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Automatic determination of the dual system width of to a bivalent system

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Abstract. A system is a set of elements that can be found in one of the following states: operating state and fault. Any system has two stable states: functioning and defect, which is why, in the theory of reliability, it is called a bivalent system. A subset of defective elements is called the system cut if all the other elements of the system are in operation and the system is defective. The width of a bivalent system is equal to the minimum number of elements the system cuts have. In this paper is presented an algorithm for automatic determination of the dual system width to a bivalent system, a Matlab script that implements the algorithm, a case study and subsequent directions of development.

Keywords: bivalent, dual, reliability, script, structure, system, width

1. Introduction

The plurality of all elements of the assembly of elements that reflect how the state of an S system depends on the states of its components is called a structure.

In the reliability analysis, structures play an important role. In a first step, the structure of the equipment is analyzed, followed by establishing the algebraic expression of the structure function and then constructing a reliability network associated with that equipment.

Considerations regarding complex systems or equipment are based on the following assumptions:

The equipment considered can only be in one of the following states: operating state or fault condition (bivalent system). The considered equipment can be decomposed into n components (elements) e_i, $i = 1,2,\cdots,n$. The set of all these elements is: $E = \{e_1, e_2, \cdots, e_n\}$.

Each component e_i, $i = 1,2,\cdots,n$, is associated with a state variable x_i defined by: $x_i = \begin{cases} 1 & \text{if } e_i \text{ is in operation} \\ 0 & \text{if } e_i \text{ does not work} \end{cases}$.

The set of states $X = \{x_1, x_2, \cdots, x_n\}$ characterizes the set of possible states of the assembly of elements. Obvious, $card(X) = 2^n$.

The system S is associated with a state variable y defined as follows:

$$y = \begin{cases} 1 & \text{if } S \text{ is in operation} \\ 0 & \text{if } S \text{ does not work} \end{cases}.$$

Let $Y = \{y\} = \{0,1\}$. The y state variable depends on the set of X states.

We can define a function $\varphi: X \to Y$, defined by: $y = \varphi(x_1, x_2, \cdots, x_n)$. The function φ is called the function of the structure. Since the function φ depends on n independent variables, it is called function of the n -order structure. The S system can be identified with the pair (E, φ): $S = (E, \varphi)$.

Let the system $S = (E, \varphi)$.

The subassembly $T = \{e_i \mid i \in I \subset \{1,2,\cdots,n\}\} \subset E$ is called a cut if for any $x_i = 0$, $i \in I$ and for any $x_i = 1$, $i \notin I$ follows: $y = \varphi(x_1, x_2, \cdots, x_n) = 0$.

In other words, a cut of a non-functioning system $(y = 0)$ is a subset (subassembly) of elements in which all elements are defective, the rest of the elements being in operation.

Reforming a cut is a subassembly of elements that are in a non-functioning state and cause the system to fail if the other elements are in operation.

Any system has at least one cut.

To determine all system cuts, calculate the values of the structure function at all points $(x_1, x_2, \cdots, x_n)$. If $f(\mathrm{x}_1, \mathrm{x}_2, \cdots, \mathrm{x}_\mathrm{n}) = 0$ for $x_{i_1} = x_{i_2} = \cdots = x_{i_p} = 0$ and $x_i = 1$ for any $i \in \{1,2,\cdots,n\} \backslash \{i_1, i_2, \cdots, i_p\}$ then the subset of elements (subassembly of elements) $\{e_{i_1}, e_{i_2}, \cdots, e_{i_p}\}$ is a cut.

Let $c_k > 0$ the number of cuts with k elements, $k \in N^*$. It is called the width of the system S number $\mu(S) = min\{k \mid c_k > 0\}$.

Let $n \in N^*$, $E = \{e_1, e_2, \cdots, e_n\}$, $X = \{x_1, x_2, \cdots, x_n\}$, function $\varphi: X \to \{0,1\}$, $y = \varphi(x_1, x_2, \cdots, x_n)$, and the system $S = (E, \varphi)$.

