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STUDYING A TWO-DIMENSIONAL NONLINEAR MODEL FOR GALLOPING OF ICED CONDUCTORS BY A MODIFIED VARIATIONAL ITERATION METHOD

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Abstract: *Galloping is a self-excited aeroelastic oscillation of slender structures, such as high voltage overhead lines or tall buildings, characterized by large-amplitudes and low-frequencies. The movement of the excited structure develops commonly transverse to the wind but other translational or rotational motions have been observed on the field. In the paper, a two-dimensional weakly nonlinear model of an iced suspended cable, having as degrees-of-freedom the vertical plunge and the rotation around the elastic axis, is introduced. The sysem is excited by a uniform wind and susceptible to galloping. A modified variational iteration method is employed to obtain a system of four amplitude-frequency modulation equations, that yields both the transient and the steady-state behaviors. The influence of wind speed on the initiation of galloping as well as on the amplitude of oscillation is analyzed in far-from resonance conditions. The theoretical results derived in the paper have been applied to a typical section model and the numerical results are contrasted with those provided by the direct integration of equations of motion.*
Keywords*: Galloping, Two-dimensional model, Modified variational iteration method*

INTRODUCTION

Galloping ia a flow-induced oscillation of a lightly damped structures having an aerodinamically unstable cross-section. It affects mainly the iced conductors of the power transmission lines, the tall and slender buildings and the bridge decks. Usually, the affected structure performs a one-degree-of-freedom motion at nearly its natural frequency, whih is in the order of 1 Hz. Galloping occurs even at low flow speeds and the explanation consista in the presence of a negative aerodynamic damping, meaning that the aerodynamic force performs positive work on the structure. Oscillations can exhibit high amplitudes involving serious implications for the structure's safety. For example, in the case of an iced conductor the amplitudes range from 0.1 to 1.0 times the sag of the span. This may cause flashover and large additional loading stress on insulators, support hardware and tower components, raising the risk of mechanical failure.

Since 1932, when den Hartog [1] introduced a condition for the vertical galloping of iced conductors to appear, a lot of theoretical research and experimental tests have been conducted aiming of understanding, controlling and preventing galloping. Different models, with finite or infinite number of degrees-of-freedom, have been developed [2-6]. Between them, a significant role in the galloping study has had the two-degrees-of-freedom vertical and torsional galloping mechanism developed by Nigol and Clarke [7].

In the next section we present briefly their model and derive the equations of motion. Because they are weakly nonlinear there exist appropriate methods, including Krylov-Bogoliubov method or Multiple time scales method, to obtain approximate analitycal expressions for their periodic or quasi-periodic solutions [8, 9]. In the paper, we employed instead a modified variational iteration method (see the third section) to derive the so-called amplitude-frequency modulation equations, that describe both the transient and steady-state behaviors in far-from resonance conditions. Finally, the approximate solutions derived in the fourth part are contrasted with those provided by the numerical integration and the conclusions are formulated.

TWO-DEGREES-OF-FREEDOM MODEL FOR GALLOPING. SHORT DESCRIPTION

Consider a slender structure (an iced cable for example) having a cross-section of an arbitrarily shape exposed to a horizontal wind field of constant velocity $\vec{V}_{\infty}$ (see Figure 1).

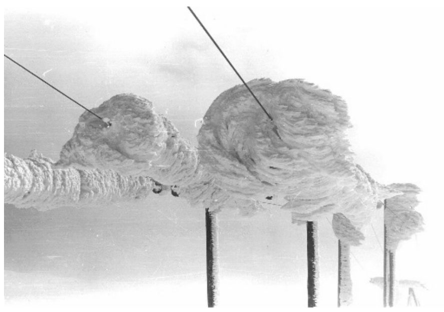

Figure 1. Ice – accretion on cables

The structure is reported to a Cartesian frame having *y* axis perpendicular to the wind direction

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and has two-degrees-of-freedom, specifically the vertical plunge *y* and the torsional angle θ around the elastic axis (see Figure 2).

