i-\Delta x, j+\Delta y} - w_{i+\Delta x, j-\Delta y} + w_{i-\Delta x, j-\Delta y}}{4(\Delta x)(\Delta y)}$	(Equation 7-2)
$\frac{\partial^2 w}{\partial y^2} \approx \frac{w_{i,j+\Delta y} - 2w_{i,j} + w_{i,j-\Delta y}}{(\Delta y)^2}$	(Equation 7-3)
$\frac{\partial^3 w}{\partial x^3} \approx \frac{w_{i+2\Delta x, y} - 2w_{i+\Delta x, j} + 2w_{i-\Delta x, j} - w_{i-2\Delta x, j}}{2(\Delta x)^3}$	(Equation 7-4)
$\frac{\partial^3 w}{\partial x \partial y^2} \approx \frac{w_{i+\Delta x, j+\Delta y} - 2w_{i+\Delta x, j} + w_{i+\Delta x - \Delta x, j-\Delta y}}{2(\Delta x)(\Delta y)^2} + \frac{-w_{i-\Delta x, j+\Delta y} + 2w_{i-\Delta x, j} - w_{i-\Delta x, j-\Delta y}}{2(\Delta y)(\Delta x)^2}$	(Equation 7-5)
$\frac{\partial^3 w}{\partial y \partial x^2} \approx \frac{w_{i+\Delta x, j+\Delta y} - 2w_{i,j+\Delta y} + w_{i+\Delta x, j-\Delta y} - w_{i-\Delta x, j+\Delta y}}{2(\Delta x)(\Delta y)^2} + \frac{2w_{i,j-\Delta y} - w_{i-\Delta x, j-\Delta y}}{2(\Delta x)(\Delta y)^2}$	(Equation 7-6)
$\frac{\partial^3 w}{\partial y^3} \approx \frac{w_{i,j+2\Delta y} - 2w_{i,j+\Delta y} + 2w_{i,j-\Delta y} - w_{i,j-2\Delta y}}{2(\Delta y)^3}$	(Equation 7-7)
$\frac{\partial^4 w}{\partial x^4} \approx \frac{w_{i+2\Delta x,j} - 4w_{i+\Delta x,j} + 6w_{i,j} - 4w_{i-\Delta x,j} + w_{i-2\Delta x,j}}{(\Delta x)^4}$	(Equation 7-8)
$\frac{\partial^4 w}{\partial y \partial x^3} \approx \frac{w_{i+2\Delta x, j+\Delta y} - w_{i+2\Delta x, j-\Delta y} - 2w_{i+\Delta x, j+\Delta y} + 2w_{i+\Delta x, j-2\Delta y}}{4(\Delta x)^3(\Delta y)} + \frac{+2w_{i-\Delta x, j+\Delta y} - 2w_{i-\Delta x, j-\Delta y} - w_{i-2\Delta x, j+\Delta y} + w_{i-2\Delta x, j-\Delta y}}{4(\Delta x)^3(\Delta y)}$	(Equation 7-9)
$\frac{\partial^4 w}{\partial y^2 \partial x^2} \approx \frac{w_{i+\Delta x, j+\Delta y} - 2w_{i,j+\Delta y} + w_{i-\Delta x, j+\Delta y} - 2w_{i+\Delta x, j} + 4w_{i,j}}{(\Delta x)^2 (\Delta y)^2}$	
$+\frac{-2w_{i-\Delta x,j}+w_{i+\Delta x,j-\Delta y}-2w_{i,j-\Delta y}+w_{i-\Delta x,j-\Delta y}}{(\Delta x)^2(\Delta y)^2}$	(Equation 7-10)

$\partial^4 w$	$\sim w_{i+\Delta x,j+2\Delta y} - 2w_{i+\Delta x,j+\Delta y} + 2w_{i+\Delta x,j-\Delta y} - w_{i+\Delta x,j-2\Delta y}$
$\partial y^3 \partial x$	$\sim$ $4(\Delta y)^3(\Delta x)$
$\perp -w_i$	$-\Delta x, j+2\Delta y+2w_{i-\Delta x, j+\Delta y}-2w_{i-\Delta x, j-\Delta y}+w_{i-\Delta x, j-2\Delta y}$
I	$4(\Delta x)^3(\Delta y)$

(Equation 7-11)

$\partial^4 w \sim$	$w_{i,j+2\Delta y} - 4w_{i,j+\Delta y} + 6w_{i,j} - 4w_{i,j-\Delta y} + w_{i,j-2\Delta y}$
$\overline{\partial y^4} \sim$	$(\Delta y)^4$

(Equation 7-12)

# 3. Results

With the use of the parameters presented in Sections 2.1- 2.6, the maximum deflection values of 24 different symmetrically laminated quasi-isotropic rectangular plates under the effect of uniformly lateral load, were found using FDM and the software-based FEM are presented in Figures. 3-6 for simply supported and in Figures.7-10 for clamped boundary conditions.

