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Optimizing the design of compression helical springs

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Abstract. The helical compression spring is calculated in the STAS 7067 standard based on the wire twist tension and considering the type of deformation, fatigue loading and vibration and buckling effect.

To avoid recalculation of the values of already calculated parameters, the proposed calculation method uses the same formulas from STAS, but sets out a set of restrictions that must be observed in the design stages.

To optimize design, start with material selection, set minimum and maximum forces, also the type of deformation as well as the admissible dimensions.

1. Introduction

The springs are machine parts to realise an elastic link between certain parts or subassemblies inside a machine. Through their special shape and special manufacturing materials, the springs have the capacity to deform under an external applied force and storing the work developed by force inside. Once the force cease to act upon the spring the stored work can be released back to the mechanical system.

The springs may be used in various applications, the most significant are:

- ❖ Damping the shocks and vibrations (car suspensions, elastic clutches)
- ❖ Storing the mechanical energy (mechanical clocks, valves etc.),
- ❖ Exerting a permanent elastic force (safety coupling, friction clutches etc.),
- ❖ Regulating and limiting forces inside the systems (air presses safety valves etc.).

The elastic load-displacement diagram for a spring is defined by the dependency of the exerted external force/torque to the elastic displacement/twist of the spring.

Generally in STAS are presented computational formulas but it is not specified how they are used. STAS 7067 provides the formulas for compression springs. Some calculations are required to determine some parameters.

This paper proposes that by imposing some major restrictions and by a series of approximations, the calculation method should be improved in order to avoid recalculating some already calculated parameters.

2. Introductory elements

The helical compression spring is calculated in the STAS 7067 standard based on the wire twist tension and considering the type of deformation, fatigue loading and vibration and buckling effect.

To avoid recalculation of the values of already calculated parameters, the proposed calculation method uses the same formulas from STAS, but sets out a set of restrictions that must be observed in the design stages.

Figure 1 presents the most important characteristics of the compression helical springs (from the STAS)

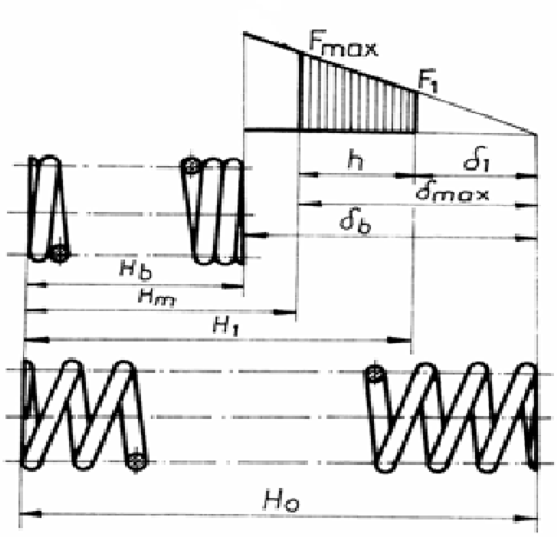

Figure 1

⇨ ***d* – diameter of spring wire**

$$d = \sqrt{\frac{8 \cdot k \cdot F \cdot i}{\pi \cdot \tau_a}} \tag{1}$$

where

i – is the spring index

$i = \frac{D_m}{D}$

k – is the form coefficient of the spring (stress factor or Wahl stress factor)

$k = 1 + \frac{1,6}{i}$

τ_a – is the allowance tension for the torsional stress

⇨ ***f* – deflection of the spring, as a result of axial load F**

$$f = 8 \cdot F \cdot \frac{D_m^3}{G \cdot d^4} \cdot n = 8 \cdot F \cdot \frac{i^3 \cdot n}{G \cdot d} \tag{2}$$

where

n – is the number of active coils

G – is shear modulus of rigidity for the spring material

D_m – is the mean diameter of the coil

⇨ ***H_0* – free length of the spring**

$H_0 = t \cdot n + (n_r - 0,5) \cdot d$ – for plain and ground ends

$H_0 = t \cdot n + (n_r + 1) \cdot d$ – for squared and ground ends (3) where

t – is the pitch of the coils

⇨ $\boldsymbol{H} = \boldsymbol{C}$ – the spring rate or stiffness of the spring

$$C = K = \frac{F}{f} = \frac{G \cdot d^4}{8 \cdot D_m^3} \cdot n = \frac{G \cdot d}{8 \cdot i^3} \cdot n \quad (4)$$

⇨ $\boldsymbol{E}$ – energy stored in the spring

$$E = \frac{1}{2} \cdot F \cdot f \quad (5)$$

The buckling of the compression helical spring is verified using Figure 2 (from STAS); the buckling don't appear if the point having $\boldsymbol{a}$ and λ coordinates is below the curves 1 and 2 in diagram.

