FUZZY INTEGER TRANSPORTATION PROBLEM

Gheorghe DOGARU¹ Ion COLTESCU²

Naval Academy "Mircea cel Batran", Constanta, Romania

Abstract. In this paper is presented an algorithm which solves the transportation problem with fuzzy values for supply and demand and with the integrability condition imposed to the solution. The algorithm is exact and calculable effective even if the problem is formulated into a general manner, i.e. the fuzzy values for supply and demand can differ one from another and they are fuzzy numbers of a certain type. **Keywords**: fuzzy, transportation problem, supply and demand, objective function **Mathematics Subject Classification 2000**: 90B06, 90C08, 90C70

1. INTRODUCTION

The transportation model has many applications, not just as the transportation problem itself, but in the production scheduling problem.

The parameters of each transportation problem are the unitary costs (the profits) and the values for supply and demand (production, storage capacity). In practice, these parameters are not always known and stable. In the next approached problem it is supposed that the unitary costs (the profits) are known exactly, but the estimation of the values for supply and demand (capabilities) is approximate. This inaccuracy results either from missing information or by a certain flexibility in planning the capabilities of the considered factory. A frequently used mean to express the inaccuracy are the fuzzy numbers.

In the classical transportation problem with integer values for supply and demand, there is always an

integer solution. This solution can be found using the simplex transportation method, which is one of the most used methods for solving the transportation problems. This property (i.e. the possibility of finding an integer solution) is not kept in the fuzzy transportation problem with fuzzy values for supply and demand, even if the characteristics of the existing fuzzy numbers in the problem are integers. To obtain an optimal integer solution (which would be necessary from flexibility rations) a special algorithm is used. Such an algorithm is presented in S. Chanas and D. Kuchta.

In this paper, we present an algorithm which determines the optimal integer solution of a fuzzy transportation problem, *more general* than that considered in S.Chanas and D.Kuchta, using only the classical (non-parametric) transportation problem.

2. THE PROBLEM FORMULATION AND USED NOTATIONS

The considered fuzzy numbers are of L-R type. An L-R type fuzzy number A is: $A = \left(\underline{a}, \overline{a}, \alpha_A, \beta_A\right)_{L-R}$ and has the characteristic function:

$$\mu_{A}(t) = \begin{cases} L\left(\frac{\underline{a}-t}{\alpha_{A}}\right), \text{ for } t \leq \underline{a} \\ 1, \quad \text{for } t \in \left(\underline{a},\overline{a}\right), t \in \Box \\ R\left(\frac{t-\overline{a}}{\beta_{A}}\right), \text{ for } t \geq \overline{a} \end{cases}$$

where $\underline{a}, \overline{a}, \alpha_A, \beta_A \in \Box_+$ and L, R are form function.

• F is a form function if F if continuous, decreasing on $[0,\infty)$, F(0)=1 and strictly decreasing on that side of the domain on which F is positive.

Example:

- Linear function: $F(y) = \max\{0, 1-y\}, y \ge 0$.
- Exponential function: $F(y) = e^{-py}, p \ge 1, y \ge 0$.
- Power function: $F(y) = \max\{0, 1-y^p\}, p \ge 1, y \ge 0$.
- Rational function: $F(y) = 1/(1+y^p), p \ge 1, y \ge 0$

There are particular cases when the functions L and R don't have any signification.

- $\underline{a} = -\infty, \ \mu_A(t) = 1, \ t \le a$.
- $\underline{a} = +\infty, \ \mu_A(t) = 1, \ t \ge \underline{a}.$
- $\alpha_A = 0, \ \mu_A(t) = 0, \ t \leq \overline{a}.$
- $\beta_A = 0, \ \mu_A(t) = 1, \ t \ge \overline{a}.$