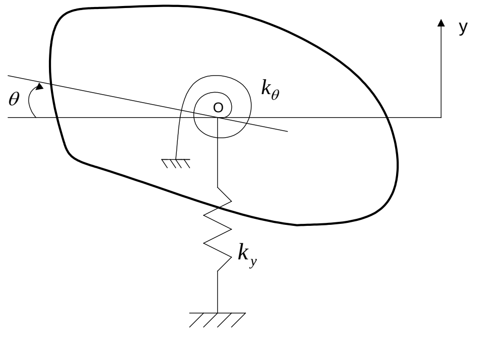

Figure 2. A two-degrees-of-freedom model for galloping

Let *m* and J_O be the body mass and the moment of inertia per unit length. Withal, the vertical and torsional stiffness coefficients are denoted by k_y and k_θ, while c_y and c_θ stand for vertical and torsional damping coefficients (the dampers are not represented in the figure). The structure will oscillate according to the following system of equations

$$\begin{cases} m\ddot{y}+c_y\dot{y}+k_y y=F_y \\ J_O\ddot{\theta}+c_\theta\dot{\theta}+k_\theta\theta=M \end{cases} \qquad (1)$$

Here, F_y and M signify the projection of the aerodynamic force on y directionand the aerodynamic momentum, respectively. They depend on the wind's angle of attack, α, as follows

$$F_y=\frac{1}{2}\rho_a V_\infty^2 d\, C_y(\alpha),\ M=\frac{1}{2}\rho_a V_\infty^2 d^2 C_M(\alpha)$$

where ρ_a is air density, *d* a suitable cross-section's reference length and C_y, C_M the aerodynamic coefficients. According to the field measurements, the last quantities can be expressed by cubic polynomials in angle α

$$C_y=a_1\alpha+a_3\alpha^3,\ C_M=b_1\alpha+b_3\alpha^3$$

At its turn, the angle α can be approximated by

$$\alpha\cong\theta-\frac{1}{V_\infty}\dot{y}-\frac{R_1}{V_\infty}\dot{\theta}$$

where R_1 is another reference length of the cross-section [7]. From the above relations, one gets

$$\begin{cases} \ddot{y}+\omega_y^2 y=-\dfrac{c_y}{m}\dot{y}+\dfrac{\rho_a V_\infty^2 d}{2m}\left(a_1\alpha+a_3\alpha^3\right) \\ \ddot{\theta}+\omega_\theta^2\theta=-\dfrac{c_\theta}{J_O}\dot{\theta}+\dfrac{\rho_a V_\infty^2 d^2}{2J_O}\left(b_1\alpha+b_3\alpha^3\right) \end{cases} \qquad (2)$$

where $\omega_y=\sqrt{\dfrac{k_y}{m}}$ and $\omega_\theta=\sqrt{\dfrac{k_\theta}{J_O}}$ are the corresponding natural frequencies. With help of notations

$$\bar{y}=\frac{y}{d}, U=\frac{V_\infty}{\omega_y d}, \tau=\omega_y t, \eta_y=\frac{\rho_a d^2}{2m}, r=\frac{\omega_\theta}{\omega_y},$$

$$\eta_\theta=\frac{\rho_a d^4}{2J_O}, \xi_y=\frac{c_y}{2m\omega_y}, \xi_\theta=\frac{c_\theta}{2J_O\omega_\theta}, \bar{R}_1=\frac{R_1}{d}$$

the system (2) is rewritten in the non-dimensional form

$$\begin{cases} \dfrac{d^2\bar{y}}{d\tau^2}+\bar{y}=-2\xi_y\dfrac{d\bar{y}}{d\tau}+\eta_y U^2\left(a_1\bar{\alpha}+a_3\bar{\alpha}^3\right) \\ \dfrac{d^2\theta}{d\tau^2}+r^2\theta=-2r\xi_\theta\dfrac{d\theta}{d\tau}+\eta_\theta U^2\left(b_1\bar{\alpha}+b_3\bar{\alpha}^3\right) \end{cases} \qquad (3)$$

where $\bar{\alpha}=\theta-\dfrac{1}{U}\dfrac{d\bar{y}}{d\tau}-\dfrac{\bar{R}_1}{U}\dfrac{d\theta}{d\tau}$.

All the terms in the right sides of system (3) are small compared with those of the left sides, so the dynamical equations (3) are weakly nonlinear. To highlight this, one uses the small parameter $\varepsilon<<1$ in rewritting the system (3) as

$$\begin{cases} \dfrac{d^2\bar{y}}{d\tau^2}+\bar{y}=\varepsilon f_y\left(\bar{y},\theta,\dfrac{d\bar{y}}{d\tau},\dfrac{d\theta}{d\tau}\right) \\ \dfrac{d^2\theta}{d\tau^2}+r^2\theta=\varepsilon f_\theta\left(\bar{y},\theta,\dfrac{d\bar{y}}{d\tau},\dfrac{d\theta}{d\tau}\right) \end{cases}$$

