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A WAVE ENERGY CONVERSION SCHEME BASED ON ROLL PARAMETRIC EXCITATION OF A FIVE-HULLED TRIMARAN BARGE

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Abstract: *As a method of obtaining power from the gravity waves that are abundant in seas, a "point-absorber" type wave energy extraction device, based on nonlinear Hill/Mathieu equation has been conceived. A special barge type device, having five hulls symmetrical both with respect to the centerline and to the waterline, and moored in a position to receive waves from the beam and thus forced to roll is the basis of this approach. At small angles of roll, the barge can be analyzed as a wall-sided hull. However, above a certain angle of roll, the hull shall become a trimaran, causing the moment of inertia and the metacentric radius and metacentric height increase rapidly. This shall induce a quadratic term to the righting arm term of the uncoupled equation of roll. Since the roll equation is periodic, by the analysis that shall be outlined in the following paper, the roll equation is a form of the Mathieu equation, causing a parametric roll phenomenon between two extreme angles. The energy of the waves as transferred to the rolling motion can be extracted by a pendulous mechanism that can be used to get electrical energy to be transmitted ashore by cables. The energy extracted shall be accounted as a term of the damping term of the equation of roll.*
The concept is examined by the numerical solution of the roll equation. An estimate of power from a typical wave at the same order of the barge's dimensions is made.
Keywords: *Parametric resonance, Mathieu equation, wave energy, rolling motion.*

Introduction

Point and line absorber types of wave energy devices that follow the contour of the wave surface are the most efficient of the wave energy conversion schemes, as witnessed by the Salter's nodding duck and Pelamis concepts (Cruz, 2008). The concept conceived by the authors is a floating barge type structure, having five hulls symmetrical bothwith respect to the centerline and to the waterline. The barge is moored in a position to receive waves from the beam and thus forced to roll.

Uncoupled rolling motion of a hull is given by the differential equation:

$$(J+J_{add})\ddot{\varphi}+C(\dot{\varphi})+\Delta\cdot GZ(\varphi)=M_w(t) \quad 1)$$

where J is the moment of inertia of the hull for rolling motion, J_{add} is the added moment of inertia for the rolling motion, $C(\dot{\varphi})$ is the damping moment, Δ is the displacement weight and $GZ(\varphi)$ is the righting moment arm. The dot notation is used for derivatives with respect to time. In nondimensional terms, realizing that the two moment of inertias can be expressed as:

$$J+J_{add}=mk^2 \quad (2)$$

and the displacement weight as:

$$\Delta=mg \quad (3)$$

The equation of roll in normalized terms becomes:

$$\ddot{\varphi}+C'(\dot{\varphi})+\frac{g}{k^2}GZ(\varphi)=M'_w(t) \quad (4)$$

The wave-induced righting moment $M'_w(t)$ for a trochoidal wave of angular frequency Ω, height ζ and wavelength λ can be given as (Sabuncu, 1993):

$$M'_w(t)=\left[\frac{g\cdot GM}{k^2}\right]\cdot\frac{2\pi\xi}{\lambda}sin\Omega t \quad (5)$$

For small angles of roll, the righting moment arm GZ_0 scan be expressed as:

$$GZ_0=GM_0 sin\varphi\cong GM_0\cdot\varphi \quad (6)$$

where GM_0 is the metacentric height at upright position. In this case, the natural frequency of roll, ω_n is expressed as:

$$\omega_n=\frac{\sqrt{g\cdot GM_0}}{k^2} \quad (7)$$

If the damping term, $(C'(\dot{\varphi}))$ is approximated to be a linear term of angular velocity, $\dot{\varphi}$, in the form $2\zeta\omega_n$, the differential equation becomes:

$$\ddot{\varphi}+2\zeta\omega_n\dot{\varphi}+\omega_n^2\varphi=M'_w(t) \quad (8)$$

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The Proposed Concept

The proposed wave energy barge shall have a wall-sided central hull which has a rather small, but positive stability. To each sides, and both above and below the waterline, there are four side hulls, symmetrical both about the calm water waterline and centerline (Figure 1). As the hull rolls above a certain angle, the side hulls shall be immersed symmetrically, with waterline areas increasing progressively. This shall impose a rapidly increasing stability and a righting moment arm, *GZ*(φ). In order to reduce resistance to rolling, underwater side hulls are connected to the central hull with streamlined struts.

