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A NOVEL BOOTSTRAP DBA-DEA APPROACH IN EVALUATING EFFICIENCY OF BANKS

M. RADOJICIC[1]
G. SAVIC[2]
S. RADOVANOVIC[2]
V. JEREMIC[2]
[1] Vojvodjanska Banka, Novi Sad, Serbia
[2] Faculty of Organizational Science, University of Belgrade, Belgrade, Serbia

Abstract: *Efficiency evaluation has long been an issue of great concern for the banking sector (financial institutions and for the banks themselves). This is particularly the case for State financial institutions which can be able to act proactively and prevent severe turbulence in the market. That is a very important task in a small market where operates a large number of banks, such as the Serbian market. The aim of paper is to present new way in calculating efficiency using data envelopment analysis (DEA), as the most widely used efficiency evaluation method. We present a novel approach in obtaining weight restrictions for DEA, based on the bootstrapping distance based analysis (DBA). Analysis was conducted on 29 banks, which have been operating in Serbia during 2010.*
Keywords: *bootstrapping, data envelopment analysis, distance based analysis, efficiency*

Introduction
Efficiency measurement in banking can be divided into two main segments:
Measuring efficiency of individual bank/branch in economy/bank,
Measuring efficiency of entire economy and comparing it with each other.
Main focus of efficiency measurement of an individual bank is to evaluate banks' efficiency compared with other banks, or in other words whether a specific bank is considered to operate efficient and whether operation of the bank can be improved. On the other side goal of measurement of efficiency of entire economy is to determine what is efficiency of one economy compared to other or comparing it to itself, during a certain period of time. Subject of this paper is to measure efficiency of individual banks and to compare them with each other. There are many scientific papers handling issues and proposing solutions how to do that. Commonly, when measuring efficiency of banks, authors rely on one very well known method: data envelopment analysis (DEA) [1, 2, 3]. Other methods are not so represented. Below, we will look on some important studies of bank efficiency measurement, first worldwide, than in case of Serbia banking sector. Main focuses are studies, which applied DEA approach using bootstrapping and new Ivanovic-Jeremic distance based analysis (DBA), but we will reflect on some other solutions of measuring bank efficiency proposed by authors. Bootstrapping method can be used in DEA and other methods as explained in [4, 5].
In this paper, results of different DEA input-oriented models will be used in order to find the one which can in best way represent the solutions. First, solution with basic DEA model such as CCR (Charnes, Cooper and Rhodes) and BCC (Banker, Chames and Cooper) will be presented. Main difference between these two models is that CCR assumes constant return to scale while BCC assumes variable return to scale. After that, four GAR (Global assurance region) models for weight restrictions DEA does not require any a-priori weights in a frontier analysis of the inputs and outputs. It can be strength, but also it can be considered as a weakness. One problem which can occur when DEA is used, is that total weight flexibility can lead to some decision-making units being evaluated, on only a small subset of their inputs and outputs, while the rests of their inputs and outputs are all but ignored. With weights constrains, authors wanted to overcome that problem. Weights which will be used in GAR models for this paper will be generated with bootstrapping I-distance. To make a range for weights mean plus/minus standard deviation formula was used. Solutions where the range is mean plus/minus six standard deviations (6σ) and mean plus/minus three standard deviations (3σ) is presented. Analysis will be conducted on the example of Serbian market.
Literature Review
DEA with bootstrap in banking
Berger and Humphrey published in 1997 comparative review of most previous studies related to topic of measuring efficiency of financial institutions [6]. Their survey includes more than 120 studies, which were conducted in more than 20 countries with at least five major efficiency frontier measurement techniques (most often it was DEA). Some of them used bootstrapping to measure efficiency score. Their main goal was to summarize empirical findings or techniques in order to find consistent results and compare results within countries. Later, Berger conducted research of more recent applications of frontier techniques applied on bank efficiency [7]. Lack of that survey was that it only cover studies, which provide international

comparisons of bank efficiency. Interesting comprehensive review of 196 studies was done in 2010 [8]. In that review of measuring bank efficiency, they have not just included studies, which employ operational research techniques, but also artificial intelligence techniques, during the period 1998-2009. From those 196 studies, 151 were about applying DEA or DEA-like techniques to estimate various measures of bank efficiency and productivity growth.