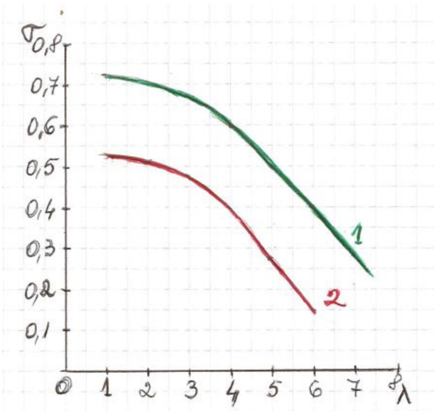

Figure 2

where

a – is the arcing coefficient

$$a = \frac{f}{H_0}$$

λ – is buckling factor

$$\lambda = \frac{H_0}{D_m}$$

The Standard (STAS) don't presents a method of design but consider that the steps of calculations depends on problem data.

3. Consideration on STAS application

- ❖ The spring index $\boldsymbol{i}$ is selected in the start of the calculation, but this selection depends on the $\boldsymbol{d}$ (diameter of the wire), which is not known at the start
- ❖ The constant $\boldsymbol{K}$ (for the calculation on $\boldsymbol{d}$) depends on $\boldsymbol{i}$ value, that is arbitraiy selected
- ❖ The wire diameter $\boldsymbol{d}$ depends on $\boldsymbol{\tau_a}$ and $\boldsymbol{i}$ (which is arbitraiy selected) and $\boldsymbol{K}$ (depending on $\boldsymbol{i}$)
- ❖ The value of spring rate $\boldsymbol{C}$ depends on $\boldsymbol{D_m}$ (which is based on disponible space) and $\boldsymbol{n}$ (which is unknown)
- ❖ If the buckling is produced, we need to modify $\boldsymbol{a}$ and λ , i.e. to modify the values just calculated

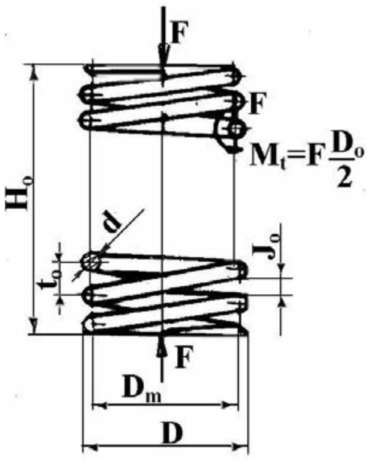

Figure 3

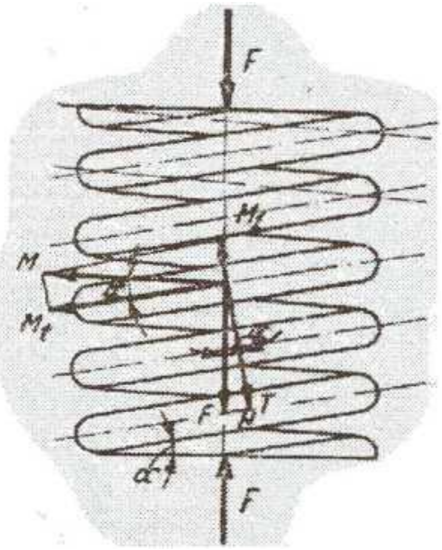

Figure 4

4. Optimizing springs

4.1. Stress of the spring wire

The basic stress in a helical springs, based on Figure 3, are:

- $\boldsymbol{\sigma_t}$ – tensile stress, produced by force $N = F \cdot sin\alpha$
- $\boldsymbol{\sigma_i}$ – bending stress, produced by bending moment $M_i = F \cdot R \cdot sin\alpha$
- $\boldsymbol{\tau_f}$ – shear stress, produced by the force $T = F \cdot cos\alpha$
- $\boldsymbol{\tau_t}; \boldsymbol{\tau_r}$ – torsional stress, produced by the torque $T = F \cdot R \cdot cos\alpha$

