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Consideration regarding tensions in a contact

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Abstract. The paper presents the existent tension at the contact level: sphere-plane. In the same time we describe the dependence of these tension by different factors of influence. The obtained relations are without dimension and in this way we can analyse better the punctual contact. For this we have made a dimensionalization of the relations used in specialized literature, and based on these, we deduced the dependencies of the tensions of the various influence factors of the contact

1.Introduction

The tension analysis in this paper takes into account a spherical-plan contact. For this we have made a dimensionalization of the relations used in specialized literature, and based on these, we deduced the dependencies of the tensions of the various influence factors of the contact.

In the study of contact we started from the quasi-static tension field determination by summing two determinations from the field of equations of the linear elasticity equation considering the limit conditions in the z = 0 plane of the semis pace z> 0 (The Hamilton Theory):

$$p_{yz} = p_{zz} = 0; p_{xz} = -\left(3\mu P / 2\pi a^3\right)\left(a^2 - r^2\right)^{1/2}, r < a \quad (1)$$

$$p_{yz} = p_{xz} = 0; p_{zz} = -\left(3P / 2\pi a^3\right)\left(a^2 - r^2\right)^{1/2}, r < a \quad (2)$$

All tractions at z = 0 are canceled for r> a and all tensions quickly become zero when points move away from the origin.

$$\left(x^2 + y^2 + z^2\right)^{-1}$$

In the (1) and (2) relations, $r = \left(x^2 + y^2\right)^{1/2}$, a-represents the radius of the loading region, P- the total normal load and μP is the total tangential force in the x-direction.

All these are necessary for determining the stresses field and writing for the state of the stresses given by the application of a point-like force (solutions of the Boussinesq and Cerutti semis) [1, 2] by integrating over the entire plane z = 0, considering the boundary conditions (1) and (2).

This approach leads to a series of integrals hard to solve. However, this analysis can be approached by extending the tangential loaded semi space , a method introduced by A.E.Green for the analysis of voltages of a normally loaded semi space. [3,4,5,6].

As a result of this expansion, the cartesian components of the movements u, v, w depend on the harmonic voltage T (x, y, z):

$$2\mu u = 2\nu\left(\delta^2 T / \delta x^2\right) + 2\left(\delta^2 T / \delta z^2\right) - z\left(\delta^3 T / \delta x^2 \delta z\right) \quad (3.a)$$

$$2\mu v = 2\nu\left(\delta^2 T / \delta x \delta y\right) - z\left(\delta^3 T / \delta x \delta y \delta z\right) \quad (3.b)$$

$$2\mu w = \left(1 - 2\nu\right)\left(\delta^2 T / \delta x \delta z\right) - z\left(\delta^3 T / \delta x \delta z^2\right) \quad (3.c)$$

μ - represents the coefficient of friction, considered constant.

ν - Poisson's coefficient

The field of equations from the linear theory of elasticity and the two limit conditions (1) and (2) are automatically satisfied.

Taking T as an imaginary part of the complex harmonic function:

$$\int_0^a t(\xi)\left\{\frac{1}{2}\left(z_1^2-\frac{1}{2}r^2\right)\ln(z_1+R_1)-\frac{3}{4}R_1z_1+\frac{1}{4}r^2\right\}d\xi \quad (4)$$

where: $z_1 = z + i\xi$ and $R_1 = \left(z_1^2 + r^2\right)^{1/2}$;

the plane z = 0 is automatically released by traction for r> a.

It remains to be shown that t (ξ) satisfies the last limit condition (1).

At z = 0, for r <a, equations (3) and (4) involve:

$$p_{xz} = \int_r^a t(\xi)\left(\xi^2 - r^2\right)^{-1/2} d\xi \qquad (5)$$

So:

$$t(\xi) = \frac{-2}{\pi}\frac{d}{d\xi}\int_\xi^a r p_{xz}\left(r^2-\xi^2\right)^{-1/2} dr \qquad (6)$$

For p_{xz} giving fot the third condition (1), $t(\xi) = -\left(3\mu P / 2\pi a^3\right)\xi$

2. The field of tensions for a plan sphere contact

The determination of the field of voltages is thus done by an elementary quadrature following (3) and (4) [7,8].