The fuzzy transportation fuzzy considered here is enounced such as follow:

$$\begin{cases} C(x) = \min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \\ \sum_{j=1}^{n} x_{ij} \cong A_{i} \quad i = \overline{1, m} \\ \sum_{i=1}^{m} x_{ij} \cong B_{j} \quad j = \overline{1, n} \\ x_{ii} \ge 0 \text{ and } x_{ii} \in \Box \quad i = \overline{1, m}, \quad j = \overline{1, n} \end{cases}$$

$$(2.1)$$

where A_i , B_i are fuzzy numbers on the form:

$$A_{i} = \left(\underline{a}_{i}, \overline{a}_{i}, \alpha_{A_{i}}, \beta_{A_{i}}\right) L_{i} - R_{i}, \quad i = \overline{1, m},$$

$$B_{j} = \left(\underline{a}_{j}, \overline{a}_{j}, \alpha_{A_{j}}, \beta_{A_{j}}\right) S_{j} - T_{j}, \quad j = \overline{1, n}.$$

• x is matrix solution (which components are the corresponding decisional variables), i.e., $x = (x_{ij})_{m \times n}$. The unitary transportation costs c_{ij} , $i = \overline{1, m}$, $j = \overline{1, n}$ are supposed to be crisp numbers.

This problem formulation showed that the result is expressed using a fuzzy number noted with G:

$$G = \left(-\infty, c_0, 0, \beta_G\right)_{L_G - R_G}$$

This problem is more complete than the one treated in S.Chanas and D.Kuchta. The complexity consist in the following: here the pairs (L_i, R_i) , $i = \overline{1, m}$ and (S_j, T_j) , $j = \overline{1, n}$ can be different, while in S.Chanas and D.Kuchta they must be identical.

In the next definition the fuzzy constraint concept and the fuzzy objective concept is defined.

Definition 2.1

Let x an arbitrary solution of the problem (2.1).

a) The value
$$\mu_c(x) = \min\left\{\mu_{A_i}\left(\sum_{j=1}^n x_{ij}\right), i = \overline{1, m}, \mu_{\beta_j}\left(\sum_{i=1}^m x_{ij}\right), j = \overline{1, n}\right\}$$
 is called the restrictions

satisfaction degree for the problem (2.1).

b)
$$\mu_G(x) = \mu_G(c(x)) = \mu_G\left(\sum_{i=1}^m \sum_{j=1}^n c_{ij}x_{ij}\right)$$
 is called the objective satisfaction degree (the (2.1) problem

result) by x.

• According to Belmann–Zadeh approach, a solution is called an optimal solution if it is a solution to a problem where the restrictions and the objective are of maximal degree. **Definition 2.2**

The maximized solution is a vector x for which $\mu_0(x) = \min \{\mu_c(x), \mu_G(x)\}$ attained the maximum. If the maximum value is 0, we say that the problem (2.1) is insoluble.

3. THE SOLUTION OF THE PROBLEM

According to Definition 2.1, solving the problem (2.1) is equivalent with solving the following mathematical programming problem:

$$\min\left\{\mu_{c}(x),\mu_{G}(x)\right\} \to \max$$

$$x_{ii} \ge 0; x_{ii} \in \Box, i = 1, m, j = 1, n$$

Solving this mathematical programming problem is equivalent with solving the following problem:

$$\begin{cases} \lambda \to \max \\ \mu_G(x) \ge \lambda \\ \mu_{A_i}\left(\sum_{j=1}^n x_{ij}\right) \ge \lambda, \quad i = \overline{1, m} \\ \mu_{B_j}\left(\sum_{i=1}^m x_{ij}\right) \ge \lambda, \quad j = \overline{1, n} \\ \lambda > 0, x_{ij} \ge 0, \quad x_{ij} \in \Box, \quad i = \overline{1, m}, \quad j = \overline{1, n} \end{cases}$$

$$(2.2)$$

To understand the following, we give the definition: **Definition 2.3**

Let A be a fuzzy number. The λ -cut of A, denoted A^{λ} , is $A^{\lambda} = \left\{ t \in \Box \mid \mu_A(t) \geq \lambda \right\}$.