(4)

where

$$\begin{cases} f_y=-2\hat{\xi}_y\dfrac{d\bar{y}}{d\tau}+\eta_y U^2\left(\hat{a}_1\bar{\alpha}+\hat{a}_3\bar{\alpha}^3\right) \\ f_\theta=-2r\hat{\xi}_\theta\dfrac{d\theta}{d\tau}+\eta_\theta U^2\left(\hat{b}_1\bar{\alpha}+\hat{b}_3\bar{\alpha}^3\right) \end{cases}$$

and

$$\xi_y=\varepsilon\hat{\xi}_y, \xi_\theta=\varepsilon\hat{\xi}_\theta, \eta_y=\varepsilon\hat{\eta}_y, \eta_\theta=\varepsilon\hat{\eta}_\theta.$$

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VARIATIONAL ITERATION METHOD. BASIC IDEA

Consider a general nonlinear system as follows

$$\mathbf{L}(\mathbf{u}(t)) + \mathbf{N}(\mathbf{u}(t)) = \mathbf{g}(t)$$

(5)

where **L** and **N** are linear and nonlinear oerators respectively, **g** is a known continuous function and *t* is the time. The basic idea of the method is to construct a correction functional for the system (5) in the form

$$\mathbf{u}_{n+1}(t) = \mathbf{u}_n(t) + \int_0^t \boldsymbol{\lambda}(s)\cdot\left[\mathbf{L}(\mathbf{u}_n) + \mathbf{N}(\tilde{\mathbf{u}}_n) - \mathbf{g}(s)\right]ds \tag{6}$$

where $\boldsymbol{\lambda}$ is a general Lagrange multiplier that can be identified optimally via the variational theory, $\mathbf{u}_n$ is the approximate solution of order *n* and $\tilde{\mathbf{u}}_n$ denotes a restricted variation, i.e $\delta\tilde{\mathbf{u}}_n = 0$ [10]. Thus, for the equation

$$\frac{d^2u}{dt^2} + \omega^2 u = g(t)$$

the Lagrange multiplier results as

$$\lambda = \frac{1}{\omega}\sin\omega(s-t).$$

Having λ determined and using a selective zeroth approximation $\mathbf{u}_0$, several approximations $\mathbf{u}_n, n \geq 1$ can be determined with iterative formula (6). Finally, the solution of problem (5) is given by

$$\mathbf{u}(t) = \lim_{n\to\infty}\mathbf{u}_n(t)$$

(7)

SOLUTION FOR SYSTEM (4) BY A MODIFIED VARIATIONAL ITERATION METHOD

The zeroth order approximations for the solution of (4) are chosen considering a small deviation from the case $\varepsilon = 0$

$$\bar{y}_0(\tau) = a_y(\tau)\cos\psi_y(\tau) + \varepsilon Y(\tau)$$
$$\theta_0(\tau) = a_\theta(\tau)\cos r\psi_\theta(\tau) + \varepsilon\Theta(\tau)$$

(8)

The amplitudes a_y, a_θ and the total phases ψ_y, ψ_θ are supossed to be slowly varying in non-dimensional time τ [11]. That is

$$\begin{cases} \dfrac{da_y}{d\tau} = \varepsilon A(a_y), \dfrac{d\psi_y}{d\tau} = 1 + \varepsilon B(a_y) \\ \dfrac{da_\theta}{d\tau} = \varepsilon C(a_\theta), \dfrac{d\psi_\theta}{d\tau} = 1 + \varepsilon D(a_\theta) \end{cases}$$

(9)

Before applying the recursive formula (6) one needs to differentiate twice with respect to τ the functions $\bar{y}_0(\tau)$ and $\theta_0(\tau)$. Doing this and retaining only the terms of $\mathrm{O}(\varepsilon)$ one has

$$\frac{d\bar{y}_0}{d\tau} = \frac{da_y}{d\tau}\cos\psi_y - a_y\sin\psi_y\frac{d\psi_y}{d\tau} + \varepsilon\frac{dY}{d\tau} \overset{(9)}{=}$$

$$= -a_y\sin\psi_y + \varepsilon\left[A\cos\psi_y - a_y B\sin\psi_y + \frac{dY}{d\tau}\right]$$

$$\frac{d\theta}{d\tau} = \frac{da_\theta}{d\tau}\cos r\psi_\theta - ra_\theta\sin r\psi_\theta\frac{d\psi_\theta}{d\tau} + \varepsilon\frac{d\Theta}{d\tau} \overset{(9)}{=}$$

$$= -ra_\theta\sin r\psi_\theta + \varepsilon\left[C\cos r\psi_\theta - ra_\theta D\sin r\psi_\theta + \frac{d\Theta}{d\tau}\right]$$

and

$$\frac{d^2\bar{y}_0}{d\tau^2} = -a_y\cos\psi_y + \varepsilon\left[-2A(a_y)\sin\psi_y + \frac{d^2Y}{d\tau^2} - \right.$$
$$\left. - 2a_y B(a_y)\cos\psi_y\right]$$

$$\frac{d^2\theta}{d\tau^2} = -r^2 a_\theta\cos r\psi_\theta + \varepsilon\left[-2rC(a_\theta)\sin r\psi_\theta + \frac{d^2\Theta}{d\tau^2} - \right.$$
$$\left. - 2r^2 a_\theta D(a_\theta)\cos r\psi_\theta\right]$$