Thus, by writing $z_2 = z + ia$ and

$R_2 = \left(z_2^2 + r^2\right)^{1/2}$ tension components are conventionally expressed in terms of the imaginary part of complex functions:

$$F = \frac{1}{2}(z-ia)R_2 + \frac{1}{2}r^2\ln(R_2+z_2) \qquad (7.a)$$

$$G = -\frac{1}{3}R_2^3 + \frac{1}{2}zz_2R_2 - \frac{1}{3}ia^3 + \frac{1}{2}zr^2\ln(R_2+z_2) \quad (7.b)$$

$$H = \frac{4}{3}ia^3z - \frac{1}{6}zR_2^3 + \frac{1}{2}iaR_2^3 - \frac{1}{4}z_2R_2r^2 - \frac{r^4}{4}\ln(R_2+z_2) \quad (7.c)$$

The Cartesian components of the tension field generated by (1) have the imaginary parts:

$$p_{xx} = \frac{3\mu P}{2\pi a^3}\frac{x}{r^4}\left[\begin{array}{l}\left(4\frac{x^2}{r^2}-3\right)\left(H\nu - \frac{1}{2}z\frac{\delta H}{\delta z}\right)+ \\ y\frac{\delta H}{\delta y} + (1-\nu)x\frac{\delta H}{\delta x} + \frac{1}{2}xz\frac{\delta^2 H}{\delta x \delta z} - 2\nu r^2 F\end{array}\right] \quad (8.a)$$

$$p_{yy} = \frac{3\mu P}{2\pi a^3}\frac{x}{r^4}\left[\begin{array}{l}\left(4\frac{y^2}{r^2}-1\right)\left(H\nu - \frac{1}{2}z\frac{\delta H}{\delta z}\right) \\ -\nu y\frac{\delta H}{\delta y} + \frac{1}{2}yz\frac{\delta^2 H}{\delta y \delta z} - 2\nu r^2 F\end{array}\right] \quad (8.b)$$

$$p_{zz} = \frac{3\mu P}{2\pi a^3}\frac{xz}{r^4}\frac{\delta F}{\delta z} \qquad (8.c)$$

$$p_{yz} = \frac{3\mu P}{2\pi a^3}\frac{xyz}{2r^4}\frac{\delta^2 H}{\delta z^2} \qquad (8.d)$$

$$p_{xz} = \frac{3\mu P}{2\pi a^3}\frac{1}{r^2}\left[2G + \frac{1}{2}\frac{\delta H}{\delta z} + z\frac{\delta}{\delta x}(xF) - 2\frac{zx^2}{r^2}F\right] \quad (8.e)$$

$$p_{xy} = \frac{3\mu P}{2\pi a^3}\frac{y}{r^4}\begin{bmatrix}\left(4\frac{x^2}{r^2}-1\right)\left(H\nu - \frac{1}{2}z\frac{\delta H}{\delta z}\right) + \\ \frac{1}{2}y\frac{\delta H}{\delta y} + \frac{1}{2}(1-2\nu)x\frac{\delta H}{\delta x} + \frac{1}{2}xz\frac{\delta^2 H}{\delta x \delta z}\end{bmatrix} \quad (8.f)$$

Along the axis z the tension component is:

$$p_{xz} = \frac{3\mu P}{2\pi a^3}\begin{bmatrix}\frac{3}{2}z\arctan\left(\frac{a}{z}\right) - \\ a - \frac{1}{2}az^2\left(z^2+a^2\right)^{-1}\end{bmatrix} \quad (9)$$