Now, it easy to observe that the hypotheses at A_i and B_j considered in this problem, the λ -cuts A_i^{λ} and B_j^{λ} are intervals, given by:

$$\begin{cases} A_{i}^{\lambda} = \left[\underline{a}_{i} - L_{i}^{-1}(\lambda) \cdot \alpha_{A_{i}}, \overline{a}_{i} + R_{i}^{-1}(\lambda)\beta_{A_{i}}\right], i = \overline{1, m} \\ B_{j}^{\lambda} = \left[\underline{b}_{j} - S_{j}^{-1}(\lambda) \cdot \alpha_{\beta_{j}}, \overline{b}_{j} + T_{j}^{-1}(\lambda)\beta_{B_{j}}\right], j = \overline{1, m} \end{cases}$$
(2.3)

• The λ - cut for the fuzzy objective G is the set:

$$G^{\lambda} = \left(-\infty, c_0 + R_G^{-1}(\lambda)\beta_G\right)$$

(2.4)

Take this into account the problem (2.2) can be rewritten as follows:

$$\begin{cases} \lambda \to \max \\ c(x) \ge G^{\lambda} \\ \sum_{j=1}^{n} x_{ij} \ge A_{i}^{\lambda}, \quad i = \overline{1, m} \\ \sum_{i=1}^{m} x_{ij} \ge B_{j}^{\lambda}, \quad j = \overline{1, n} \\ \lambda > 0 \\ x_{ij} \ge 0, \quad x_{ij} \in \Box \quad , i = \overline{1, m}, \quad j = \overline{1, n} \end{cases}$$

$$(2.5)$$

The above problem is not a transportation problem because of its objective function and the first condition. This is why we cannot use the transportation algorithms to solve it. But, we can associate an interval transportation problem. Further, the solution of this auxiliary problem allows finding the solution of the problem (2.5), and from here, of the problem (2.1).

This auxiliary problem is given by:

$$\begin{cases} \sum_{j=1}^{n} x_{ij} \in A_{i}^{\lambda}, \quad i = \overline{1, m} \\ \sum_{i=1}^{m} x_{ij} \in B_{j}^{\lambda}, \quad j = \overline{1, n} \\ x_{ij} \ge 0, \quad x_{ij} \in \Box, \quad i = \overline{1, m}, \quad j = \overline{1, n} \end{cases}$$
(2.6)

- Solving the problem (2.6) for a fixed $\lambda > 0$, allows to find if (2.5) is possible for this value of λ . Is suffices to verify if the value of the objective function value of the problem (2.6) verifies the first condition from (2.5). It is necessary that the maximum value of λ for which (2.5) is possible and the corresponding solution of (2.5).
- The end points of the intervals that appear in the conditions of the problem (2.6) can be non-integers. This means that after we pass at the classical transportation problem described in (2.6), we could have a classical transportation problem with non-integers values for supply and demand, and that's the reason why the classical algorithm does not guarantee obtaining of an integer solution. However, one can be replace the problem (2.6) with a problem (2.7), which has already the integer end points for the supply and demand intervals without changing the set of the possible and optimal solutions.

For defining the problem (2.7) we use the following notation:

Definition 2.4

Let A be an interval. [A] is the largest interval with integer end points contained in A, i.e. [A] = [a,b],

where
$$a = \min \{t \mid t \in A \cap \Box\}$$
, $b = \max \{t \mid t \in A \cap \Box\}$.