It's called the dual system of the system $S = (E, \varphi)$ the system $S^d = (E, \varphi^d)$ where the structure function $\varphi^d: X \to \{0,1\}$ is defined by $\varphi^d(x_1, x_2, \cdots, x_n) = 1 - \varphi(1 - x_1, 1 - x_2, \cdots, 1 - x_n)$. Through duality, series connections become parallel connections and parallel connections become serial connections.

The study of the cuts and width of a system brings important information about the operation of the system under the condition that some of the component elements are defective.

The difficulty of manually determining cuts and the width of a system increases as the number of elements in the system increases. There is a need to simplify and automate the determination of system cuts and system width. For this reason, the authors wrote the Matlab functions presented in Section 2.

2. Matlab implementation

The Matlab main function with the signature *function cuts(n)* receives the n-value of the input. The function generates all cuts to the reliability network. The function calculates the width of the dual system.

The function *function w=conversion(n,v)* converts the value of the input argument n from base 10 to base 2 and stores the result of the conversion in the variable w.

The function *function val=fstruct(x)* receives the input vector x of the structure function variables and returns the value of the structure function.

The function function val = fdstruct (x) receives the vector x and returns the value of the structure function of the dual system.

The display of cuts is made by the function *function write(n,w,tip)*.

The function *function w=inverse(v)* inverts the vector v and stores the result in the vector w.

The function *function printnl(nb)* displays the number of cuts with k elements, $k = 1,2,\cdots$

We present below the Matlab code for implementing the algorithm.

```
% The main function
function cuts(n)
clc;
k=2^n;
for i=1:n
v(i)=0;
end
```

```
ival=1;
for i=0:k-1
w=conversion(i,v);
val=fdstruct(w);
if val==0
vval(ival)=i;
fprintf(' The value %d :\n y = f ( ',i);
nb(ival)=0;
for j=1:length(w)-1
fprintf(' %d , ',w(j));
if w(j)==0
nb(ival)=nb(ival)+1;
end
end
if w(length(w))==0
nb(ival)=nb(ival)+1;
end
fprintf(' %d ',w(length(w)));
fprintf(' ) = %d \n',val);
write(i,w,'t');
ival=ival+1;
end
end
printnt(nb);
fprintf(' The dual system''s width is %d \n',min(nb));
end
% Conversion function
% from base 10 in base 2
function w=conversion(n,v)
i=1;
while n ~= 0
uc=mod(n,2);
v(i)=uc;
i=i+1;
n=floor(n/2);
end
w=inverse(v);
end
% Function for reversing vector v
function w=inverse(v)
n=length(v);
for i=1:n
w(i)=v(n-i+1);
end
end
% Function for displaying cuts
function write(n,w,tip)
p=length(w);
if tip=='t'
fprintf(' Subassembly T = { ');
for i=1:p
```

```
if w(i)==0
fprintf(' e%d ',i);
end
end
fprintf(' } is a cut \n');
end
end
% Function to display the number of cuts
% with k elements
% k=1,2,...
function printnt(nb)
kmin=min(nb);
kmax=max(nb);
for i=kmin:1:kmax
s=0;
for j=1:length(nb)
if nb(j)==i & nb(j)~=0
s=s+1;
end
end
fprintf(' Number of %d - element cuts = %d \n',i,s);
end
end
% Function of the dual system structure
function val=fdstruct(x)
y=1-x;
val=1-fstruct(y);
end
% Function of system structure
function val=fstruct(x)
val=x(3)*(1-(1-x(1)*x(2))*(1-x(4)*x(5)));
end
```

3. Case study

Consider the system $S=(E,\varphi)$ with the properties: $n=5$, $E=\{e_1,e_2,e_3,e_4,e_5\}$, $X=\{x_1,x_2,x_3,x_4,x_5\}$. The graph of the system is shown in figure 1. The graph of the dual system is shown in figure 2.