On the surface, inside the contact area, z = 0 and r <a, the voltage component will be:

$$p_{fyy} = \left[\frac{3\nu}{(4+\nu)}\right]p_{fxx} = \frac{x}{y}\left[\frac{3\nu}{(2-\nu)}\right] \quad (10)$$

$$p_{fxy} = -\frac{3\mu P}{2\pi a^3}\frac{3}{8}x\pi\nu$$

and outside of the contact area:

$$p_{fxx} = -\frac{3\mu P}{2\pi a^3}\frac{x}{r^4}\left[2\left(r^2+\nu y^2\right)F_0 + \nu\left(3-4x^2r^{-2}\right)H_0\right] \quad (11.a)$$

$$p_{fyy} = -\frac{3\mu P}{2\pi a^3}\frac{\nu x}{r^4}\left[2x^2F_0 + \left(1-4y^2r^{-2}\right)H_0\right] \quad (11.b)$$

$$p_{fxy} = -\frac{3\mu P}{2\pi a^3}\frac{y}{r^4}\left[\left(r^2-2\nu x^2\right)F_0 + \nu\left(1-4x^2r^{-2}\right)H_0\right] \quad (11.c)$$

where:

$$F_0 = -\frac{1}{2}a\left(r^2-a^2\right)^{1/2} + \frac{1}{2}r^2\arctan\left[a\left(r^2-a^2\right)^{-1/2}\right] \quad (11.d)$$

$$H_0 = \frac{1}{2}a\left(r^2-a^2\right)^{3/2} - \frac{1}{4}r^4\arctan\left[a\left(r^2-a^2\right)^{-1/2}\right] - \frac{1}{4}ar^2\left(r^2-a^2\right)^{1/2} \quad (11.e)$$

Inside, respectively, outside the charged surface, the adimensioned pressure relationships can be written as follows:

- For the inside of the loaded surface (z = 0, r <1) of relations (3.10) taking into account the condition (1) results:

$$\overline{p}_{fyy} = -\mu\frac{3\pi}{8}x\nu, \quad (12.a)$$

$$\overline{p}_{fxx} = -\mu\frac{\pi}{8}x(4+\nu) \quad (12.b)$$

$$\overline{p}_{fxy} = -\mu\frac{\pi}{8}y(2-\nu) \quad (12.c)$$

$$\overline{p}_{fyz} = \overline{p}_{fzz} = 0 \quad (12.d)$$

with: $y = \left(r^2 - x^2\right)^{1/2}$

The graphical representation of these pressures is given in figure1, figure 2 and figure 3 for a coefficient of friction μ = 0.6. [9]

It can be observed that the pressure $\overline{p}_{xx}$ has the highest values; respectively a restricted area distribution for pressure $\overline{p}_{xy}$

For a coefficient of friction μ = 0.8[10] the pressure representations within the loaded surface are given in figure 4, figure 5, figure 6.

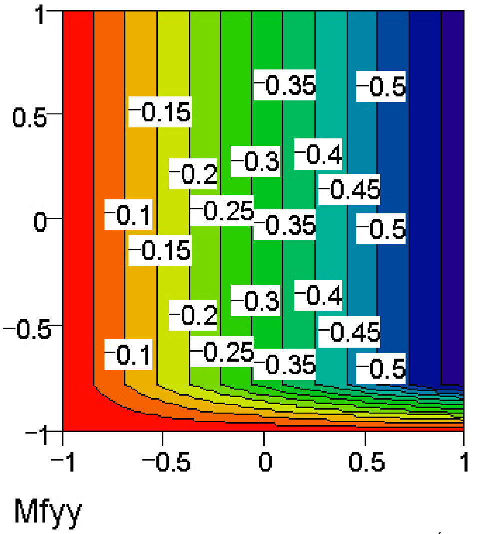

Figure 1.Dependence of pressure $\overline{p}_{yy}\left(x_i, r_j, \mu\right)$ ($x_i = 0 + 0.1i$, $r_j = 0 + 0.1j$,i=0..30,j=1..9)