With this notation the problem (2.7) which replaces the problem (2.6) is given by:

$$\begin{cases} c(x) \rightarrow \min \\ \sum_{j=1}^{n} x_{ij} \in [A_i^{\lambda}], \quad i = \overline{1, m} \\ \sum_{i=1}^{m} x_{ij} \in [B_j^{\lambda}], \quad j = \overline{1, n} \\ x_{ij} \ge 0, \quad x_{ij} \in \Box, \quad i = \overline{1, m}, \quad j = \overline{1, n} \end{cases}$$

$$(2.7)$$

- The sets of the possible and optimal solutions of (2.6) and (2.7) are identical because of the integrability condition imposed to x.
- For $\lambda > 0$ fixed, it can be resolved (2.5) by replacing it with a classical integer values for supply and demand transportation problem and it can be applied, for example, the simplex algorithm transportation problem.
- If one can solve (2.7) (or the corresponding classical transportation problem) as a problem with parameter λ , also one can be solved and the initial problem.
- However, the coefficients of the problem depend nonlinear on λ which determine some difficulty. To avoid solving a parameter transportation problem we propose the following algorithm which implies just solving a few classical transportation problems.
- The algorithm begins from the largest values of λ , i.e., $\lambda = 0$ and $\lambda = 1$. Is investigated for which values of λ , the problem (2.5) is admissible.
 - If it is impossible for $\lambda = 0$, (2.1) is impossible.

If (2.5) is possible for $\lambda = 1$, then $\lambda = 1$ the optimal value of the objective function in (2.5) and the corresponding solution of (2.5) is, again, a solution for the problem (2.1).

If the problem (2.5) is possible for $\lambda = 0$ and impossible for $\lambda = 1$ (the most frequent case), we consider $\lambda = 1/2$, and then [0, 1/2].

Acting in this manner we will approach from both sides to the optimal value of the objective function of (2.5). Thus, at each step one consider $[\lambda_1, \lambda_2]$ such that (2.5) is possible for $\lambda = \lambda_1$ and impossible for $\lambda = \lambda_2$. It's not necessary to divide this interval, when (2.7), for $\lambda = \lambda_1$, is a minimal extension of (2.7) for $\lambda = \lambda_2$ (just like in the definition 2.5).

Definition 2.5

The problem (2.7) for $\lambda = \lambda_1$ is a minimal extension when the problem (2.7) for $\lambda = \lambda_1$ is identical with the problem (2.7) for $\lambda = \lambda^*$ where:

$$\lambda^* = \max\left\{\max_{1 \le i \le m, t \notin A_j^{\lambda}} \mu_{A_i}\left(t\right), \max_{1 \le j \le n, t \notin B_j^{\lambda}} \mu_{B_j}\left(t\right)\right\}, \ t \in \Box.$$

From above section it follows that the corresponding algorithm (where the consecutive intervals of the values is divided in two) is finished after a finite number of steps, if we check at every step if the problem (2.7) for $\lambda = \lambda_1$ is a minimal extension of the problem (2.7) for $\lambda = \lambda_2$.

The user can determine $[\lambda_1, \lambda_2]$, starting with the conditions that must be verified. The *algorithm* is given by:

Step 1

Set $\lambda(1) = 0$, $\lambda(2) = 1$.

Step 2

We solve the problem (2.7) for $\lambda = \lambda(1)$. If the problem is possible and $C(x(\lambda(1))) \in G^{\lambda(1)}$, then GO to Step 3. Else, the problem (2.1) is impossible.

Therefore, STOP (the problem (2.1) impossible and $\mu_D(x) = 0$, for all x).

Step 3

We solve the problem (2.7) for $\lambda = \lambda(2)$. If the problem is possible and $c(x(\lambda(2))) \in G^{\lambda(2)}$, then STOP, and $x(\lambda(2))$ is the optimal solution of the problem (2.1) and $\mu_D(x(\lambda(2))) = 1$. Else, GO to Step 4.

Step 4

Set $\lambda_m := (\lambda(1) + \lambda(2))/2$ and GO to Step 5.