Ferrier and Hirschberg in 1997, published their work regarding bootstrapping method on efficiency measurement in banking [9]. They used DEA to find efficient banks from Italian market, which were operating in 1986. Inputs were labor (number of employee), capital, consumer deposit accounts, commercial deposit accounts and industrial deposit accounts. The outputs were loans (consumer, commercial and industrial), deposits at other financial institutions, investments, and the number of branches operated by each bank. Without bootstrapping, measuring only Farrell technical efficiency, they found that 77 from 94 banks were on the efficiency border. Because of that result, which does not have practical use for decision makers to distinguish bank performance among other banks, they used a bootstrap to obtain more information on bank performance. Original sample was resample by modification of bootstrapping method in a way, they produce more samples combining pseudo-efficiency with new efficiency scores. They found that only 28 banks are constantly on the efficiency border. It can be concluded that by using bootstrap decision makers have more info about bank performance and behavior of a single bank.

Because of few previous studies on efficiency of Australian banks, which have not examined statistical properties of efficiency estimates, in [10] it is provided statistical insights into pure technical efficiency estimates of individual Australian banks using the bootstrap DEA technique. Those recent studies used interest expense and non-interest expense as an input and net interest income (NII) and non-interest income as an output, but authors of [10] have chosen to replace NII with interest income because NII ignores the impact of unaccounted interactions between endogenous and exogenous and adding unnecessary duplications. They used same time period and same banks which as in [11]. First, it was showed that 23% of DMU were efficient, opposite to 83% which were found efficient in [11]. That was due to use of interest income instead net interest income, which improved the discrimination power of efficiency estimates. Second, with bootstrapping, in [10] it was found that all fully efficient banks are not operating at the same level of efficiency. There was shown that the estimated confidence intervals are quite wide for a number of banks and rather narrow for some others. Therefore it is possible to make distinction between them.

Another recent quality study was conducted in 2014 [12]. There have been measured efficiency of 44 major banks in China, during 2007-2011. Deposits, fixed assets and number of employees was treated as an input and total net loan and other earning assets as an output. To measured efficiency, it was used two-stage DEA double bootstrap procedure, which improves statistical efficiency in the second-stage regression [13]. In [13] Simar and Wison proposed single and double bootstrap procedures, but Monte Carlo experiment reported in the work showed that the double bootstrap procedure performs very well in terms of both coverage for estimated confidence intervals, and root mean square error. Result that was obtained in [12], is that average efficiency for Chinese commercial banks were 89%. They have used parametric and non-parametric tests to determine whether the means of the estimates of the bias-corrected technical efficiency, and the conventional technical efficiency are systematically different. Results showed that difference exists between the bias-corrected technical efficiency scores and the conventional technical efficiency scores, at the 1% significance level.

Chinese banks were ranked according to their efficiency over the period 1998–2008, with the Inverse B-convex model which is derived from B-convex model [14, 15]. Advantage of this approach is that it is not necessary to suppose the nature of returns to scale in the technology, and it allows us to take into account the possible complementarities of inputs.

Savic and authors measured efficiency of Serbian banks using Window DEA analysis [16]. They have estimated profit efficiency of the bank and operating efficiency of the bank. For profit efficiency model input was interest expense and non-interest expense and output were interest income and non-interest income. For operating efficiency model input was number of employees, fixed assets and intangible investments, capital and deposits and output was granted loans and deposits, and non-interest income. They examine 28 banks during period 2005-2011. They used super-efficiency according to [17], to rank efficient banks. Thus, this approach is better in understanding the difference between efficient units than regular DEA model introduced in [18]. Similar to Savic, Czech banking sector was analyzed [19]. Technical efficiency of the banks was measured also with DEA window approach. Labor and deposit were inputs, and loans and NII were outputs.