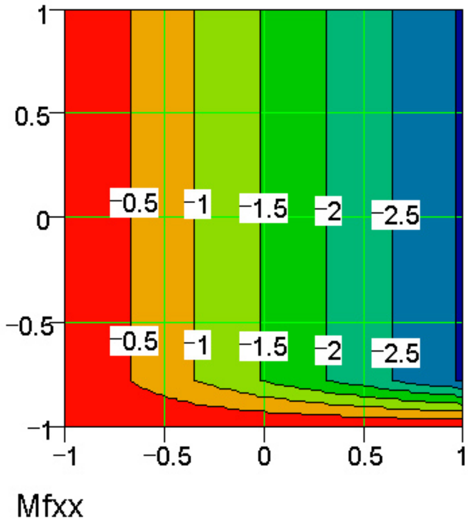

Figure 2. Dependence of pressure $\overline{p}_{xx}\left(x_i, r_j, \mu\right)$ ($x_i = 0 + 0.1i$, $r_j = 0 + 0.1j$,i=0..30,j=1..9)

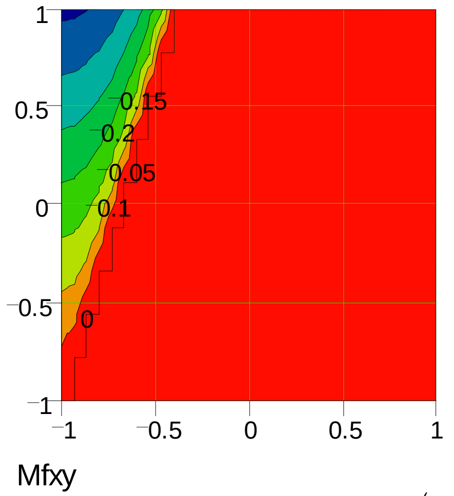

Figure 3. Dependence of pressure $\overline{p}_{xy}\left(x_i, r_j, \mu\right)$ ($x_i = 0 + 0.1i$, $r_j = 0 + 0.1j$,i=0..30,j=1..9)

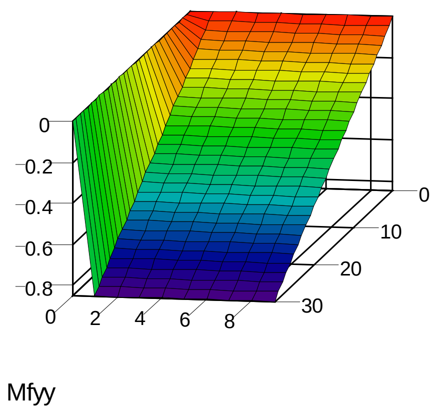

Figure 4. Dependence of pressure $p_{yy}\left(x_i, r_j, \mu\right)$ ($x_i = 0 + 0.1i$, $r_j = 0 + 0.1j$,i=0..30,j=1..9)

For the zone from the outside loaded surface (z = 0, r> 1) of the relations (11) and taking into account the condition (1), it results:

$$\overline{p}_{xxe} = -\mu \frac{x_e}{r^4}\left[2\left(r^2 + \nu y_e{}^2\right)F_0 + \nu\left(3 - 4x_e^2 r^{-2}\right)H_0\right] \qquad (13.a)$$

$$\overline{p}_{yye} = -\mu \frac{\nu x_e}{r^4}\left[2x_e^2 F_0 + \left(1 - 4y_e^2 r^{-2}\right)H_0\right] \qquad (13.b)$$

$$\overline{p}_{xye} = -\mu \frac{y_e}{r^4}\left[\left(r^2 - 2\nu x_e^2\right)F_0 + \nu\left(1 - 4x_e^2 r^{-2}\right)H_0\right] \qquad (13.c)$$

with:

$$F_0 = -\frac{1}{2}\left(r^2-1\right)^{1/2} + \frac{1}{2}r^2 \arctan\left[\left(r^2-1\right)^{-1/2}\right] \qquad (13.d)$$

$$H_0 = \frac{1}{2}\left(r^2-1\right)^{3/2} - \frac{1}{4}r^4 \arctan\left[\left(r^2-1\right)^{-1/2}\right] - \frac{1}{4}r^2\left(r^2-1\right)^{1/2} \qquad (13.e)$$