First corrections for the zeroth order approximations $\bar{y}_0(\tau)$ and $\theta_0(\tau)$ are obtained by selecting *n* = 0 in (6). That yields

$$\bar{y}_1(\tau) = \bar{y}_0(\tau) + \int_0^\tau \sin(s-\tau)\left\{\frac{d^2\bar{y}_0}{d\tau^2} + \bar{y}_0 - \right.$$
$$\left. - \varepsilon f_y\left(\bar{y}_0, \theta_0, \frac{d\bar{y}_0}{d\tau}, \frac{d\theta_0}{d\tau}\right)\right\}ds \tag{10}$$

$$\theta_1(\tau) = \theta_0(\tau) + \frac{1}{r}\int_0^\tau \sin r(s-\tau)\left\{\frac{d^2\theta_0}{d\tau^2} + r^2\theta_0 - \right.$$
$$\left. - \varepsilon f_\theta\left(\bar{y}_0, \theta_0, \frac{d\bar{y}_0}{d\tau}, \frac{d\theta_0}{d\tau}\right)\right\}ds \tag{11}$$

Let us concentrate on the equation (10). After some algebra, it becomes

$$\bar{y}_1(\tau) = \bar{y}_0(\tau) + \varepsilon\int_0^\tau \sin(s-\tau)\left\{\frac{d^2Y}{d\tau^2} + Y - 2A(a_y)\cdot\right.$$
$$\cdot\sin\psi_y - 2a_y B(a_y)\cos\psi_y - 2\hat{\xi}_y a_y\sin\psi_y -$$

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$$-\hat{\eta}_y U^2 a_1\left[a_\theta \cos r\psi_\theta + \frac{a_y}{U}\sin\psi_y + \frac{\overline{R}_1 r a_\theta}{U}\sin r\psi_\theta\right]$$
$$-\hat{\eta}_y U^2 a_3\left[a_\theta \cos r\psi_\theta + \frac{a_y}{U}\sin\psi_y + \frac{\overline{R}_1 r a_\theta}{U}\sin r\psi_\theta\right]^3 \}ds$$

Functions like $-2A(a_y)\sin\psi_y$ or $-2A(a_y)\sin\psi_y$ provide the so-called *secular terms* , because they increase slowly in time and became important after large intervals of time. Indeed,

$$\int_0^t \sin(s-\tau)\sin\psi_y\, ds = \int_0^\tau \sin(s-\tau)\sin(s+\varphi_y)ds =$$
$$=\frac{1}{2}\int_0^\tau \left[\cos(\tau+\varphi_y)-\cos(2s-\tau+\varphi_y)\right]ds =$$
$$=\frac{1}{2}\tau\cos(\tau+\varphi_y)-\frac{1}{4}\left(\sin(\tau+\varphi_y)+\sin(\tau-\varphi_y)\right)$$

with φ_y a constant. Because galloping is a periodic oscillation, one needs to avoid the secular terms or, with other words, to cancel the coefficients of $\sin\psi_y$ and $\cos\psi_y$.

These conditions produce the following two equations for findind $A(a_y)$ and $B(a_y)$

$$\sin\psi_y :-2A(a_y)-2\hat{\xi}_y a_y - \hat{\eta}_y U a_1 a_y -$$
$$-\frac{3\hat{\eta}_y a_3}{4U}a_y^3 - \frac{3\hat{\eta}_y U a_3}{2}a_y a_\theta^2 - \frac{3\hat{\eta}_y \overline{R}_1 r a_3}{2U}a_y a_\theta^2 = 0$$
$$\cos\psi_y :-2a_y B(a_y)=0$$

Solving them and replacing $A(a_y)$ and $B(a_y)$ in (9) one obtains

$$\begin{cases} \dfrac{da_y}{d\tau} = -\left(\xi_y + \dfrac{\eta_y U}{2}a_1\right)a_y - \\ \qquad -\dfrac{3\eta_y a_3}{8U}\left[a_y^2 + 2\left(U^2+\overline{R}_1^2 r^2\right)a_\theta^2\right]a_y \\ \dfrac{d\psi_y}{d\tau} = 1 \end{cases}$$

(12)