Equivalent tension, based on the 3 rd theory of the failure is:

$$\sigma_{ech} = \sqrt{(\sigma_t + \sigma_i)^2 + 4 \cdot \left(\tau_f + \tau_r\right)^2} =$$

$$= \sqrt{\left(\frac{N}{A} + \frac{M_i}{W_z}\right)^2 + 4 \cdot \left(\frac{4 \cdot T}{3 \cdot A} + \frac{M_t}{W_p}\right)^2} \tag{6}$$

and,

$$A = \frac{\pi \cdot d^2}{4} = \pi \cdot r^2 \tag{7}$$

$$W_z = \frac{\pi \cdot d^3}{32} = \frac{\pi \cdot r^3}{4} \tag{8}$$

$$W_p = \frac{\pi \cdot d^3}{16} = \frac{\pi \cdot r^3}{2} \tag{9}$$

And if we substituting the values of $N, T, M_i, M_t, A, W_z, W_p$, results:

$$\sigma_{ech} = \sqrt{\left(\frac{Fsin\alpha}{\pi r^3} + \frac{4FRsin\alpha}{\pi r^3}\right)^2 + 4\left(\frac{4Fcos\alpha}{3\pi r^3} + \frac{2FRcos\alpha}{\pi r^3}\right)^2}$$

$$\sigma_{ech} = \frac{F}{\pi \cdot r^2}\sqrt{sin^2\alpha(1+4i)^2 + 4cos^2\alpha(1{,}33+2i)^2} \qquad (10)$$

If we consider that $cos^2\alpha = 1 - sin^2\alpha$ and transform relation (10), neglecting the terms containing $sin^2\alpha$, we may approximate:

$$\sigma_{ech} \approx \frac{8F(1{,}33+2i)}{\pi d^2} \leq \sigma_a \qquad (11)$$

And, based on the equivalence

$$\tau_{ech} \approx 0{,}75\sigma_{ech} = 0{,}75\frac{8F(1{,}33+2i)}{\pi d^2} \leq \tau_a \qquad (12)$$

And denominate

$$\rho = \frac{d}{D_a}$$

Where,

$D_a -$ the diameter of guidance axis

We obtain a restriction condition

$$i + 0{,}665 + 0{,}196\frac{D_a^2 \cdot \sigma_a}{F} \cdot \rho^2 \leq 0 \qquad (13)$$

Considering $F = F_M$ maximum value of axial load

From the transformations of relations, 0,196 is the value of $\frac{\pi}{16}$ and 0,665 is $\frac{1{,}33}{2}$

For the σ_a values, is possible to use the approximate value

$$\sigma_a = \frac{1{,}3 \cdot \sigma}{3 - R_S} \qquad (14)$$

where

$R_S = \frac{F_{min}}{F_{max}}$ is the coefficient of varying force

4.2. The spring stiffness

The spring stiffness is directly influenced by the free length of the arc H_0 and the effective length H produced by the force F_{max} (Fig. 3 and 4) and is determined by equations 2 and 4.

If f_m is the deflection produced by the maximum force F_m , the following conditions must be met:

$$H_0 - f_m \leq H$$
$$H_0 - f_M \geq n \cdot d \qquad (15)$$

$$n \leq n_0 = \frac{G \cdot H \cdot d}{8(F_M - F_m) \cdot i^3 + G \cdot d^2} \qquad (16)$$

where,
n_0 – Is the number of coils, corresponding to $(F_M - F_m)$ force

If it is imposed a value of deflection $= f_M - f_m$, the conditions are:

$$n \cdot d + f \leq H$$
$$n \leq n_0 = \frac{(H-f)}{d} \tag{17}$$

If the spring must store an imposed amount of energy E,

$$E = \frac{(F_M - F_m)(f_M - f_m)}{2} \tag{18}$$

$$c_0(f_M^2 - f_m^2) = 2E$$
$$=> \; c_0 = \frac{2E}{(f_M^2 - f_m^2)} \tag{19}$$

$$n \leq n_0 = \frac{G \cdot d}{8 \cdot i^3 \cdot c_0} \tag{20}$$

4.3. Dimensional restrictions

If D_c is the disponibile diameter to fit the spring and $\gamma = \frac{D_c}{D_a}$

$$D_m + d \leq D_c$$
$$or$$
$$d \cdot (i+1) \leq D_c \tag{21}$$

and results the condition

$$i + 1 - \frac{\gamma}{\rho} \leq 0 \tag{22}$$

For the inner space D_a, the condition are

$$D_m - d \geq D_a$$
$$or$$
$$d \cdot (i-1) \geq D_a \tag{23}$$

and results the condition

$$i - 1 - \frac{1}{\rho} \leq 0 \tag{24}$$

4.4. Stability restrictions

The value of λ from relation $\lambda = \frac{H_0}{D_0}$ must be lower then λ_{cr} (critical value)

$$\lambda \leq \lambda_{cr} = \frac{2{,}62}{\beta} \tag{25}$$

with β - buckling factor, depending on the form of end connections of the spring and of the guidance form (spring located in the tube, or mounted on a central rod).