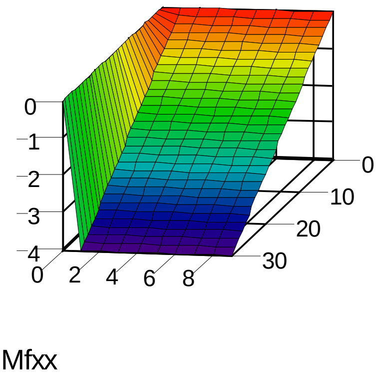

Figure 5. Dependence of pressure $p_{xx}\left(x_i, r_j, \mu\right)$ ($x_i = 0 + 0.1i$, $r_j = 0 + 0.1j$,i=0..30,j=1..9)

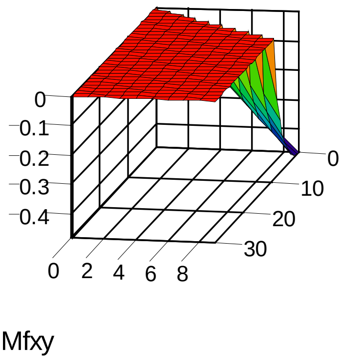

Figure 6. Dependence of pressure $p_{xy}\left(x_i, r_j, \mu\right)$ ($x_i = 0 + 0.1i$, $r_j = 0 + 0.1j$,i=0..30,j=1..9)

The graphical representation of these pressures is given in figure 7, figure 8, figure 9 for a coefficient of friction μ= 0.6. Note that pressures in the outside zone of the loaded area are higher than the pressure p_{xxe}.

For a coefficient of friction μ = 0,8 the pressure representations in the outside area of the loaded surface are given in figure 10, figure 11, figure 12.

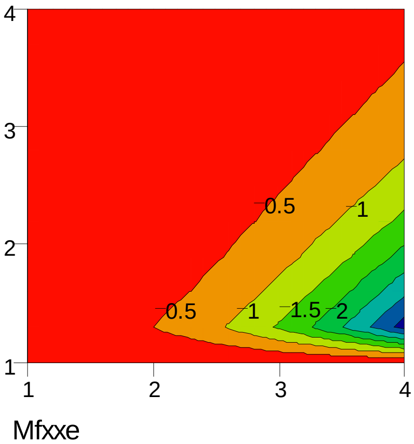

Figure 7. Dependence of pressure $\overline{p}_{xxe}\left(x_i, r_j, \mu\right)$ ($x_i = 0 + 0.1i$, $r_j = 1 + 0.1j$,i=0..30,j=1..10)

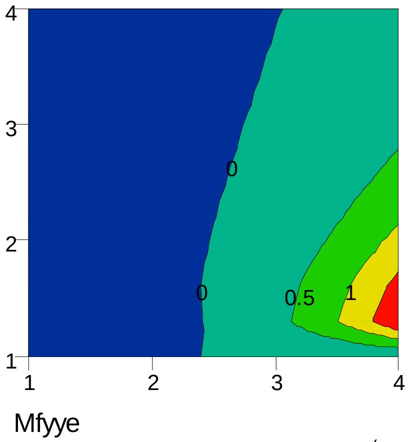

Figure 8. Dependence of pressure $\overline{p}_{yye}\left(x_i, r_j, \mu\right)$ ($x_i = 0 + 0.1i$, $r_j = 1 + 0.1j$,i=0..30,j=1..10)