Doing the same for the equation (11) results in

$$\begin{cases} \dfrac{da_\theta}{d\tau} = -\left(r\xi_\theta + \dfrac{\eta_\theta U \overline{R}_1 b_1}{2}\right)a_\theta - \\ \qquad -\dfrac{3\eta_\theta \overline{R}_1 b_3}{8U}\left[2a_y^2 + \left(U^2+\overline{R}_1^2 r^2\right)a_\theta^2\right]a_\theta \\ \dfrac{d\psi_\theta}{d\tau} = 1 - \dfrac{\eta_\theta U^2 b_1}{2r^2}a_\theta - \\ \qquad -\dfrac{3\eta_\theta b_3}{8r^2}\left[2a_y^2 + \left(U^2+\overline{R}_1^2 r^2\right)a_\theta^2\right]a_\theta \end{cases} \quad (13)$$

The equations (12) and (13) are called *amplitude-frequency modulation equations* and describe both the transient and the steady state motions of the structure. The last are obtained by setting $\dfrac{da_y}{d\tau} = \dfrac{da_\theta}{d\tau} = 0$. There are four possible behaviors:

(**I**) Equilibrium : $a_y = a_\theta = 0$
(14)

(**II**) Periodic plunge oscillation:

$$a_y = \sqrt{-\frac{4U}{3a_3}\left(\frac{2\xi_y}{\eta_y} + U a_1\right)},\ a_\theta = 0$$

(15)

(**III**) Periodic torsional oscillation

$$a_y = 0,\ a_\theta = \sqrt{-\frac{4}{3\overline{R}_1 b_3}\frac{2r\xi_\theta/\eta_\theta + U\overline{R}_1 b_1}{U+\overline{R}_1^2 r^2/U}}$$

(16)

(**IV**) Quasi-periodic plunge-torsional oscillation

$$a_y = \sqrt{\frac{A_2C_1 - A_1C_2}{B_1C_2 - B_2C_1}},\ a_\theta = \sqrt{\frac{A_1B_2 - A_2B_1}{B_1C_2 - B_2C_1}}$$

(17)

where

$$A_1 = -\left(\xi_y + \frac{\eta_y U a_1}{2}\right),\ A_2 = -\left(r\xi_\theta + \frac{\eta_\theta U \overline{R}_1 b_1}{2}\right),$$
$$B_1 = -\frac{3a_3\eta_y}{8U},\ C_1 = -\frac{3a_3\eta_y}{4}\left(U + \frac{\overline{R}_1^2 r^2}{U}\right),$$
$$B_2 = -\frac{3b_3\eta_\theta \overline{R}_1}{4U},\ C_2 = -\frac{3b_3\eta_\theta\overline{R}_1}{8}\left(U + \frac{\overline{R}_1^2 r^2}{U}\right).$$

Which of these behaviors is realized in a practical situation depends on the non-dimensional wind speed and on the initial conditions [9]. The existence and stability of solutions (**I**) to (**IV**) can be established by studying the signs of the real parts of the eigenvalues for the Jacobian matrix

$$J = \begin{pmatrix} A_1 + 3B_1 a_y^2 + C_1 a_\theta^2 & 2C_1 a_y a_\theta \\ 2B_2 a_y a_\theta & A_2 + B_2 a_y^2 + 3C_2 a_\theta^2 \end{pmatrix} \quad (18)$$

A solution is stable if all the eigenvalues of *J* have negative real parts. The bifurcation from one solution to another depends on the parameters

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$$U_y = \frac{\eta_y U a_1}{2\xi_y}\ ,\ U_\theta = \frac{\eta_\theta U \bar{R}_1 b_1}{2r\xi_\theta} \quad (19)$$

For a given structure $a_1, b_1, r, \bar{R}_1, \xi_y, \xi_\theta, \eta_y$ and η_θ are fixed, so the behavior of that structure will depends entirely on the wind speed. Thus, solution (**I**) is stable if $U_y > -1$ and $U_\theta > -1$. It bifurcates to solution (**II**) if $U_y < -1$ and towards solution (**III**) if
$U_\theta < -1$. In their turn, solutions (**III**) and (**IV**) lose their stability and transform into solution (**IV**) if the conditions

$$2\frac{a_1}{a_3}\frac{b_3}{b_1}\frac{U_\theta}{U_y}\frac{(1+U_y)}{(1+U_\theta)} < 1 \text{ or } \frac{a_1}{a_3}\frac{b_3}{b_1}\frac{U_\theta}{U_y}\frac{(1+U_y)}{(1+U_\theta)} > 2$$