Step 5

We solve the problem (2.7) for $\lambda := \lambda_m$. If the problem is impossible, then $\lambda(2) := \lambda_m$ and GO to Step 6. Else, there are 3 possible cases:

i) $\mu_G(x(\lambda_m)) = \mu_C(x(\lambda_m))$, result that $x(\lambda_m)$ is an optimal solution of the problem (2.1) and STOP.

ii) $\mu_G(x(\lambda_m)) > \mu_C(x(\lambda_m))$, then we set $\lambda(1) = \mu_C x(\lambda_m)$ and GO to Step 6.

ii)
$$\mu_G(x(\lambda_m)) < \mu_C(x(\lambda_m))$$
, one set $\lambda(2) \coloneqq \mu_C(x(\lambda_m))$ or, if $\lambda(2) = \mu_C(x(\lambda_m))$ GO to Step

6. Step 6

If $\lambda(2) - \lambda(1) > \varepsilon$, then GO to Step 4. Else, we verify if the problem (2.7) for $\lambda = \lambda(1)$ is an minimal extension of the problem (2.7) for $\lambda = \lambda(2)$. If it is not, GO to Step 4. Else, STOP. A solution $x(\lambda(1))$ or $x(\lambda(2))$ is optimal for the problem (2.1).

If the problem (2.5) is impossible for $\lambda = \lambda(2)$, then $x(\lambda(1))$ is an optimal solution for the problem (2.1). ε is given by the user $(0.05 \le \varepsilon \le 0.1)$.

4. NUMERICAL EXAMPLE

The above algorithm is illustrated with the following example.

$$\begin{cases} 10x_{11} + 20x_{12} + 30x_{13} + 20x_{21} + 50x_{22} + 60x_{23} \rightarrow \min \\ x_{11} + x_{12} + x_{13} \cong (10, 10, 5, 5)_{L-E} \\ x_{21} + x_{22} + x_{23} \cong (16, 16, 5, 5)_{E-L} \\ x_{11} + x_{21} \cong (10, 10, 5, 5)_{P-E} \\ x_{12} + x_{22} \cong (9, 9, 4, 4)_{L-E} \\ x_{13} + x_{23} \cong (1, 1, 1, 1)_{P-R} \\ x_{ij} \ge 0, \ x_{ij} \in \Box, \ i = \overline{1, 2, j} = \overline{1, 3} \end{cases}$$

The fuzzy objective is determined by the following fuzzy number $G = (0,300,0,500)_{L-L}$. *L* replace the linear form function , *E* the exponential function with parameter p = 1, *P* the power form function with parameter p = 2, *R* the rational function with parameter p = 1. The λ -cuts for the fuzzy values of the supply and demand, and for the fuzzy objective with the considering form function are (taking account relation (2.3) and (2.4)):

$$A_{1}^{\lambda} = \left[10 - 5(1 - \lambda), 10 - 5 \cdot \ln \lambda\right],$$

$$A_{2}^{\lambda} = \left[16 - 5\ln \lambda, 16 + 5 \cdot \ln(1 - \lambda)\right],$$

$$B_{1}^{\lambda} = \left[10 - 5\sqrt{1 - \lambda}, 10 - 5\ln \lambda\right],$$

$$B_{2}^{\lambda} = \left[9 - 4(1 - \lambda), 9 - 4\ln \lambda\right],$$

$$B_{3}^{\lambda} = \left[1 - \sqrt{1 - \lambda}, 1 + (1 - \lambda)/\lambda\right],$$

$$G^{\lambda} = \left[0, 300 + (1 - \lambda) \cdot 500\right].$$

The steps of the algorithm are:

Step 1

 $\lambda(1) = 0, \lambda(2) = 1.$

Step 2

The problem (2.7) is possible for $\lambda = 0$ and $c(x(0)) \in G^0$, where $G^0 = [0, 800]$.

Step 3

The problem (2.7) is impossible for $\lambda = 1$.

(

Step 4

 $\lambda_m = (0+1)/2 = 0.5$

Step 5

The problem (2.7) is possible for $\lambda = 0.5$ and $\mu_G(x(0.5)) = 0.74 > \mu_C(x(0.5)) = 0.5$.

Step 6

 $\lambda(2) - \lambda(1) = 1 - 0.5 = 0.5 > 0.05$.

Step 4

 $\lambda_m = (0.5 + 1)/2 = 0.75$

Step 5

The problem (2.7) is possible for $\lambda = 0.75$. Thus, $\lambda(2) := 0.75$.