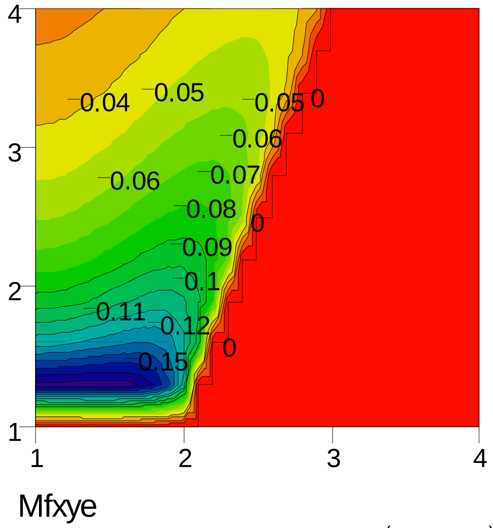

Figure 9. Dependence of pressure $\overline{p}_{xye}\left(x_i, r_j, \mu\right)$
($x_i = 0 + 0.1i$, $r_j = 1 + 0.1j$,i=0..30,j=1..10

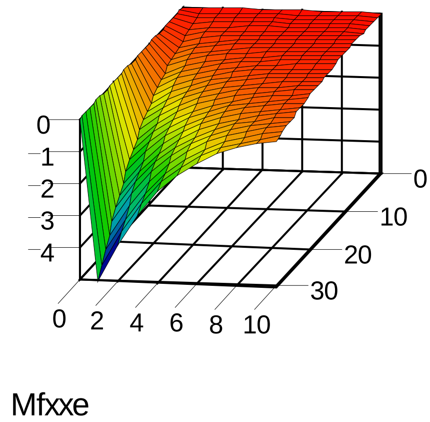

Figure 10. Dependence of pressure $p_{xxe}\left(x_i, r_j, \mu\right)$
($x_i = 0 + 0.1i$, $r_j = 1 + 0.1j$,i=0..30,j=1..10)

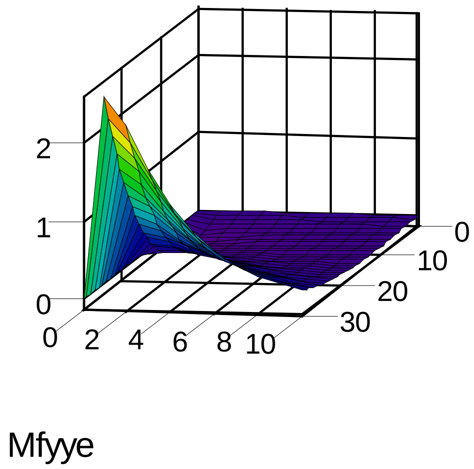

Figure 11. Dependence of pressure $p_{yye}\left(x_i, r_j, \mu\right)$
($x_i = 0 + 0.1i$, $r_j = 1 + 0.1j$,i=0..30,j=1..10)

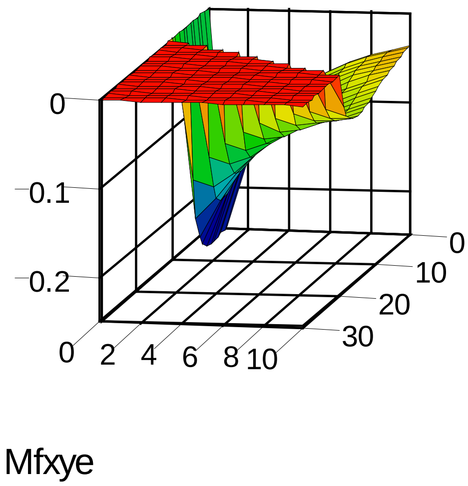

Figure12.Dependence of pressure $p_{xye}\left(x_i, r_j, \mu\right)$
($x_i = 0 + 0.1i$, $r_j = 1 + 0.1j$,i=0..30,j=1..10)

4.Conclusions

Determination of the stresses inside and outside the loaded surface allows for an adequate analysis of the contacts with a plan plane taking into account a coefficient of friction between surfaces.

The analysis allows the representation and determination of pressures for the considered contact.

The field of tension created for a sliding circular contact can be related to material fracture issues. Quantitatively showing interest in the position of pressures, in order to be able to establish criteria to predict the cracking and deterioration of such contacts.

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