A novel approach in measuring efficiency was introduced in [20], by employing a statistical Ivanovic-Jeremic Distance Based Analysis (DBA) on various health indicators in order to determine the efficiency of EU countries' health system. Successful implementation of this method of measuring efficiency of the banks was demonstrated in [21] where it was measured efficiency of banks in India. Significance of this work lies in the fact that it compared DBA and DEA methods on same data. 34 banks were analyzed throughout 2005-2012 with equity, borrow funds, number of employees and number of bank branches as input and deployed funds and non-interest income as output. Profit model of DEA was used to rank banks instead AP (Andersen Petersen) model because the profit model uses both input costs and output prices into consideration while measuring the performance of the banks. Results showed that there is minor variation in the ranking of banks between DBA and DEA. In general, DBA tends to give lower rank to banks, which are efficient according to DEA.

Weight restrictions in DEA

One of the most important papers on weight restriction in DEA was conducted in 1997 [22]. It provided a review on the evolution, development and future research directions for the use of weight restrictions in DEA.

One of the first papers which discussed weightings in DEA was [23], where authors have reduced weight flexibility. They made a model which can be used when DMUs (Decision Making Units) are evaluated with only a single input. They used regression analysis to construct lower bounds of the output weights.

In contrast to them, the method for restricting weight flexibility, which can be applied when DMU being evaluated having multiple inputs and outputs is presented in [24]. It is based on proportions.

In [25] AR (Assurance regions) consisting of separate linear homogeneous restrictions on the input and output multipliers are used. They wanted to avoid large differences in terms of the weight values among all DMUs. Using AR in DEA, they managed to reduce the number of extreme-efficient candidates for overall efficiency. Another study which included AR in DEA is [26]. They have measured and ranked nations Olympic achievement. Their assumption was that different nations valued medals differently. It is common that ARs apply uniformly across all DMUs, but they establish the model where multiple sets of DMUs specific ARs are incorporated in DEA. A recent use of AR with DEA can be seen in [27, 28].

Efficiency of Spanish Courts using DEA were measured in [29]. In order to ensure correct comparison between the Courts, authors used restriction on weights. The idea was that efficient units should not obtain their scores based on only a single ratio output-input and independently of their performance on the inputs and outputs taken as a whole. Zero weights should not be attached to any of input or output because they all have some importance. They considered that some weights should be higher than other, because higher consumption of resources used in them.

Chilingerian and Sherman used cone ratio model to enhance the analysis of best practices by incorporating managerial philosophy and strategic intent into the model by placing bounds on the virtual multipliers, based upon the ratios between weights or the rate of technical substitution of certain inputs [30].

One research which measured efficiency of Mexican banks and also addressing to importance of weight restrictions is [31]. Authors have used only one output (total income) for ranking banks. They also implemented AR like in [25] for calculating efficiency.

Podinovski has demonstrated the role of weight bounds in DEA [32]. He suggested that weight bounds can be assessed using production trade-offs between inputs and outputs. First, it is necessary to transform DEA model to special form in which the weight bounds are explicitly linked to production trade-offs.

In [33], it is showed how LoOP-based weight restrictions can be incorporated in DEA. The Law of One Price (LoOP) states that all firms face the same prices for their inputs and outputs under market equilibrium. They proposed applying a set of input prices that is common for all firms, and that maximizes the cost efficiency within the industry.

In [34], it is proposed mathematical programming approach to constructing CI (composite indicators). The proposed approach uses two sets of weights that are most and least favorable for each entity to be evaluated and therefore, could provide a more reasonable and encompassing CI. Latter, it is extended and a multiplicative optimization approach to constructing CIs is proposed [35]. The proposed approach requires no prior knowledge of the weights for sub-indicators. The weights can be generated by solving a series of multiplicative DEA type models that can be transformed into equivalent linear programs. If additional information on the relative importance of sub-indicators is available, it can be incorporated into the proposed models. Since the proposed approach uses two sets of weights that are the most and the least favorable for each entity, it provides a more reasonable and encompassing CI.