Step 6

 $\lambda(2) - \lambda(1) = 0.75 - 0.5 = 0.25 > 0.05$.

Step 4

$$\lambda_m = (0.5 + 0.75)/2 = 0.625$$

Step 5

The problem (2.7) is possible for $\lambda = 0.625$ and $\mu_G(x(0.625)) = 0.54 < \mu_C(x(0.625)) = 0.64$.

Thus, $\lambda(2) := 0.64$.

Step 6

 $\lambda(2) - \lambda(1) = 0.64 - 0.5 = 0.14 > 0.05$.

Step 4

 $\lambda_m = (0.5 + 0.64)/2 = 0.57$.

Step 5

The problem (2.7) is possible for $\lambda = 0.57$ and $\mu_G(x(0.57)) = 0.58 < \mu_C(x(0.57)) = 0.6$. Thus,

 $\lambda(2) \coloneqq 0.6$.

Step 6

$$\lambda(2) - \lambda(1) = 0.6 - 0.5 = 0.1 > 0.05$$

Step 4

 $\lambda_m = (0.5 + 0.6)/2 = 0.55$.

Step 5

The problem (2.7) is possible for $\lambda = 0.55$ and $\mu_G(x(0.55)) = 0.58 < \mu_C(x(0.55)) = 0.6$. Since $\lambda(2) = \mu_C(x(0.55)) = 0.6$, $\lambda(2) := 0.55$. Step 6

 $\lambda(2) - \lambda(1) = 0.55 - 0.5 = 0.05 \le 0.05$.

But the problem (2.7) for $\lambda = \lambda(1) = 0.5$ is not a minimal extension of the problem (2.7) for $\lambda = \lambda(2) := 0.55$ and Step 4 is executed one more time.

Step 4

 $\lambda_m = (0.5 + 0.55)/2 = 0.525.$

Step 5

The problem (2.7) is possible for $\lambda = 0.525$ and $\mu_G(x(0.525)) = 0.7 > \mu_C(x(0.525)) = 0.5488$. Thus, $\lambda(1) := 0.5488$.

Step 6

 $\lambda(2) - \lambda(1) = 0.55 - 0.5488 = 0.012 < 0.05$. Therefore, we verify if the problem (2.7) for $\lambda = \lambda(1) := 0.5488$ it is minimal extension of the problem (2.7) for $\lambda = \lambda(1) := 0.55$. Since the answer is positive, the algorithm ends here. One of the solutions x(0.5488) and x(0.55) is an optimal solution for the problem (2.1). Since $\mu_D(x(0.55)) = 0.58 > \mu_D(x(0.5488)) = 0.5488$, the second solution is better than the first one. The complete form of this solution is given by the following table.



Total cost = 510

$$\lambda_{C}(x) = 0.6$$
, $\mu_{G}(x) = 0.58$, $\mu_{D}(x) = 0.58$.

To illustrate the entire Step 5 of the algorithm is showed how the problem (2.5) is constructed and solved for a certain value of the parameter. Let $\lambda = 0.55$. In this case:

$$\begin{aligned} A_1^{0.55} &= \begin{bmatrix} 7.55, 12.99 \end{bmatrix}, \ A_2^{0.55} &= \begin{bmatrix} 13.01, 18.25 \end{bmatrix}, \\ B_1^{0.55} &= \begin{bmatrix} 6.46, 12.99 \end{bmatrix}, \ B_2^{0.55} &= \begin{bmatrix} 7.2, 11.39 \end{bmatrix}, \ B_3^{0.55} &= \begin{bmatrix} 0.33, 1.82 \end{bmatrix}, \\ & \begin{bmatrix} A_1^{0.55} \end{bmatrix} = \begin{bmatrix} 8, 12 \end{bmatrix}, \ \begin{bmatrix} A_2^{0.55} \end{bmatrix} = \begin{bmatrix} 14, 18 \end{bmatrix}, \ \begin{bmatrix} B_1^{0.55} \end{bmatrix} = \begin{bmatrix} 7, 12 \end{bmatrix}, \ \begin{bmatrix} B_2^{0.55} \end{bmatrix} = \begin{bmatrix} 8, 11 \end{bmatrix}, \ \begin{bmatrix} B_3^{0.55} \end{bmatrix} = \begin{bmatrix} 1, 1 \end{bmatrix}. \end{aligned}$$

