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The exact and asymptotically value of the failure rate of a parallel system

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Abstract. The problem of determining the failure rate of a series system according to the failure rates of the component elements is solved in the literature, a reference in the field being the work [4]. There is no direct relationship between the failure rate of a parallel system and the failure rates of the component elements. In this paper we will deduce an exact relation between the failure rate of a parallel system and the failure rates of the component elements, their reliability and the reliability of the system. Starting from the exact relationship we will deduce a simpler relation of the failure rate of a parallel system by studying the asymptotic failure rate behaviour.

Keywords: reliability, asymptotically, system, parallel

1. Introduction

Let $r(t)$ the failure rate of a simple element, $r:[0,\infty)\to[0,\infty)$ and $R(t)$ its reliability defined by: $R:[0,\infty)\to[0,1]$.

In the work [4] demonstrates that: $r(t)=-\frac{1}{R(t)}\frac{dR}{dt}$ and $R(t)=e^{-\int_0^t r(\tau)\cdot d\tau}$.

Let it be a series system with n components $e_1,e_2,\cdots,e_n$ having fault rates, respectively $r_1(t),r_2(t),\cdots,r_n(t)$.

In the work [4] it is shown that between the failure rate of the series system $r(t)$ and the failure rates of its components $\mathrm{r_k(t)}$ there is equality: $r(t)=\sum_{k=1}^{n} r_k(t)$.

In the works [1], [2], [3], [4] and [5] there is no relation to calculate the failure rate of the parallel system depending on the failure rates of the component elements and their reliability.

We will deduce a similar relationship for the case of parallel systems.

In Section 2 we will briefly describe the results for the series systems. In Section 3, we will deduce the exact relationship between the failure rate of the parallel system and the component failure rates, their reliability, and the reliability of the parallel system. We will also deduce from asymptotic considerations a simplified expression of the failure rate of the parallel system. In Section 4 we will indicate future development directions.

2. Series system

Consider the series system with n components $e_1, e_2, \cdots, e_n$ with the respective reliability $R_k(t)$, with $k = 1,2,\cdots,n$, connected in series (Figure 1).

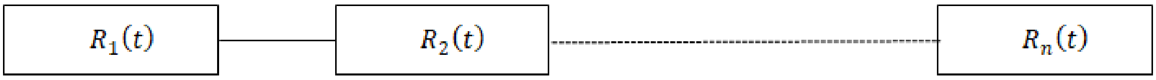

Figure 1. Series system

By definition, a series system is in operation if all of its elements are in working order. The serial system is faulty if at least one of the elements is faulty.

Let's note with:

$R: [0,\infty) \to [0,1]$, $R(t)$ the system reliability;

$r: [0,\infty) \to [0,\infty)$, $r(t)$ the system failure rate, $r(t) = -\frac{1}{R(t)}\frac{dR}{dt}$;

$R_k: [0,\infty) \to [0,1]$, $R_k(t)$ the e_k element reliability for $k = 1,2,\cdots,n$;

$r_k: [0,\infty) \to [0,\infty)$, $r(t)$ the e_k element failure rate, $r_k(t) = -\frac{1}{R_k(t)}\frac{dR_k}{dt}$ and $R_k(t) = e^{-\int_0^t r_k(\tau)\cdot d\tau}$ for $k = 1,2,\cdots,n$.

In the works [1], [2], [3], [4] and [5] it is demonstrated that equality takes place:

$$R(t) = R_1(t) \cdot R_2(t) \cdot \cdots \cdot R_n(t) \quad (1)$$

and

$$r(t) = r_1(t) + r_2(t) + \cdots + r_n(t) \quad (2)$$

We will deduce in Section 3 a relationship similar to the relation (2) for parallel systems.

3. Parallel systems

We consider a system with n components $e_1, e_2, \cdots, e_n$ with the respective reliability $R_k(t)$, with $k = 1,2,\cdots,n$, connected in parallel (Figure 2).

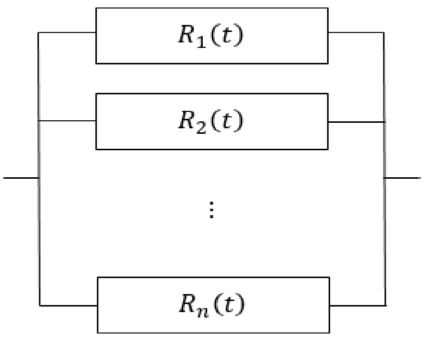

Figure 2. Parallel system

By definition a parallel system is in operation if at least one of the components is in operation.