MCDA-DEA (multi-criterion decision analysis-data envelopment analysis) approach to construct CI is used in [36,37]. MOLP (multiple objective linear programming) was used to generate common set of weights in order to provide a common base for ranking the DMUs [38].

Dimitrov and Sutton proposed the SWAT (symmetric weight assignment technique), that does not affect feasibility and rewards decision making units (DMUs) that make a symmetric selection of weights [39]. That allows for a method of weight restrictions that does not require preference constraints among the variables.

Interesting way of restricting weights was developed in [40]. It is based on correlations between input and output variables. The efficiency scores are calculated at a given level of correlation between the input and output variables in the model. If the current relationship is taken into account in assigning the weights of the variables when calculating the efficiency score with the suggested approach, the weights of the variable are balanced. If a balanced concept is based on the degree of importance of a variable in the production process, this variable should be placed with a weight at that level in production.

Canonical correlation was used in [41] to construct weight restrictions. The goal was to maximize the correlation of the linear combinations of the sets of inputs and outputs in DEA model. The canonical correlation is then used to establish bounds for the proportional virtual weight restrictions on inputs and outputs, seeking to reflect a judgment on the value of each variable within the DEA model. It was managed to increases the consistency of the estimated DEA scores and that these limits do not present mathematical infeasibility problems while avoiding the need for subjectively restricting weight variation in DEA.

Genetic algorithm for weight restrictions in DEA was implemented in [42]. The approach involves finding a set of weights, which are at a minimum distance from all the decision makers' preferences. The approach is flexible and was able to generate a common set of weights and DMU specific weight restrictions simultaneously. It guarantees feasibility at all times.

Methodology

DBA

The need for an impartial ranking was greatly emphasized in the 1960s when countries had to be ranked by the level of their development based on several socio-economic indicators. One of the devised methodologies, which could answer such a task, was the I-distance method developed by Ivanovic [43].His metric easily solves the problem of incorporating various indicators of different measurement units into a single synthetic indicator which thereafter represents the rank [44].Since it is able to overcome the problem of subjectivity in a composite indicator, the I-distance method was frequently used as the aggregation method [45, 46, 47].

In order to apply the I-distance method, it is necessary to fix one entity as a reference in the observed data set. The fixed or referent entity is the entity with the minimal value for each indicator. If not applicable, it can be a fictive entity with the minimal value of each indicator. The ranking of entities in the data set is founded on the calculated distance from the referent entity [46, 48]. The construction of the I-distance is an iterative process, which can consist of several steps. The first step calculates the amount of discriminate effect of the first variable (the most significant variable that provides the most information on the ranking phenomenon); the second step calculates the value of the discriminate effect of the second variable, not included in the first. This procedure is repeated for all the variables in the observed data set [49].

Let $X^T = \left(X_1, X_2, ...X_k\right)$ be a set of variables chosen to characterize the entities. I-distance between two entities $e_r = \left(x_{1r}, x_{2r}, ...x_{kr}\right)$ and $e_s = \left(x_{1s}, x_{2s}, ...x_{ks}\right)$ is defined as

$$D(r,s) = \sum_{i=1}^{k} \frac{\left|d_i\left(r,s\right)\right|}{\sigma_i} \prod_{j=1}^{i-1}\left(1 - r_{ji.12...j-1}\right), \qquad (1)$$

where $d_i(r,s)$ is the discriminative effect, ,the distance between the values of variable X_i for e_r and e_s

$$d_i(r,s) = x_{ir} - x_{is}, i \in \{1,...,k\} \qquad (2)$$

σ_i is the standard deviation of X_i and $r_{ji.12...j-1}$ is the partial coefficient of the correlation between X_i and X_j, $\left(j < i\right)$ [47, 50].