are fulfilled, respectively.
To complete the solution (8) one needs to determine *Y* and Θ. In reference [11] the authors proposed a constant sequence $\mathbf{u}_n, n \geq 0$ for the approximate solution of (5), meaning the only one iteration will solve the problem. In our case, that involes $\bar{y}_1 = \bar{y}_0$ and $\theta_1 = \theta_0$.
If we restrict our attention to the variable $\bar{y}$, it is sufficient for the function $Y(\tau)$ to be a solution of differential equation

$$\begin{aligned}
&\frac{d^2Y}{d\tau^2} + Y = -\frac{\hat{\eta}_y a_3}{4U} a_y^3 \sin 3\psi_y + \hat{\eta}_y \Big\{ U^2 a_1 + \frac{3a_3}{4} \cdot \\
&\left[2a_y^2 + \left(U^2 + \bar{R}_1^2 r^2\right)a_\theta^2\right] a_\theta \Big\} \cos r\psi_\theta + \hat{\eta}_y \Big\{ \bar{R}_1 r U a_1 + \\
&+ \frac{3\bar{R}_1 r a_3}{4U}\left[2a_y^2 + \left(U^2 + \bar{R}_1^2 r^2\right)a_\theta^2\right] a_\theta \Big\} \sin r\psi_\theta + \\
&+ \frac{\hat{\eta}_y a_3}{4}\left(U^2 - 3\bar{R}_1^2 r^2\right)a_\theta^3 + \frac{\hat{\eta}_y \bar{R}_1 r a_3}{4U}\left(3U^2 - \bar{R}_1^2 r^2\right) \cdot \\
&a_\theta^2 + \frac{3\hat{\eta}_y a_3 U}{4} a_y a_\theta^2 \left[\cos(\psi_y + r\psi_\theta) + \cos(\psi_y - r\psi_\theta)\right] \\
&- \frac{3\hat{\eta}_y \bar{R}_1^2 r^2 a_3}{2U} a_y a_\theta^2 \left[\sin(\psi_y + r\psi_\theta) + \sin(\psi_y - r\psi_\theta)\right] \\
&+ \frac{3\hat{\eta}_y a_3}{4} a_\theta a_y^2 \left[\cos(2\psi_y + r\psi_\theta) + \cos(2\psi_y - r\psi_\theta)\right] \\
&- \frac{3\hat{\eta}_y \bar{R}_1 r a_3}{4U} a_\theta a_y^2 \left[\sin(2\psi_y + r\psi_\theta) - \sin(2\psi_y - r\psi_\theta)\right]
\end{aligned} \quad (20)$$

Its solution is discussed in the next section for a particular case.

NUMERICAL SIMULATIONS

In the first part of this section, the approximate solution (8) with $Y = \Theta = 0$ is contrasted with its counterpart obtained by direct numerical integration of equations (3). The required parameters have been selected from reference [7] and they come from wind tunnel tests performed on a typical angle section model. Their values are

$$a_1 = -0.656, a_3 = 7.83, b_1 = -0.105, b_3 = 9.37, \xi_y = 0.0041,$$

$$\xi_\theta = 0.00513, \eta_y = 0.003, \eta_\theta = 0.01952, r = 2.92, \bar{R}_1 = 0.5$$

It follows that $U_y = -0.24U$ and $U_\theta = -0.0342U$, so the equilibrium state is stable for $U < 4.1667$.
Figure 3 presents a comparison between analytical and numerical solutions for $U = 1$ and the initial conditions $\left(\bar{y}(0), \frac{d\bar{y}}{d\tau}(0), \theta(0), \frac{d\theta}{d\tau}(0)\right) = (0.5, 0, 0.5, 0)$
The structure performs plunge and torsional oscillations with decreasing amplitudes and periods $T_y = 2\pi, T_\theta = 2\pi / r$. It is worth noting the excellent agreement between the two solutions.

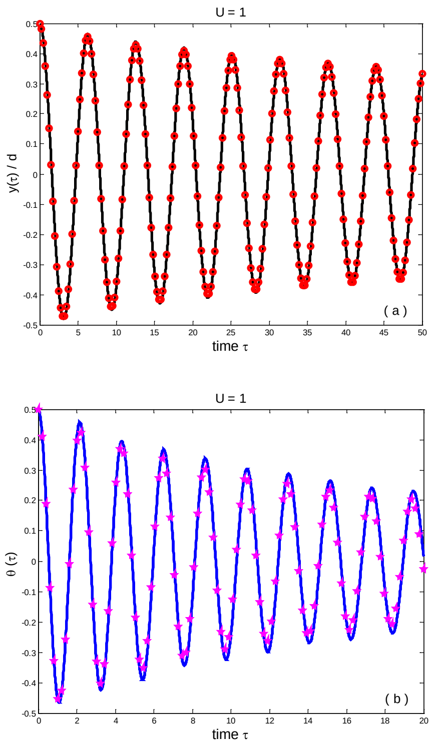

Figure 3. The comparison between the time series solutions $y(\tau)/d$ and $\theta(\tau)$ *obtained with exact and approximate equations (3) and (8) for* $U = 1$*. The red*

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circles and magenta stars stand for approximate solution.