The problem (2.5) with interval values for request and offering can be reduced, according to Section 4, to a transportation problem with forbidden routes (given in table 2) which optimal is given in table 3.

| 838382 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Supply |
|--------|----|----|----|----|----|----|----|----------|
| 1 | 10 | 20 | 30 | 10 | 20 | 30 | ٠ | 8 |
| 2 | 20 | 50 | 60 | 20 | 50 | 60 | ٠ | 14 |
| 3 | 10 | 20 | 30 | 10 | 20 | 30 | 0 | 4 |
| 4 | 20 | 50 | 60 | 20 | 50 | 60 | 0 | 4 |
| 5 | ٠ | ٠ | ٠ | 0 | 0 | 0 | 0 | 8 |
| Demand | 7 | 8 | 1 | 5 | 3 | 0 | 14 | \times |



| \times | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Supply | | | |
|----------|---|---|---|---|---|---|----|--------|--|--|--|
| 1 | | 7 | | | 1 | | | 8 | | | |
| 2 | 7 | 1 | 1 | 5 | | | | 14 | | | |
| 3 | | | | | | | 4 | 4 | | | |
| 4 | | | | | | | 4 | 4 | | | |
| 5 | | | | | 2 | | 6 | 8 | | | |
| Demand | 7 | 8 | 1 | 5 | 3 | 0 | 14 | ***** | | | |
| Table 2 | | | | | | | | | | | |

Table 3

The optimal solution of the problem is reduced, after the application of the formulas from the end of section 4, to the solution of the initial problem (2.7) (which is given in table 1). Along to the last application of Step 6,

during verification of the optimal condition, λ^* (defined in Definition 2.5) is calculated from:

$$\lambda^{*} = \max \left\{ \begin{array}{l} \mu_{A_{1}}(7), \mu_{A_{1}}(13), \mu_{A_{2}}(13), \mu_{B_{1}}(6), \mu_{B_{1}}(6) \\ \mu_{B_{1}}(13), \mu_{B_{2}}(7), \mu_{B_{2}}(13), \mu_{B_{3}}(0), \mu_{B_{3}}(2) \end{array} \right\} = 0.5488.$$

The problem (2.7) for $\lambda = \lambda^*$ is analogue with that for $\lambda = \lambda(1)$ which goes to the stop of the algorithm.

CONCLUSIONS

In this paper it was given an algorithm for solving the fuzzy transportation problem (with fuzzy values for supply and demand and also for the objective) in the sense of maximizing the satisfaction both objectives and restrictions. This algorithm demand beside a simple transformations, just solving the classical transportation problem (particulary is not necessary solving the parameter problem). The fuzzy numbers which defines the problem must not be trapezoidal.

BIBLIOGRAPHY:

[1] R.R. Bellman and L.A.Zadeh, *Decision – making in a fuzzy environment*, Management Sci. B17, 1970, pp. 203 – 218;

[2] S.Chanas, *Parametric techniques in fuzzy linear programming problems,* in: J.L Verdegay and M. Delgado, Eds., The Interface between Artificial Intellingence and Operations Research in Fuzzy Environment (Verlag TÜV Rheinland, Köln, 1989), pp. 105–116;

[3] R. Dombrowschi, *Zagadnienia trasnportowe z parametryc – znymi ograniczeniami*, Przeglad Statystyczny, XV, 1968, pp. 103–117;

[4] D.Dubois and H. Prade, Operations on fuzzy numbers, Internat. J. Systems Sci. 6, 1978, pp. 613–626.