In addition, frequently used square I-distance provides additional benefits [44]. It is given as:

$$D^2\left(r,s\right) = \sum_{i=1}^{k} \frac{d_i^2\left(r,s\right)}{\sigma_i^2} \prod_{j=1}^{i-1}\left(1 - r_{ji.12...j-1}^2\right). \qquad (3)$$

With bootstrap method, in every iteration, I - distance values are calculated. Then Pearson correlation coefficients are calculated for input and output variable. Weights are formed by weighting the Pearson correlation. Values of correlations are divided by the sum of correlations. The final sum equals 1, thus forming a novel appropriate weighting system:

$$w_i = \frac{r_i}{\sum_{r=1}^{k} r_j}. \qquad (3)$$

where r_i (i = 1,... k) is a Pearson correlation between i-th input variable and I-distance value [50].

These obtained weights for every input-output variable is then used for generating lower and upper bounds for weights restrictions in DEA using mean ± SD (standard deviation - σ).

DEA

DEA has been used for performance evaluation in the wide spread areas in the last 30 years, from non-profit sector like as in [45, 51, 52], to profit sector like as in [16, 53, 54]. DEA was introduced in [18]. Suppose that DM (j = 1,…, n) uses inputs (i = 1,…, m) to produce outputs (r = 1,…,s):

$$
\begin{aligned}
&(max)\ h_k = \sum_{r=1}^{s} \mu_r y_{rk} \\
&s.t. \\
&\sum_{i=1}^{m} v_i x_{ij} = 1 \\
&\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1,\dots,n, \qquad j \ne k \\
&\mu_r \ge \varepsilon, \qquad r = 1,\dots,s \\
&v_i \ge \varepsilon, \qquad i = 1,\dots,m
\end{aligned}
\tag{4}
$$

where is relative efficiency of DMUk, is weight assigned to output r, and is weight assigned to input i.

Model (4) is input oriented model with constant return to scale. By adding u^* (it represent position of auxiliary hyperplane which lies at or above each DMU included in analysis) into objective function and second constraints, model (4) becomes input oriented model with variable return to scale.

In this paper for weight restrictions it is used Global assurance region method (GAR) [55]. It introduces designated constraints on virtual inputs (outputs) which are common for all DMUs. In fact, these constraints indicate that share of observed virtual input or output (weight multiplied by input or weight multiplied by output value) must be in a certain L-U range, compared with the total virtual input or output.

$$L \le \frac{v_1 x_{1k}}{v_1 x_{1k} + v_2 x_{2k} + \dots + v_m x_{mk}} \le U \tag{5}$$

TABLE III.
SIX SIGMA WEIGHT CONSTRAINS

Variable	Lower bound	Upper bound
Number of employee	0.239	0.326
Fixed assets	0.216	0.296
Capital	0.180	0.294
Deposits	0.180	0.269
Loans	0.285	0.416
Other placement	0.283	0.389
Noninterest income	0.217	0.409

where L is lower bound and U is upper bound, and m represent number of weight restrictions which are imposed on virtual weights.

Findings and analysis

TABLE I.
EFFICIENCY WITH CCR AND BCC MODEL

Bank	CCR score	CCR Rank	BCC score	BCC rank
AIK banka	1	1	1	1
Alpha Bank	0.8335	21	1	1
Banca Intesa	1	1	1	1
Banka Poštanska štedionica	1	1	1	1
Credy banka	0.7175	24	1	1
Čačanska banka	0.9754	13	1	1
Erste Bank	1	1	1	1
Eurobank EFG	0.7863	23	0.8065	26
Findomestic banka	0.8249	22	1	1
Hypo Alpe-Adria-Bank	0.9169	15	0.9278	21
JUBMES banka	1	1	1	1
KBC banka	0.8627	20	0.8885	24
Komercijalna banka	0.8644	18	1	1
Marfin Bank	1	1	1	1
Crédit Agricole banka	0.8938	17	0.8994	23
Razvojna banka Vojvodine	0.5659	27	0.6507	27
NLB banka	1	1	1	1
OTP banka	0.8641	19	0.9095	22
Piraeus Bank	0.9085	16	0.9405	20
Agrobanka	0.5751	26	0.6456	28
Privredna banka	1	1	1	1
ProCredit Bank	0.9740	14	0.9778	19
Raiffeisen banka	0.9869	12	1	1
Société Générale banka	1	1	1	1
Srpska banka	0.4706	29	1	1
Unicredit Bank	1	1	1	1
Univerzal banka	0.6401	25	0.8310	25
Vojvođanska banka	0.4828	28	0.5008	29
Volksbank	1	1	1	1

Data

As it was mentioned in introduction, we used data of banking sector in Serbia from the end of the 2010. There were 29 banks operating on the market.