By definition, the system does not work if all of its components are defective. The system works until the last component fails. They are supposed to fail independently of each other.

Let's note with:

$R:[0,\infty)\to[0,1]$, $R(t)$ the system reliability;

$r:[0,\infty)\to[0,\infty)$, $r(t)$ the system failure rate, $r(t)=-\frac{1}{R(t)}\frac{dR}{dt}$;

$R_k:[0,\infty)\to[0,1]$, $R_k(t)$ the e_k element reliability, $k=1,2,\cdots,n$;

$r_k:[0,\infty)\to[0,\infty)$, $r(t)$ the e_k element failure rate, $r_k(t)=-\frac{1}{R_k(t)}\frac{dR_k}{dt}$, $k=1,2,\cdots,n$;

Events are considered: Let it be considered the following events:

E event of good system operation at probability $P(E)=R(t)$;

$E_1,E_2,\cdots,E_n$ the good functioning events of the components $e_1,e_2,\cdots,e_n$, with probabilities: $P(E_k)=R_k(t)$.

Because $E\cap\bar{E}=\emptyset$ (ϕ the impossible event) and $E\cup\bar{E}=\Omega$ (Ω the safe event) it follows that events E and $\bar{\text{E}}$ form a complete system of events and therefore: $P(E)+P(\bar{E})=1$.

Let it be considered the following random variables:

T the random variable of the operating time until the first system failure;

T_k the random variable of the operating time to the first component failure e_k, $k=1,2,\cdots,n$.

Under these conditions equality takes place: $T=max\{T_1,T_2,\cdots,T_n\}$.

We consider $\{\mathrm{T}<t\}$ the event that the system will work until the moment t and $\{T_k<t\}$ the operation event of the component e_k up to the moment t, $k=1,2,\cdots,n$.

It is obtained:

$\{\mathrm{T}<t\}=\bigcup_{k=1}^{n}\{T_k<t\}$ and from here $\overline{\{T\geq t\}}=\bigcap_{k=1}^{n}\overline{\{T_k\geq t\}}$. In other words, equality is achieved: $\bar{E}=\bigcap_{k=1}^{n}\overline{E_k}$.

Applying the probability is obtained: $P(\bar{E})=P\left(\bigcap_{k=1}^{n}\overline{(E_k)}\right)$ from which it follows that: $1-P(E)=P\left(\bigcap_{k=1}^{n}\overline{(E_k)}\right)$ and $P(E)=1-P\left(\bigcap_{k=1}^{n}\overline{(E_k)}\right)$.

Because $P(E)=R(t)$ and $P(E_k)=R_k(t)$ follows:

$P(E)=R(t)=1-\prod_{k=1}^{n}P(\overline{E_k})=1-\prod_{k=1}^{n}\left(1-P(E_k)\right)=1-\prod_{k=1}^{n}\left(1-R_k(t)\right)$ and therefore: $R(t)=1-\prod_{k=1}^{n}\left(1-R_k(t)\right)$.

Deriving in both members of equality:

$R(t)=1-\prod_{k=1}^{n}\left(1-R_k(t)\right)$ there is obtained: $\frac{dR}{dt}=\sum_{k=1}^{n}\prod_{\substack{p=1\\p\neq k}}^{n}\left(1-R_p(t)\right)\cdot\frac{dR_k}{dt}$.

Because: $\prod_{\substack{p=1\\p\neq k}}^{n}\left(1-R_p(t)\right)=\frac{(1-R_1(t))\cdot(1-R_2(t))\cdot\cdots\cdot(1-R_n(t))}{1-R_k(t)}=\frac{1-R(t)}{1-R_k(t)}$ it is obtained that:

$\frac{dR}{dt}=\sum_{k=1}^{n}\frac{dR_k}{dt}\cdot\frac{1-R(t)}{1-R_k(t)}=\left(1-R(t)\right)\cdot\sum_{k=1}^{n}\frac{1}{1-R_k(t)}\cdot\frac{dR_k}{dt}$.

The above relationship is amplified with: $\frac{-1}{R(t)}$ and equality is achieved:

$$-\frac{1}{R(t)}\cdot\frac{dR}{dt}=-\frac{1-R(t)}{R(t)}\cdot\sum_{k=1}^{n}\frac{1}{1-R_k(t)}\cdot\frac{dR_k}{dt}\tag{3}$$

The term $\frac{1}{1-R_k(t)}\cdot\frac{dR_k}{dt}$, by amplifying the counter and the denominator with $R_k(t)$ becomes:

$\frac{R_k(t)}{1-R_k(t)} \cdot \frac{1}{R_k(t)} \cdot \frac{dR_k}{dt}$.

