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CANONICAL MATHEMATICAL MODELS USED IN HARBOR ACTIVITIES

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***Abstract**: The paper aims is to prove, once again, that the canonical-mathematical models could provide better solutions for the management of different activities in the harbour framework.*
***Keywords**: Graph, mathematical model, Hamiltonian path, transportation problem.*

Introduction

The evolution of the harbor managerial processes, technical or economic, needs a scientific planning of activities, at each moment, which can lead to good results, properly correlated with the material costs invested and with the time used. The use of mathematics in harbor activities involves specific problems, in fact, that means accessing the most appropriate mathematical models which is not very simple but it can help solve the issues.

The situations that occurred in economic practice, continuously stimulates the use of mathematical methods with benefits for both economists and mathematicians. The use of mathematical language within the framework of management of harbor activities offers the possibility of using rigor and clarity and those can give rise for an optimal solution.

The aim of this paper is to present an easy access to some mathematical results frequently used in describing the phenomena from harbor branch, on short we wish that this paper emphasizes the most known economic-mathematical models which could provide solutions for the management problems occurring in the harbor activities.

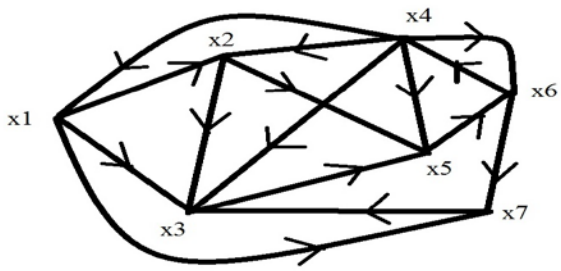
In the first section we used from the graph theory „Latin matrix algorithm"[1] to determine the Hamiltonian paths that means all elementary paths which use all nodes.

In the second section we used the North-west corner method [1], [3], [4]; the Minimum element method [1], [3], [4]; the Mixed differences method [1], [3] to obtain an initial solution for the transport problem that could appear in a real situation.

Canonical mathematical models used in harbor activity

The problem that we have considered is about a general cargo shipping company which operates a port terminal that has 7 cargo warehouses and 3 tractor-trailers for carrying the cargo to the warehouses, each trailer having the following capacity of transport : t1 = 1000 kg, t2 = 2000 kg and t3 = 3000 kg.

The warehouses, x_1, x_2, x_3, x_4, x_5, x_6, x_7, are located as follows:



Some issues could appear for such a company, like those that we imagine:

a) Determining between what warehouses, two by two, are paths for cargo carrying, the paths where tractors can go, passing once in a warehouse, this means Hamiltonian paths, and the circuits where the tractors can carry cargo between the warehouses;

b) Which is the time required to carry the cargo in 4 warehouses in the following amounts: A_1 = 1500 kg, A_2 = 500 kg, A_3 = 2500 kg, A_4 = 1500 kg, knowing the carrying time 3, 2, 2, 1; 2, 3, 2, 1; 2, 1, 2, 1(minutes).

a) First we want to apply Yu Chen's algorithm for determining the matrix paths. This algorithm needs the Boolean matrix, B = (b_{ij}), to be written, where by definition

$$b_{ij}=\begin{cases} 1, \ if\ (x_i, x_j) \in \ \Gamma \\ 0, \ if\ (x_i, x_j) \notin \ \Gamma; \end{cases} \quad i, j = 1,2,\dots,n,$$

where (X, Γ) the graph corresponding to the problem.

Therefore we have the Boolean matrix:

B	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_1	0	1	1	0	0	0	1
x_2	0	0	1	0	1	0	0
x_3	0	0	0	0	1	0	0
x_4	1	1	1	0	1	1	0
x_5	0	0	0	0	0	1	0
x_6	0	0	0	1	0	0	1
x_7	0	0	1	0	0	0	0

To apply Yu Chen's algorithm it is necessary to remember that the steps are:

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Step 1 - It is written The Boolean matrix, B, of the graph;
Step 2- It is added Boolean to the first line all the corresponding lines at nodes that have 1 on the first line. The new 1 that appears is marked by *;
Step 3 - It is added Boolean to the first line all the corresponding lines at nodes that have 1* on the first line. The new 1 that appears it is marked by **; this step will continue until no longer appears any new 1 on the first line;
Step 4 – Steps 2 and 3 are applied to each of the Boolean matrix lines.
In the end are obtained the paths matrix, D, which for this problem is written below:

D	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_1	1****	1	1	1***	1*	1**	1
x_2	1***	1***	1	1**	1	1*	1**
x_3	1***	1***	1***	1**	1	1*	1**
x_4	1	1	1	1*	1	1	1*
x_5	1**	1**	1**	1*	1**	1	1*
x_6	1*	1*	1*	1	1*	1*	1
x_7	1****	1****	1	1***	1*	1**	1***

The paths for cargo carrying are between any two warehouses as we can see from the table before.
Next, it will be presented a method through which all the Hamiltonian paths can be found and this method is named, in mathematical terms, „Latin matrix algorithm"[1]. Let (X, Γ) the graph corresponding to the problem. It will be used Latin matrix L = (l_{ij}), where

$$l_{ij}=\begin{cases} x_i\, x_j, \ if\ (x_i, x_j) \in \Gamma \\ 0,\ if\ (x_i, x_j) \notin \Gamma; \end{cases} \quad i,j = 1,2,\ldots,n$$

and, for our problem, the corresponding Latine matrix is:

L	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_1	0	x_1x_2	x_1x_3	0	0	0	x_1x_7
x_2	0	0	x_2x_3	0	x_2x_5	0	0
x_3	0	0	0	0	x_3x_5	0	0
x_4	x_4x_1	x_4x_2	x_4x_3	0	x_4x_5	x_4x_6	0
x_5	0	0	0	0	0	x_5x_6	0
x_6	0	0	0	x_6x_4	0	0	x_6x_7
x_7	0	0	x_7x_3	0	0	0	0

With this matrix we determine now the matrix $\tilde{L}$, which is built from L, by removing x_i from the sequence $x_i x_j$, when this exists. For our problem, that matrix is:

$\tilde{L}$	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_1	0	x_2	x_3	0	0	0	x_7
x_2	0	0	x_3	0	x_5	0	0
x_3	0	0	0	0	x_5	0	0
x_4	x_1	x_2	x_3	0	x_5	x_6	0
x_5	0	0	0	0	0	x_6	0
x_6	0	0	0	x_4	0	0	x_7
x_7	0	0	x_3	0	0	0	0

Now, in the following, it is used a special matrix multiplication named Latin multiplication[1], [2] and written as " $*$ " and defined by:
a) the multiplying is done rows by columns;
b) instead of the usual multiplication the join of elements is used, if these don't repeat or zero is written in the opposite case;
c) instead of the usual addition one considers the groups obtained in b), when we have such groups.
By short, we will write L $*$ $\tilde{L}$ = L^2 . Similarly we calculate $L^2 * \tilde{L} = L^3$, . . . , $L^{k-1} * \tilde{L} = L^k$. It is noticed that L^k containes all the elementary paths of k length.
Consequently, in L^{n-1} matrix all Hamiltonian paths appear and in this case n=7, so using L^6 we obtain the paths where tractors can go, passing once through a warehouse i.e. Hamiltonian paths: $x_1x_7x_3x_5x_6x_4x_2$, $x_2x_5x_6x_4x_1x_7x_3$, $x_2x_3x_5x_6x_4x_1x_7$, $x_4x_1x_2x_3x_5x_6x_7$, $x_7x_3x_5x_6x_4x_1x_2$.
The circuits where the tractors can carry cargo between the warehouses are : $x_1x_2x_3x_5x_6x_4x_1$; $x_1x_7x_3x_5x_6x_4x_1$; $x_2x_3x_5x_6x_4x_1x_2$; $x_3x_5x_6x_4x_1x_7x_3$; $x_3x_5x_6x_4x_1x_2x_3$; $x_4x_1x_2x_3x_5x_6x_4$; $x_4x_1x_7x_3x_5x_6x_4$; $x_5x_6x_4x_1x_2x_3x_5$; $x_5x_6x_4x_1x_7x_3x_5$; $x_6x_4x_1x_2x_3x_5x_6$; $x_6x_4x_1x_7x_3x_5x_6$; $x_7x_3x_5x_6x_4x_1x_7$.
b) The second issue that we considered, has as target the determination of the time required to carry the cargo in the chosen warehouses in the amounts : A_1 = 1500 kg , A_2 = 500 kg , A_3 = 2500 kg , A_4 = 1500 kg , knowing the carrying time 3, 2, 2, 1 ; 2, 3, 2, 1, ; 2, 1, 2, 1(minutes) and x_{ij} being the amount of products that will be transported with the tractor-trailers T_i, i = 1,2,3, to the warehouses W_j, j = 1,2,3,4. For this problem we can obtain the following mathematical model:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1000$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 2000$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 3000$$