There is no unique procedure in selection of inputs and outputs for purpose of measuring efficiency of the banks.

In this paper, total of seven variables were used - four for input and three for output. Table II shows used inputs and outputs and their descriptive statistics:

Results

Analysis was conducted with six different DEA models. First efficiency of bank was measured without any weight restrictions using basic DEA models - CCR and BCC. Later, using bootstrap DBA method lower and upper bound were generated. Assurance region of 6 sigma and three sigma was used to create bounds for weights, which will be used in DEA model. In both analysis results of DEA models under constant return to scale were compared to DEA models under variable return to scale.

TABLE IV.
EFFICIENCY WITH GAR MODEL SIX SIGMA

Bank	6σC score	6σC Rank	6σV score	6σV Rank
AIK banka	0.6621	13	0.7053	19
Alpha Bank	0.5754	17	0.6077	22
Banca Intesa	0.7814	9	1	1
Banka Poštanska štedionica	0.7179	11	0.7390	17
Credy banka	0.4254	22	1	1
Čačanska banka	0.6533	15	0.9600	9
Erste Bank	0.8253	4	0.9180	10
Eurobank EFG	0.5155	18	0.5223	25
Findomestic banka	0.5832	16	1	1
Hypo Alpe-Adria-Bank	0.8081	5	0.8144	15
JUBMES banka	0.4998	19	1	1
KBC banka	0.1294	29	0.5756	23
Komercijalna banka	0.6596	14	0.7150	18
Marfin Bank	0.9121	3	1	1
Crédit Agricole banka	0.6946	12	0.7801	16
Razvojna banka Vojvodine	0.4440	21	0.5619	24
NLB banka	0.7550	10	0.8650	13
OTP banka	0.3016	25	0.4476	26
Piraeus Bank	0.4853	20	0.6695	21
Agrobanka	0.2677	26	0.3310	27
Privredna banka	0.7881	8	1	1
ProCredit Bank	0.1542	28	0.3102	29
Raiffeisen banka	0.7896	7	0.9012	11
Société Générale banka	0.7943	6	0.8230	14
Srpska banka	0.3065	24	0.8662	12
Unicredit Bank	1	1	1	1
Univerzal banka	0.4077	23	0.6908	20
Vojvođanska banka	0.2659	27	0.3115	28
Volksbank	0.9872	2	1	1

From Table I it can be seen, that with CCR model there are 7 efficient banks, while BCC model gave only 11 banks to be inefficient while all others are considered as an efficient. Average score in CCR model is 0.867 while in BCC it is 0.9302. It can be concluded that using constant return to scale instead variable we can get better distinction between the banks.

Now if we want to make a further distinction between efficient banks, we need to use assurance regions. That is achieved with GAR model. As it is mentioned, bounds are formed by calculating mean of weighted Pearson correlations generated thorough bootstrapping I-distance, and then adding and subtracting *m* standard deviations. In our first and second GAR models *m* equals 6 and in our third and fourth *m* is equal to 3. In Table IV lower and upper bound for 6σ models are shown.

The efficiency scores given in Table IV are generated by using 6σ and 3σ weight constraints in GAR DEA model with constant (3σC and 6σC) and GAR DEA model with variable return to scale (3σV and 6σV). As it can be seen, 6σ DEA model with constant return to scale gives only one efficient bank, while 6σ DEA model with variable return to scale considers eight banks to be efficient. Average efficiency score in 6σC model is 0.5928 while in 6σV model it is 0.7626. It can be drawn a conclusion that implementing weight constraints we managed to lower number of efficient banks (in constant return to scale from 7 to 1, and in variable return to scale from 18 to 8).