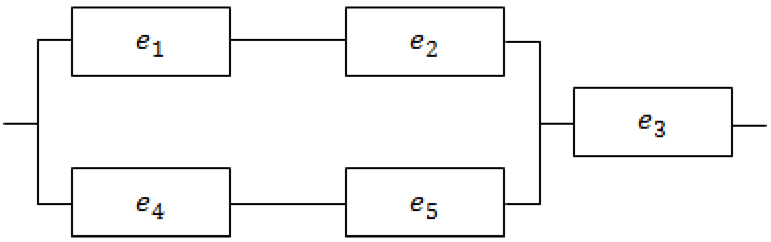

Figure 1. The system $S=(E,\varphi)$.

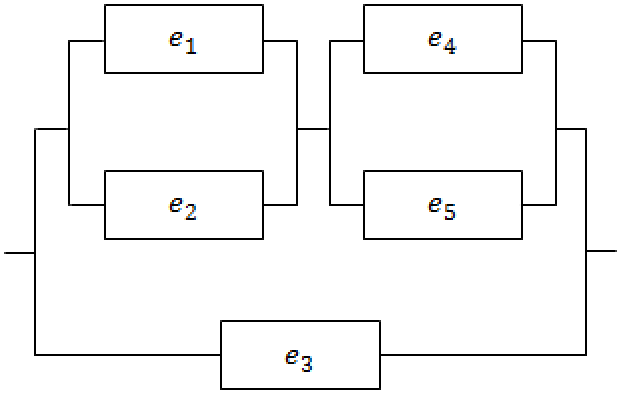

Figure 2. The dual system $S^d = (E, \varphi^d)$.

Obviously, the structure function of the system is the function: $\varphi: X \to \{0,1\}$,
$\varphi(x_1, x_2, x_3, x_4, x_5) = x_3 \cdot (1 - (1 - x_1 \cdot x_2) \cdot (1 - x_4 \cdot x_5))$,

The function of the dual system structure is the function:
$\varphi^d: X \to \{0,1\}$ defined by:
$\varphi^d(x_1, x_2, x_3, x_4, x_5) = 1 - \varphi(1 - x_1, 1 - x_2, 1 - x_3, 1 - x_4, 1 - x_5)$,
$\varphi^d(x_1, x_2, x_3, x_4, x_5) = 1 - (1 - x_3) \cdot \Big(1 - \big(1 - (1 - x_1) \cdot (1 - x_2)\big) \cdot \big(1 - (1 - x_4) \cdot (1 - x_5)\big)\Big)$.

For manual determination of all system cuts, the values of the structure function are calculated at all points $(x_1, x_2, x_3, x_4, x_5)$.

If for $x_{i_1} = x_{i_2} = \cdots = x_{i_p} = 0$ and $x_i = 1$ for any $i \in \{1,2,\cdots,n\} \backslash \{i_1, i_2, \cdots, i_p\}$ and $\varphi(x_1, x_2, \cdots, x_n) = 0$ then the subassembly of elements $\{e_{i_1}, e_{i_2}, \cdots, e_{i_p}\}$ is a cut.

Table 1 is completed and all system cuts are obtained.

Table 1. Dual system cuts $S^d = (E, \varphi^d)$.