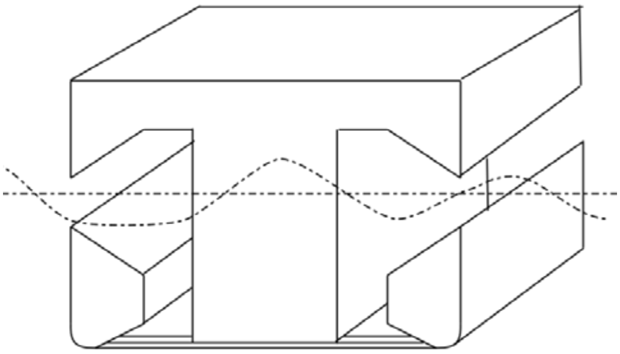

Figure 1. The proposed wave-energy barge

Three regimes of stability as the hull rolls can be observed (Figure 2):

1. $\varphi \leq tan^{-1}\frac{2h}{B} \Rightarrow$

$$GZ(\varphi) = sin\varphi\left(GM + \frac{1}{2}BMtan^2\varphi\right),$$

with single hull

2. $tan^{-1}\frac{2h}{B} < \varphi \leq tan^{-1}\left[\frac{2(h+wtan\theta)}{B-2w_1}\right] \Rightarrow$

GZ, as calculated in the Appendix 1

3. $\varphi > tan^{-1}\left[\frac{2(h+wtan\theta)}{B-2w_1}\right] \Rightarrow$

$$GZ(\varphi) = sin\varphi\left(GM + \frac{1}{2}BMtan^2\varphi\right),$$

with three hull configuration

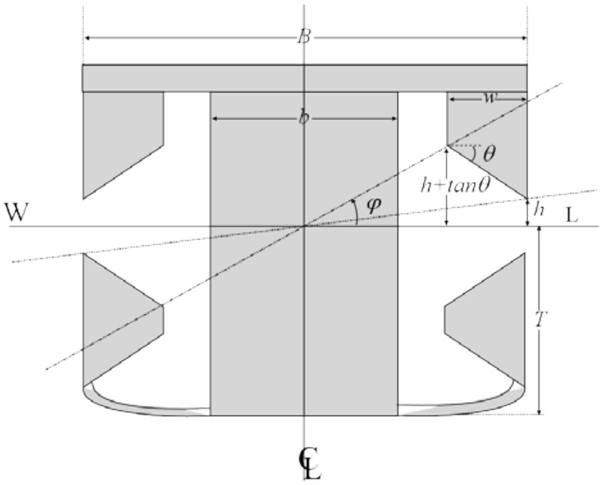

Figure 2. Geometric nomenclature and regimes of roll of the barge

Stability analysis of the barge in the second region has been made in Appendix 1. The calculations have revealed a change in the righting moment arm *GZ*(φ) at heel angles of regimes 2 and 3 can be approximated as a quadratic function:

$$GZ(\varphi) = A\varphi|\varphi| \quad (10)$$

This results in a differential equation in the form:

$$\ddot{\varphi} + 2\zeta\omega_n\dot{\varphi} + \varphi(\omega_n^2 + A|\varphi|) = M'_w(t) \quad (11)$$

In the literature, the solution of homogeneous differential equations in the form:

$$x'' + ax' + bc + cx|x| = 0 \quad (12)$$

Have been given in terms of Jacobi elliptic functions, for example, Tamura and Li (1990). However, for the purpose of this paper, an approximate solution of the general solution has been adopted.