The effect of change in orientation angle and aspect ratio for simply supported condition is shown in Figures. 3-4, when the short edge of the plate is at y-axis. The results obtained are given in Figures. 5-6., when the short edge of the plate is at x-axis.

For the clamped cases, the effect of change in orientation angle and aspect ratio is shown in Figures. 7-8, when the short edge of the plate is at y-axis. The results obtained are given in Figures. 9-10 when the short edge of plate is at x-axis.

The maximum deflections for clamped conditions are lower than those for simply supported conditions as expected.

It is seen that from the results at the simply supported boundary condition, the maximum deflection values of 24 different symmetrically laminated quasi-isotropic plates increase with the increase of the aspect ratio. It is also observed that in cases where the short edge of the plate is a or b, the maximum deflection values of the plates change (Figures. 3-6). The same tendency is also seen with the clamped boundary condition (Figures. 7-10).

It can be noted that by considering the Figures.3-4 and Figures.7-8, which denote simply supported hinge and a clamped one, respectively, that the plates LT7 ( $[0_2/-45_2/45_2/90_2]_s$ ) and LT9 ( $[0_2/45_2/-45_2/90_2]_s$ ) have the highest maximum deflection (for a/b=2), when the short edge "a" of the plate is selected. This construction coincides longitudinal structure system.

It can be noted that by considering the Figures.5-6 and Figures.9-10, which denote simply supported hinge and clamped one, respectively, that the plates LT20 ( $[90_2/-45_2/45_2/0_2]_s$ ) and LT23 ( $[90_2/45_2/-45_2/0_2]_s$ ) have the highest maximum deflection (for b/a=2), when the short edge "b" of the plate is selected. This construction coincides with the transverse structure system.



Figure 3. Maximum deflection of the quasi-isotropic plates in 24 types with simply supported hinge a/b=1, 1.4, 2- longitudinal structure system solved using FDM and FEM



Figure 4. Maximum deflection of the quasi-isotropic plates in 24 types with simply supported hinge a/b= 1.2, 1.6, 1.8 – longitudinal structure system solved using FDM and FEM



Figure 5. Maximum deflection of the quasi-isotropic plates in 24 types with simply supported hinge b/a= 1, 1.4, 2 – transversal structure system solved using FDM and FEM



Figure 6. Maximum deflection of the quasi-isotropic plates in 24 types with simply supported hinge b/a= 1.2, 1.6, 1.8 – transversal structure system solved using FDM and FEM



Figure 7. Maximum deflection of the quasi-isotropic plates in 24 types with clamped hinge a/b=1, 1.4, 2 – longitudinal structure system solved using FDM and FEM



Figure 8. Maximum deflection of the quasi-isotropic plates in 24 types with clamped hinge a/b= 1.2, 1.6, 1.8 – longitudinal structure system solved using FDM and FEM



Figure 9. Maximum deflection of the quasi-isotropic plates in 24 types with clamped hinge b/a= 1, 1.4, 2 - transversal structure system solved using FDM and FEM



Figure 10. Maximum deflection of the quasi-isotropic plates in 24 types with clamped hinge b/a= 1.2, 1.6, 1.8 - transversal structure system solved using FDM and FEM

## 4. Conclusions

Static bending analysis of uniformly lateral loaded composite thin plates has been investigated for different boundary conditions as clamped and simply supported hinge and also for lamination types, aspect ratios (a/b, b/a). Calculations were performed based on the governing equations of Classical Lamination Plate Theory (CLPT) using Finite Difference Method (FDM). The obtained results compared with those of the software ANSYS based on the FEM received from the open literature were found close to each other. With this parametric study using the FDM, most suitable lamination types can be determined in preliminary design stage of a composite ship, where numerous design parameters such as sizes, boundary conditions, lamination types, aspect ratios, transversal or longitudinal structural systems etc., are considered. It may be concluded that the FDM is a suitable and fast method that can be applied with the aid of the MATLAB software used for creating a computer code in composite ship design.

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