In the next analysis lower and upper bound for virtual weights will be generated on the 3σ level (table V).

Comparing Table V with Table III it is obvious how scope of weight is narrowed.

Again, GAR model with constraints from Table V is implemented, and results from Table VI are obtained. For 3σC model again, only one bank is considered as an efficient, but now average score is little less than in 3σC model 0.5508. For 3σV model average score is 0.7349 and it has one less efficient bank than 3σV model.

TABLE VI.
EFFICIENCY WITH GAR MODEL THREE SIGMA

Bank	3σC score	3σC Rank	3σV score	3σV Rank
AIK banka	0.6229	11	0.6688	19
Alpha Bank	0.5227	17	0.5545	23
Banca Intesa	0.7256	9	1	1
Banka Poštanska štedionica	0.6226	12	0.6499	21
Credy banka	0.3828	22	0.9667	8
Čačanska banka	0.6024	15	0.9229	9
Erste Bank	0.7617	5	0.8646	10
Eurobank EFG	0.4855	18	0.4948	25
Findomestic banka	0.5561	16	1	1
Hypo Alpe-Adria-Bank	0.7892	4	0.7998	13
JUBMES banka	0.4650	19	1	1
KBC banka	0.1131	29	0.5582	22
Komercijalna banka	0.6123	14	0.6909	17
Marfin Bank	0.8452	3	1	1
Crédit Agricole banka	0.6220	13	0.7300	16
Razvojna banka Vojvodine	0.4242	21	0.5358	24
NLB banka	0.7084	10	0.7964	14
OTP banka	0.2649	25	0.4094	26
Piraeus Bank	0.4565	20	0.6516	20
Agrobanka	0.2431	26	0.3059	27
Privredna banka	0.7283	7	1	1
ProCredit Bank	0.1344	28	0.2944	28
Raiffeisen banka	0.7390	6	0.8537	11
Société Générale banka	0.7260	8	0.7666	15
Srpska banka	0.2765	24	0.8439	12
Unicredit Bank	1	1	1	1
Univerzal banka	0.3760	23	0.6690	18
Vojvođanska banka	0.2364	27	0.2845	29
Volksbank	0.9305	2	1	1

This article has provided comparison of the results for six different DEA models. Three models assumed constant return to scale, and three assumed a variable return to scale. Two models did not use any weight restriction while four models have used it. The analysis was conducted on the example of banking market in Serbia. The paper also gives a detailed review of how other authors measured efficiency in banking, either in the markets of other countries or in Serbia market. Different ways of restricting weights in DEA are presented through paper. The greatest contribution of this study is that it successfully combines the DEA method with DBA method. Using bootstrapped I-distance method weights are generated and then that weights are later used for DEA models with assurance regions.

It was shown that models with a variable return to scale have tendency that large number of DMUs consider as an efficient. Basic BCC model give result that 18 out of 29 banks are efficient. CCR model gave notably lower number - 11 banks, which is slightly more than a third of the total number of banks. It is a solution that can be accepted in the case of banking market in Serbia, where it is considered that it is not realistic that there is such large number of banks in such a small market. However, with results obtained with CCR models do not give a picture of what the difference is between banks that are considered to be efficient. Much clearer picture is shown by GAR models. Introducing a weight restrictions significantly reduce the number of effective units. Both 6σ and 3σ model with constant return to scale give only one efficient bank - Unicredit. 3σ with variable return to scale consider seven banks as an efficient, which differ from 6σ with variable return to scale only in a way that Credy bank is not considered efficient by 3σ.

TABLE V.
THREE SIGMA WEIGHT CONSTRAINS

Variable	Lower bound	Upper bound
Number of employee	0.261	0.304
Fixed assets	0.236	0.276
Capital	0.208	0.265
Deposits	0.203	0.247
Loans	0.318	0.383
Other placement	0.310	0.363
Noninterest income	0.265	0.361

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