The solution for φ shall be harmonic with frequency ϖ. Substituting a trial solution $\varphi = \Phi\, cos\varpi t$, and using the Fourier approximation the term $|cos\varpi t|$:

$$|cos\varpi t| = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n cos2n\varpi t}{4n^2-1}$$

$$|cos\varpi t| = \frac{2}{\pi}\left(1 + \frac{2}{3}cos2\varpi t - \frac{2}{15}cos4\varpi t + \frac{2}{35}cos6\varpi t - \cdots\right) \quad (13)$$

Approximating the Fourier series above with its first two terms, and substituting into the roll equation,

$$\ddot{\varphi} + 2\zeta\omega_n\dot{\varphi} + \varphi[(\omega_n^2 + 0.5366A\Phi) + 0.4244A\Phi cos2\omega_n t] = M'_w cos\Omega t \quad (14)$$

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This last equation is the damped Mathieu equation. Discussion of this equation to ship stability has been made in various publications, for example, Francescutto et al, (2004). For a wave energy extraction device, the energy extracted can also be expressed in terms of an additional damping. However, to investigate the condition for the threshold value of damping above which there shall be parametrically excited rolling, a rough estimate has been made, as can be seen in the Appendix 2.. The barge encountering beam waves with frequency $\Omega = \omega_n$ shall experience parametric periodic motion if the equivalent damping coefficient is less than the threshold value:

$$\zeta < \frac{h}{4} \quad \text{or} \quad \zeta < \frac{0.1061A\Phi}{\omega_n^2+0.5366A\Phi} \tag{15}$$

where *h* is the term of the Mathieu equation:

$$\ddot{\varphi} + 2\zeta\varpi\dot{\varphi} + \varpi_n^2(1 + 2hcos2\varpi t)\varphi = M_w cos\varpi t \tag{16}$$

Implying that with high values of $A\Phi$ or low values of ω_n, ζ< 0.2 is the condition of parametric rolling, from which energy can be extracted.

Energy Extraction from Rolling

Energy extracted from rolling motion can be expressed as:

$$P = \frac{1}{T}\int_0^T M_D(t)\cdot\dot{\varphi}\cdot dt = \frac{C}{T}\int_0^T \dot{\varphi}^2\cdot dt \tag{17}$$

if the damping moment M_D is linear with the angular velocity of roll, $\dot{\varphi}$.

For harmonically rolling ship, substituting $\varphi = \Phi cos\ \omega t$; and integrating,

$$P = \frac{1}{2}C\omega^2\Phi^2 \tag{18}$$

In terms of the linear damping coefficient $\zeta = \frac{C}{2\omega_n(J+J_{add})}$, power extracted from rolling is:

$$P = (J + J_{add})\omega_n^3\Phi^2\zeta \tag{19}$$

Part of this power is accounted by the "natural" damping, with viscous and potential components, and part is by the energy extracted to the energy grid.

Energy can be expressed by a pendulous mechanism, which can activate a hydraulic pumps to obtain a rotating shaft through a hydraulic motor or, directly by a linear generator concept.

A conceptual barge

A representative barge, whose dimensions are from Figure 2, has been analyzed. Principal dimensions are:

Central hull beam	b = 2.0 m
Side hull width	w = 1.0 m
Total beam	B = 4.0 m
Side hull clearance from WL	h = 0.25 m
Side hull taper angle	θ = 30°
Draught	T_d= 1.80 m
Displacement	∇= 63 m^3
Length	L = 10 m
Center of buoyancy height	KB = 0.90 m
Center of gravity height	KG = 0.80 m

The natural period of roll of this barge has been found through standard procedures to be 10.58 seconds, or, ω_n =0.594 rad/s.