Equality is achieved:

$-\frac{1}{R(t)} \cdot \frac{dR}{dt} = -\frac{1-R(t)}{R(t)} \cdot \sum_{k=1}^{n} \frac{R_k(t)}{1-R_k(t)} \cdot \frac{1}{R_k(t)} \cdot \frac{dR_k}{dt}$

Because $r(t) = -\frac{1}{R(t)} \cdot \frac{dR}{dt}$ and $r_k(t) = -\frac{1}{R_k(t)} \cdot \frac{dR_k}{dt}$ the relationship (3) becomes:

$$r(t) = \frac{1-R(t)}{R(t)} \cdot \sum_{k=1}^{n} \frac{R_k(t)}{1-R_k(t)} \cdot r_k(t) \quad (4)$$

and hence:

$$\frac{R(t)}{1-R(t)} \cdot r(t) = \sum_{k=1}^{n} \frac{R_k(t)}{1-R_k(t)} \cdot r_k(t) \quad (5)$$

Next we will use the geometric series.

As is well known for any real x with the property $|x| < 1$ equality occurs: $\frac{1}{1-x} = \sum_{i=0}^{\infty} x^i = 1 + x + x^2 + \cdots + x^n + \cdots$, in which $n \in N^*$.

Because $|R_k(t)| < 1$ development is achieved: $\frac{1}{1-R_k(t)} = 1 + R_k(t) + \left(R_k(t)\right)^2 + \cdots + \left(R_k(t)\right)^n + \cdots$.

Each term $\left(R_k(t)\right)^p$ can be approximated by 0 for any $p \geq 2$, $p \in N^*$. It can thus approximate $\frac{1}{1-R_k(t)}$ by $1 + R_k(t)$ and from here we can approximate: $\frac{R_k(t)}{1-R_k(t)} \cong R_k(t) + \left(R_k(t)\right)^2 \cong R_k(t)$.

Because $|R(t)| < 1$ development is achieved: $\frac{1}{1-R(t)} = 1 + R(t) + \left(R(t)\right)^2 + \cdots + \left(R(t)\right)^n + \cdots$.

Each term $\left(\mathrm{R(t)}\right)^{\mathrm{p}}$ can be approximated by 0 for any $p \geq 2$, $p \in N^*$. It can thus approximate $\frac{1}{1-R(t)}$ by $1 + R(t)$ and from here we can approximate: $\frac{R(t)}{1-R(t)} \cong R(t) + \left(R(t)\right)^2 \cong R(t)$.

Using these approximations the relationship (5) becomes:

$$R(t) \cdot r(t) = \sum_{k=1}^{n} R_k(t) \cdot r_k(t) \quad (6)$$

from which it is deduced the asymptotic value of the failure rate of the parallel system according to the failure rates of the component elements, their reliability and the reliability of the system:

$$r(t) = \frac{1}{R(t)} \cdot \sum_{k=1}^{n} R_k(t) \cdot r_k(t) \quad (7)$$

4. Conclusions

In this paper we determined the exact expression of the failure rate of a parallel system according to the component failure rates, the component reliability and the parallel system reliability (relation 4) and the asymptotic value of the failure rate of a parallel system according to failure rates of component elements and their reliability (relationship 7).

The authors intend to obtain a refinement of the relationship (7) through further research.

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References

[1] Cătuneanu V.M., Mihalache A., Bazele teoretice ale fiabilităţii, Ed. Academiei, Bucureşti, 1983
[2] Ganciu T., Fiabilitate, Universitatea Tehnică "Gh. Asachi", Iaşi, 1994
[3] Hohan I., Tehnologia şi fiabilitatea sistemelor, Ed. Didactică şi Pedagogică, Bucureşti, 1982
[4] Kaufmann A., Grouchko D., Cruon R., Mathematical Models for the Study of the Reliability of Systems, Mathematics in Science and Engineering, vol 124, Academic Press, New York San Francisco London 1977
[5] Ţârcolea C., Filipoiu A., Bontaş S., Tehnici actuale în teoria fiabilităţii, Ed. ştiinţifică şi Enciclopedică, Bucureşti, 1989