The torsional motions vanishes faster than the plunge motion, as illustrated in Figure 4.

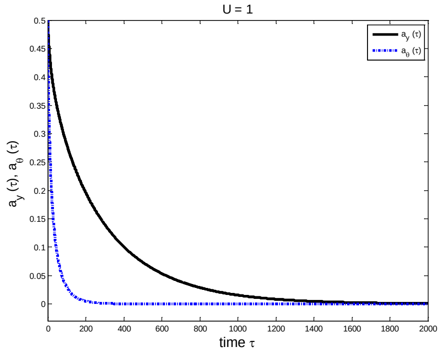

*Figure 4. Dissipation of an initial disturbance for $U = 1$ proves that solution (**I**) is stable*

By increasing the non-dimensional speed at $U = 10$, then $U_y = -2.4 < -1, U_\theta = -0.342 > -1$ and the structure evolves into a plunge oscillation with the steady amplitude $a_y = 2.5527$. Figure 5 shows, on the one hand, extracts of the transitional and stationary phases of the translational motion and, on the other hand, a high accuracy of the zeroth order approximations (8). The same is true for the rotational motion, as depicted in Figure 6.

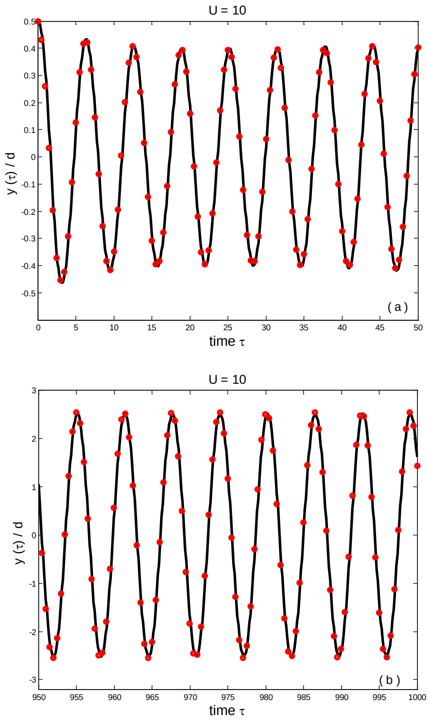

Figure 5. The comparison between the time series solutions $y(\tau)/d$ obtained with exact and approximate equations (3) and (8) for $U = 10$. The red dots are for solution (8): (a) transitional phase; (b) stationary phase

The transition from the initial displacement $\bar{y}(0)$ toward the steady-state is rather slowly, beeing necessary almost 1,000 units of time (see Figure 7). In fact, the closer is the speed *U* to the bifurcation point $U_b = 4.1617$ the larger is the time required to reach the stationary behavior [9].

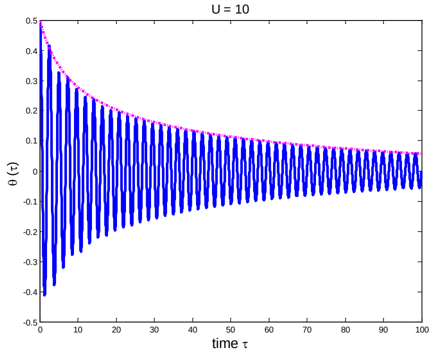

Figure 6. The comparison between the time series solutions $\theta(\tau)$ obtained with exact and approximate equations (3) and (8) for $U = 10$. The approximate solution is reported only by its amplitude $a_\theta(\tau)$ (the magenta interrupted curve)

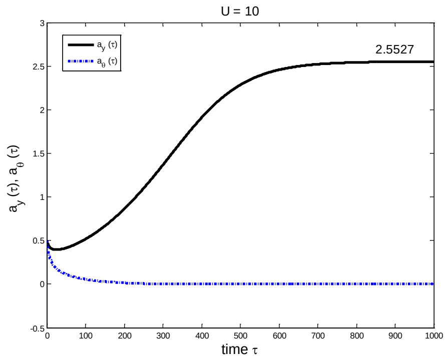

Figure 7. Transition toward the stationary state for $U = 10$

The theoretical results derived above predict that the periodic plunge solution (15) will lose its stability at $U = 29.2398$ and a periodic torsional oscillation, described by (16), will appear instead.
It is worth to remember that the amplitude-frequency modulation equations have been obtained by assuming no secular terms in (10) and (11). Such a presumption implies bounded solutions. For the analized cross-section model the numerical solution becomes unbounded for $U_{\max} \cong 13.9$ so any comparison between solutions of (3) and (8) above this limit has no physical meanings. In conclusion, after an initial excitation,