The righting moment arm of the barge, GZ is found to be approximated by:

$$GZ \cong 3.475\varphi|\varphi| \tag{20}$$

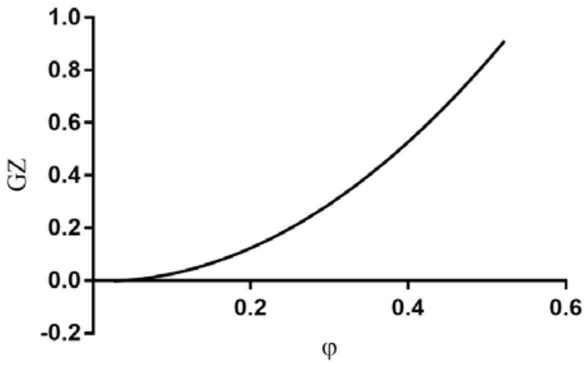

Figure 3. Righting moment arm, GZ as a function of angle of heel, φ

The motion of the hull is analyzed when waves equal to the natural period of roll are encountered. Oscillations of the hull are solved through the Runge-Kutta method. Solutions are shown in Figures 4 and 5 for damping factor ζ= 0.3 and for a wave having amplitude 0.5 m and wavelength 20 m, the barge encountering waves with a period 10.6 seconds and initially at rest. The roll amplitude is 0.170 radians, or 9.7 degrees.

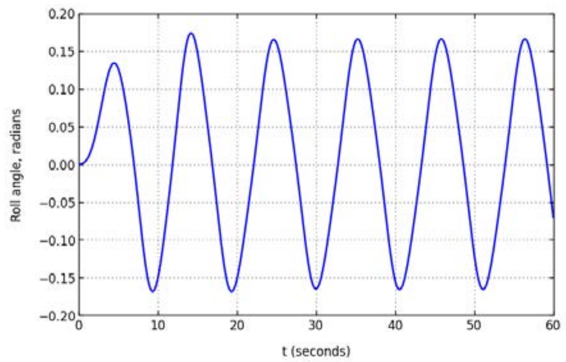

Figure 4. Rolling of the barge for damping factor ζ= 0.3 and wave amplitude ξ=0.5 m, wavelength λ=10 m, frequency of encounter ω=0.594 rad/s.

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The phase plot or rolling motion is in Figure 5.

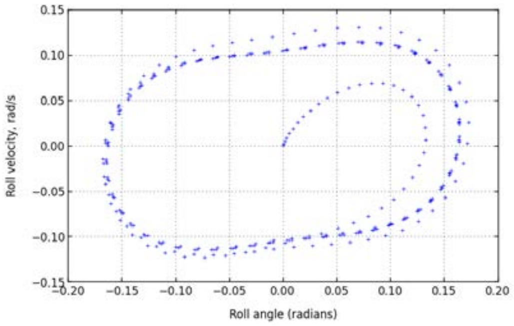


Figure 5. Phase plot of the rolling motion

Energy of the system can be estimated from equation (19) to be 0.6 kW assuming half of the dissipated energy is converted into electrical energy. On the other hand, the energy of the incoming wave (per meter of wavefront) is (Cornett, 2008):

$$E = \frac{\rho g^2 T \xi^2}{32\pi} \quad (21)$$

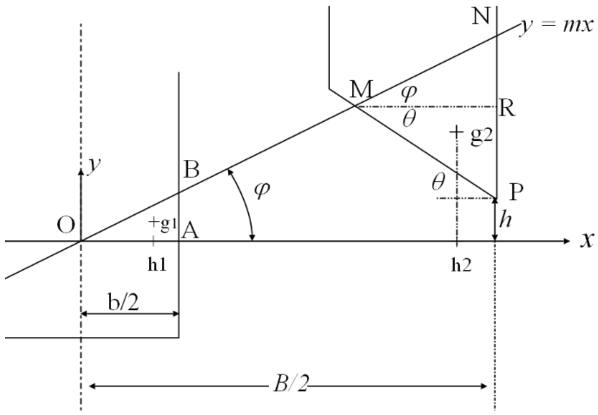
from which the energy of the wave is estimated to be 26 kW, for a 10 m wavefront.

Conclusions

A wave energy system based on parametric resonance is proposed. The feasibility of the system is believed to be enhanced by further research into the form, and by the proper selection of the power takeoff system. It is recommended to perform further research on tis or similar concepts that exploit large amplitude oscillations of the parametric roll phenomena.