Value	x_1	x_2	x_3	x_4	x_5	φ	φ^d	Cuts in $S^d = (E, \varphi^d)$
0	0	0	0	0	0	0	1	$\{e_1, e_2, e_3, e_4, e_5\}$
1	0	0	0	0	1	0	1	$\{e_1, e_2, e_3, e_4\}$
2	0	0	0	1	0	0	1	$\{e_1, e_2, e_3, e_5\}$
3	0	0	0	1	1	0	1	$\{e_1, e_2, e_3\}$
4	0	0	1	0	0	1	0	
5	0	0	1	0	1	1	0	
6	0	0	1	1	0	1	0	
7	0	0	1	1	1	1	0	
8	0	1	0	0	0	0	1	$\{e_1, e_3, e_4, e_5\}$
9	0	1	0	0	1	1	0	
10	0	1	0	1	0	1	0	
11	0	1	0	1	1	1	0	
12	0	1	1	0	0	1	0	
13	0	1	1	0	1	1	0	
14	0	1	1	1	0	1	0	
15	0	1	1	1	1	1	0	

16	1	0	0	0	0	0	1	$\{e_2, e_3, e_4, e_5\}$
17	1	0	0	0	1	1	0	
18	1	0	0	1	0	1	0	
19	1	0	0	1	1	1	0	
20	1	0	1	0	0	1	0	
21	1	0	1	0	1	1	0	
22	1	0	1	1	0	1	0	
23	1	0	1	1	1	1	0	
24	1	1	0	0	0	0	1	$\{e_3, e_4, e_5\}$
25	1	1	0	0	1	1	0	
26	1	1	0	1	0	1	0	
27	1	1	0	1	1	1	0	
28	1	1	1	0	0	1	0	
29	1	1	1	0	1	1	0	
30	1	1	1	1	0	1	0	
31	1	1	1	1	1	1	0	

To determine the width of the dual system, the numbers are determined:
$c_3 = 2$ because there are two cuts with three elements (cuts: $\{e_1, e_2, e_3\}$ and $\{e_3, e_4, e_5\}$);
$c_4 = 4$ because there are four cuts each with four elements (cuts: $\{e_1, e_2, e_3, e_4\}$, $\{e_1, e_2, e_3, e_5\}$, $\{e_1, e_3, e_4, e_5\}$ and $\{e_2, e_3, e_4, e_5\}$);
$c_5 = 1$ because there is only one cut with five elements (cut: $\{e_1, e_2, e_3, e_4, e_5\}$).

The S^d dual system's width is equal to: $\mu(S) = min\{k \mid c_k > 0\} = min\{3,4,5\} = 3$.

Using the Matlab function above, we will determine all the cuts of this system and the width of the dual system.

The structure function denoted by φ coincides with the function denoted by f in the Matlab code.

An example of execution is:

```
The value 0 :
y = f (  0 ,  0 ,  0 ,  0 ,  0  ) = 0
Subassembly T = {  e1  e2  e3  e4  e5  } is a cut
The value 1 :
y = f (  0 ,  0 ,  0 ,  0 ,  1  ) = 0
Subassembly T = {  e1  e2  e3  e4  } is a cut
The value 2 :
y = f (  0 ,  0 ,  0 ,  1 ,  0  ) = 0
Subassembly T = {  e1  e2  e3  e5  } is a cut
The value 3 :
y = f (  0 ,  0 ,  0 ,  1 ,  1  ) = 0
Subassembly T = {  e1  e2  e3  } is a cut
The value 8 :
y = f (  0 ,  1 ,  0 ,  0 ,  0  ) = 0
Subassembly T = {  e1  e3  e4  e5  } is a cut
The value 16 :
y = f (  1 ,  0 ,  0 ,  0 ,  0  ) = 0
Subassembly T = {  e2  e3  e4  e5  } is a cut
The value 24 :
y = f (  1 ,  1 ,  0 ,  0 ,  0  ) = 0
```

Subassembly T = { e3 e4 e5 } is a cut
Number of 3 - element cuts = 2
Number of 4 - element cuts = 4
Number of 5 - element cuts = 1
The dual system's width is 3

It is easy to see that the cuts generated by the matlab function coincide with the cuts determined manually in table 1.

Also, the system width with the matlab function coincides with the one determined manually.

4. Conclusions

One of the future development directions is to determine the minimal cuts of the $S=(E,\varphi)$ system and its dual system $S^d=\left(E,\varphi^d\right)$.

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