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the studied structure will come to rest if $U<U_b$ or will perform a plunge oscillation for $U\in(U_b,U_{\max})$ Table 1 reports the approximate and numerical amplitudes a_y and the relative errors *ERR* between them, recorded after $\Delta\tau=3000$ units of time. The errors of estimation, defined in (21), are founded to be limited to less than 5% for all the U values, excepting the small region near $U_{\max}$. This is an interesting result, given that it refers to the zeroth order approximation (without *Y*). Even more, the two solutions remain extremely close even after a long period of time, as Figure 8 proves.

$ERR=100\cdot\left|\left(a_y^{num}-a_y^{appx}\right)/a_y^{num}\right|\ (\%)$ (21)

Table 1. The aproximate and numerical amplitudes a_y for different wind speeds U

U	a_y^{appx}	a_y^{num}	*ERR (%)*
4.6	0.4719	0.4700	0.40
5	0.6823	0.6880	0.83
6	1.1085	1.1498	3.59
7	1.4885	1.5434	3.56
8	1.8508	1.9004	2.61
9	2.2043	2.2250	0.93
10	2.5526	2.5161	1.45
11	2.8976	2.7705	4.59
12	3.2404	2.9836	8.59
13	3.5815	3.1504	13.65
13.8	3.8535	3.2461	18.46

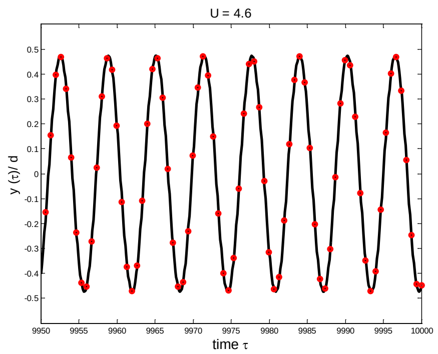

Figure 8. The closeness between the analyical and numerical solutions is kept even after a long period of time

Theoretically, the above reported results may be improved by including the term $\varepsilon Y(\tau)$. For the speed *U* range where there exist a numerical solution, the long-term behavior of the structure is characterized by $a_\theta=0$. The only term in the right hand side of the equation (20) which may be of some importance is the first one. The truncated equation

$$\frac{d^2Y}{d\tau^2}+Y=-\frac{\hat{\eta}_y a_3}{4U}a_y^3\sin 3\psi_y$$

(22)

has the solution $Y(\tau)=\frac{\hat{\eta}_y a_3}{32U}a_y^3\sin 3\psi_y$, such as

$$\bar{y}(\tau)=a_y\cos\psi_y+\frac{\eta_y a_3}{32U}a_y^3\sin 3\psi_y$$

(23)

with $\psi(\tau)=\tau+\phi,\ \phi$ some constant. For our particular cross- section the ratio between the amplitudes of the two terms in (23) is $\frac{U}{0.000734a_y^2}$

As an example, if *U* = 10 than the ratio is 2 091, meaning that nothing relevant is gaining if the term $\varepsilon Y(\tau)$ is added in the approximation for the plunge oscillation.

Conclusions

In the paper, a two-degrees-of-freedom model for the aero-elastic galloping of aslender structure in far-from resonance condition, excited by a transversal wind flow, has been analysed by means of a modified variational iteration method. The degrees-of-freedom are the vertical plunge and the torsional angle around the elastic axis.The method's algorithm provided a system of four differential equations for the amplitudes and frequencies of the zeroth order solutions, applicable to both the transient and stationary stages. Four possible behaviors of the system have been deduced and they can be either an equilibrium state, a periodic plunge oscillation, a periodic torsional vibration, or a motion on a two-dimensional torus. Using a typical cross-section of an electrical transmission line and the wind speed as a parameter, we founded an excellent agreement between the analytical solution, delivered by the variational iteration method, and its numerical counterpart. There exist a minimum (critical) wind speed required for the galloping's initiation. Below it any

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initial displacement is cancelled by damping mechanism. Above the critical wind speed, the analyzed cross-section performed just a plunge oscillation, in full agreement with the theoretical results. The torsional oscillation and the coupled plunge-rotational vibration were not found in the simulations, because the numerical scheme became unstable at some level of the wind speed. For other structures, it is conceivable that these behaviors may be present.

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