Appendix 1

Calculation of Moment Arm, *GZ* for Region 2 of Heeling



In the Figure A-1 above, calling $m = \tan\varphi$ and $s = \tan\theta$,
Triangle OAB (central hull immersed wedge):
Area: $v_1 = \frac{1}{8} b^2 m$ Ordinates of center of area: Oh1 = 1/6 *b* , g1h1=1/6 *bm*

Triangle MNP (side hull immersed wedge):
Line MN : $y = mx$;
Line MP : $y = h - s\left(\frac{B}{2} - x\right)$
Point M :($\frac{h+\frac{B}{2}s}{m+s}$, $m\left(\frac{h+\frac{B}{2}s}{m+s}\right)$
Point P: (*B/2, h*),
Point N (*B/2, mB/2*)
Side *PN= mB/2 - h* ,
Side $MR = \frac{B}{2} - \frac{h+\frac{B}{2}s}{m+s} = \frac{\frac{B}{2}m-h}{m+s}$

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Area : $v_2 = \frac{\left(\frac{B}{2}m-h\right)^2}{2(m+s)}$

Ordinates of center of area of triangle MNP:

$Oh2 = \frac{B}{2} - \frac{1}{3}\left(\frac{\frac{B}{2}m-h}{m+s}\right)$, $g2h2 = \frac{B}{2}m + \frac{2h}{3} + \left(\frac{\frac{B}{2}m-h}{m+s}\right)cos\theta$

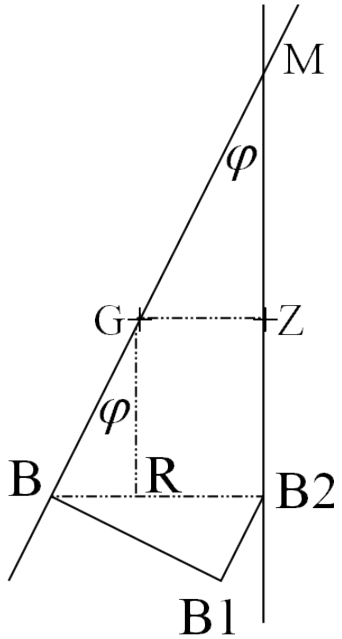

Figure A2. Calculation of moment arm, GZ

Distance BB1: $BB1 = \frac{2}{\nabla}(v_1 \cdot Oh1 + v_2 \cdot Oh2)$

Distance B1B2: $B1B2 = \frac{2}{\nabla}(v_1 \cdot g1h1 + v_2 \cdot g2h2)$

Distance BB2: $BB2 = BB1cos\varphi + B1B2sin\varphi$

Moment arm GZ: $GZ = BB2 - BGsin\varphi$

Appendix 2:
Calculation of critical damping coefficient

Starting with the canonical form of the Mathieu equation, forced by the resonant forcing term:

$$\ddot{\varphi} + 2\zeta\omega_n\dot{\varphi} + \omega_n^2(1 + hsin2\omega_n t)\varphi = Esin\omega_n t$$

Substituting $\varphi = \phi cos\omega_n t$, $\dot{\varphi} = -\omega_n sin\omega_n t$, $\ddot{\varphi} = -\omega_n^2\varphi$

$-\omega_n^2\varphi - 2\zeta\omega_n\dot{\varphi} + \omega_n^2(1 + hsin2\omega_n t)\varphi = Esin\omega_n t$

Simplifying,

$$-2\zeta\omega_n\phi sin\omega_n t + h\omega_n^2 cos\omega_n t \cdot sin2\omega_n t = Esin\omega_n t$$

Using the trigonometric identity for $\sin\alpha \times \cos\beta$,

$-2\varsigma\omega_n\Phi\sin\omega_n t + \frac{1}{2}h\omega_n^2\left(\sin\omega_n t + \sin 3\omega_n t\right) = E\sin\omega_n t$ The third harmonic term with $3\omega_n t$ does not contribute to the resonant motion and is eliminated. Therefore,

$$\Phi = \frac{2E}{(h - 4\zeta)\omega_n^2}$$

Above which when $h > 4\zeta$ the wave amplitude shall be positive and the system shall be resonant.

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