It exists two cases:

- ► Determine

$$\beta = \frac{2{,}62 \cdot D_m}{H_0} \tag{26}$$

and establish the need of the guidance and form of end connections.

- ► If β is imposed,

$$=> H_0 = \frac{2{,}62 \cdot D_m}{\beta}$$

and determine the number of active coil as

$$n = \frac{\left(\frac{2{,}62 \cdot D_m}{\beta} - H\right) \cdot G \cdot d}{8 \cdot F_m \cdot i^3} \tag{27}$$

4.5. Other restrictions

Based on manufacture technology of the spring, finally results $i_{min} > 5$, to avoid supplementaiy tensions in the wire, in the manufacturing process.

5. Optimum design algoritm

5.1. Imposed data

- The allowable dimensions: D_c, D_a, H
- Minimum force F_m; maximum force F_M
- Energy to be stored E
- Minimum deflection f_m; maximum deflection f_M

5.2. The materials

- STAS provide the characteristics of the steels for springs.
- based on relation (14), we determine σa, with $R_S = \frac{F_m}{F_M}$
- we can select the material based on σ_a, considering the costs of material and other economical criteria.

5.3. Undimensional coefficients

- ► The A parameters, defined from relation (13)

$$A = 0{,}196 \frac{D_a^2 \cdot \sigma_a}{F_m} \tag{28}$$

And resulting from relation (28)

$$i = \lambda \cdot \rho^2 - 0{,}665 \tag{29}$$

For the case of an imposed material, with known σ_a value

- ► If the material is not imposed, we can select him for another value of σ_a in calcuiation of A and imposing the conditions (22) and (13), resulting the approximate value

$$\rho = \frac{\gamma}{i+1}$$

$$i = \frac{A \cdot \gamma^2}{(i+1)^2} - 0{,}665 \tag{30}$$

and

$$(i+1)^3 \approx A \cdot \gamma^2 \tag{31}$$

► From relation (30)

$$=> A \cdot \gamma^2 = (i + 0{,}665) \cdot (i+1)^2 \tag{32}$$

Approximately

$$A \cdot \gamma^2 \approx (i+1)^3$$

$$A = (i-1)^3$$

$$i = \sqrt[3]{A \cdot \gamma^2} - 1 \tag{33}$$

5.4. The calculation steps

⇨ The compatibility of the problem results from

$$\sqrt[3]{A} \geq \left\{ \frac{1{,}6}{\sqrt[3]{\gamma^2 - 1}} ; \frac{6}{\sqrt[3]{\gamma^2}} \right\} \tag{34}$$

⇨ If the relation (34) is accomplished,

$$i = \sqrt[3]{A \cdot \gamma^2} - 1 \tag{35}$$

And from relation (30) results ρ and from $\rho = \frac{d}{D_a}$, results d

⇨ If relation (29) is accomplished, the case is of imposed material and results another value of γ from relation (30)

⇨ If $i = \frac{\gamma}{\rho} - 1$, the material is to be selected using relations (30) and (28)

$$\sigma_a \approx \frac{5 \cdot F_M (i + 0{,}665)}{D_a^2 \cdot \rho^2} \tag{36}$$

⇨ The number of the active coils is determined by relations (16) or (17) and H_0, and H by relation (15)

⇨ If it is imposed a value of energy stored by the spring relation (18), the number of coils is determined by relation (20)

⇨ The stability (buckling) – is ensured by β determination from relation (25) and modifying the form of end connections and guidance of the spring, or determining the number of the coils by relation (27)

6. Conclusions

a) To avoid recalculation of the values of already calculated parameters, the proposed calculation method uses the same formulas from STAS, but sets out a set of restrictions that must be observed in the design stages.
b) To optimize design, start with material selection, set minimum and maximum forces, also the type of deformation as well as the admissible dimensions.
c) The method eliminates the re-calcuJations resulting from standard method, based on the restriction analysis.
d) This comment has not analyzed the spring vibrations (because of the complexity of the investigation) and the fatigue calculation (same motivation).

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