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COMPUTER SIMULATION A RANDOM VARIABLES BY COMPOSITION METHOD

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Abstract: *In building-up simulation models, as well as in various problems it is necessary to use the algorithms for computer generation of different types of random variables.*
In this paper are presented two algorithms for simulation of random variables with values of Pearson XI and Burr distribution, using the composition method.
Keywords: random variables, simulation algorithm, distribution function, generate.

INTRODUCTION

In their evolution, real systems are influenced by random factors, which should be taken into account when making their studies. Components of any system follow some probability distribution. To study them, we need algorithms to generate sampling values with a given probability distribution, corresponding to each component. One of the methods used to simulate different values of probability distributions is composition method.. By this method, the simulation values that are specific probability distributions is reduced to simulate the values of known distributions, for which are known simulation performance algorithms.

THE COMPOSITION METHOD

Let X be a random variable whose probability distribution function $F(x)$ can be written as the composition of a family of distribution function $\{G(x,y),\ y \in \mathbf{R}\}$ by the distribution function $H(y)$, such

$$F(x) = \int_{-\infty}^{\infty} G(x,y)dH(y)$$

The family distribution function $G(x,Y)$ depends on random parameter Y which has distribution function $H(y)$.

The simulation process of the values of the random variables X with distribution function $F(x)$, when generating processes are known to Y with distribution function $H(y)$ and Z_Y having distribution function $G(x,Y)$ is as follows:

Generate a random variate Y with distribution function $H(y)$.

Generate a random variate Z_Y with distribution function $G(x,Y)$.

$X = Z_Y$.

SIMULATED VALUES OF THE RANDOM VARIABLE DISTRIBUTION WITH PEARSON XI

A random variable is Pearson XI distribution if its probability distribution function is given by

$$F(x) = \begin{cases} 0 & , x \le 0 \\ 1 - (\dfrac{\alpha}{x+\alpha})^{\nu}, & x > 0,\ \alpha > 0, \nu > 0 \end{cases}$$

To simulate the random variable values with Pearson XI distribution is applied the composition method by mixing a family of distribution functions gama(ν,λ), where λ is a random parameter that has a negative exponential distribution.

A random variable is gama(ν,λ) distribution if its probability density function is given by

$$g(x) = \begin{cases} 0 & , x \le 0 \\ \dfrac{\lambda^{\nu}}{\Gamma(\nu)} x^{\nu-1} e^{-\lambda x}, & x > 0,\ \nu > 0,\ \lambda > 0 \end{cases}$$

and distribution function

$$G(x,\nu,\lambda) = \frac{\lambda^{\nu}}{\Gamma(\nu)} \int_0^x t^{\nu-1} e^{-\lambda t} dt$$

A random variable is negative exponential distribution $Exp(\mu)$ if its distribution function is given by

$$H(\lambda) = \begin{cases} 0 & , \lambda \le 0 \\ 1 - e^{-\lambda\mu} & , \lambda > 0, \mu > 0 \end{cases}$$

The mixing family of distribution functions gama(ν,λ), where λ is a random parameter that has a negative exponential distribution, is as follows:

$$F(x) = \int_0^{\infty} G(x,\nu;\lambda) dH(\lambda) = \int_0^{\infty} (\frac{\lambda^{\nu}}{\Gamma(\nu)} \int_0^x t^{\nu-1} e^{-\lambda t} dt)\, \mu e^{-\lambda\mu} d\lambda$$

Hence

$$F(x) = 1 - (\frac{\mu}{x+\mu})^{\nu+1}$$

is the distribution function of Pearson XI.

Simulation algorithm

Input ν, α

Generate a negative exponential(ν) random variate Y.

Generate a gama (α, Y) random variate Z.

$X = Z$.

SIMULATED VALUES OF THE RANDOM VARIABLE DISTRIBUTION WITH BURR

A random variable is Burr distribution if its probability distribution function is given by

$$F(x) = \begin{cases} 0 & , x \le 0 \\ 1 - \dfrac{1}{(1+x^c)^k}, & x > 0,\ c, k \in \Re_+ \end{cases}$$

To simulate the random variable values with Burr distribution is applied the composition method by mixing a family of distribution functions Weibull (θ), where θ is a random parameter that has a gama distribution.

Distribution function of a Weibull random variable is given by

$$G(x,\theta) = \begin{cases} 0 & , x \le 0 \\ 1 - e^{-\theta x^c} & , x > 0, \theta > 0, c > 0 \end{cases}$$

The parameter θ has gama distribution, which means that the probability density function is given by

$$h(\theta) = \begin{cases} 0 & , \theta \le 0 \\ \dfrac{\theta^{k-1} e^{-\theta}}{\Gamma(k)}, & \theta > 0,\ k > 0 \end{cases}$$

The mixing family of distribution functions Weibull (θ), where θ is a random parameter that has a gama distribution, is as follows:

$$F(x) = \int_0^\infty G(x,\theta) dH(\theta) = \int_0^\infty G(x,\theta) h(\theta) d\theta$$

$$F(x) = \int_0^\infty (1 - e^{-\theta x^c}) \frac{\theta^{k-1} e^{-\theta}}{\Gamma(k)} d\theta = \frac{1}{\Gamma(k)} [\int_0^\infty \theta^{k-1} e^{-\theta} d\theta - \int_0^\infty \theta^{k-1} e^{-\theta(1+x^c)} d\theta$$

$$F(x) = \frac{1}{\Gamma(k)} [\Gamma(k) - \frac{1}{(1+x^c)^k} \Gamma(k)] = 1 - \frac{1}{(1+x^c)^k}$$

$F(x)$ is the distribution function of Burr.

Simulation algorithm

Input $c,\ k$.

Generate a gama (k) random variate Y.

Generate a Weibull (c, Y) random variate Z.

$X = Z$.

Remark.

In order to obtain sampling values having Pearson XI or Burr distribution, the algorithms are repeated the number of times that required.

CONCLUSION

The performances of the algorithms presented above depend of the performance of the algorithms for random generating variables having negative exponential distribution, Weibull distribution and gama distribution.

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