$$x_{11} + x_{21} + x_{31} = 1000$$
$$x_{12} + x_{22} + x_{32} = 500$$
$$x_{13} + x_{23} + x_{33} = 2500$$
$$x_{14} + x_{24} + x_{34} = 1500$$
$$x_{ij} \geq 0 \; ; \; i = 1,2,3 \; ; \; j = 1,2,3,4$$

$$(\min)\ f = 3x_{11} + 2x_{12} + 2x_{13} + x_{14} + 2x_{21} + 3x_{22} + 2x_{23} + x_{24} + 2x_{31} + x_{32} + 2x_{33} + x_{34}$$

Starting by applying the North-West corner method [1], [3], [4] we will find an initial solution that is written in the table below:

T_i / D_j	W_1	W_2	W_3	W_4	Capacity
T_1	3 | 1000 x_{11}	2 | 0 x_{12}	2 | 0 x_{13}	1 | 0 x_{14}	1000
T_2	2 | 500 x_{21}	3 | 500 x_{22}	2 | 1000 x_{23}	1 | 0 x_{24}	2000
T_3	2 | 0 x_{31}	1 | 0 x_{32}	2 | 1500 x_{33}	1 | 1500 x_{34}	3000
Required	1500	500	2500	1500	6000

With this initial solution obtained, the time required to carry the cargo is found:
(min) f = 3x1000 + 2x500 + 3x500 + 2x1000 + 2x1500 + 1500 = 12000 minutes.
Now, using the minimal element method [1], [3], [4] we will find another initial solution to this problem.
Arranging the values obtained by this method in a table, we have:

T_i / D_j	W_1	W_2	W_3	W_4	Capacity
T_1	3 | 1000 x_{11}	2 | 0 x_{12}	2 | 0 x_{13}	1 | 0 x_{14}	1000
T_2	2 | 500 x_{21}	3 | 0 x_{22}	2 | 0 x_{23}	1 | 1500 x_{24}	2000
T_3	2 | 0 x_{31}	1 | 500 x_{32}	2 | 2500 x_{33}	1 | 0 x_{34}	3000
Required	1500	500	2500	1500	6000

For these values we calculated the times required:
(min) f = 3x1000 + 2x500 + 1500 + 500 + 2x2500 = 10500 minutes
Another method that we want to use is the maximum differences method [1], [3]. This method starts by determining the smallest difference between two elements (times) and is calculated for each row, each column respectively. Then on the row or column with the maximum difference, variables are determined in the box with minimal time. If the maximum difference and the minimum time are repeated, the algorithm asks to choose the variable with maximum allocation.
Further on we will find out an initial solution with the maximum difference method for the transportation problem stated above.
Finally, respecting the algorithm and arranging in a table the values obtained for the variables, we will determine the solution:

T_i / D_j	W_1	W_2	W_3	W_4	Capacity
T_1	3 | 0 x_{11}	2 | 0 x_{12}	2 | 1000 x_{13}	1 | 0 x_{14}	1000
T_2	2 | 0 x_{21}	3 | 0 x_{22}	2 | 500 x_{23}	1 | 1500 x_{24}	2000
T_3	2 | 1500 x_{31}	1 | 500 x_{32}	2 | 1000 x_{33}	1 | 0 x_{34}	3000
Required	1500	500	2500	1500	

The time required for the transport that could be carried out is:
(min) f = 2 x 1000 + 2x500 + 1500 + 2x1500 + 500 + 2x1000 = 10000 minutes

CONCLUSIONS
We conclude that specific issues that could appear in the harbor framework could be solved by using graph theory (Yu Chen's algorithm) and the classical transportation problem.
It is better to bring a mathematical solution for some of the real problems faced by managers when they are planning a transportation complex activity.
The use of specific graph theory algorithms lead much quicker to all solutions that the institution could use in developing the plan of transportation.
It is easy to observe that by applying the north-west corner method we got a less efficient solution than that obtained by the minimum element method, and that of the mixed differences method.
We want to emphasize that the times required for the transports that could be carried out, were used instead of costs, in the algorithms for the classical transport problem.

These solutions can be improved and optimized by using the algorithm of potentials, and the results can confirm once again the fact that mathematical methods could be used more in practice.

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