"Mircea cel Batran" Naval Academy Scientific Bulletin, Volume XVII – 2014 – Issue 2
Published by "Mircea cel Batran" Naval Academy Press, Constanta, Romania // The journal is indexed in: PROQUEST SciTech Journals, PROQUEST Engineering Journals, PROQUEST Illustrata: Technology, PROQUEST Technology Journals, PROQUEST Military Collection PROQUEST Advanced Technologies & Aerospace

CALCULUS OF THE FEEDBACK ELECTROMAGNETIC FIELD OF A WALL FROM A SHIP

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Abstract: *Calculus of the feedback electromagnetic field of a wall from a ship refers to a calculus method of the electromagnetic field reflected by a metallic structure/wall from a ship. The motivation behind this research consists in the fact that, according to EMC, a special interest concerning the protection of the electronic equipment and human body on board is given to the determination of the secondary radiation created by the plane metallic structures of a navy ship. This radiation is due to the reflection of electromagnetic waves of the radio and radar antennas. For this purpose we propose a method based on the calculus of the plan-parallel shield, using Helmholtz propagation equation. The results of the modeling are useful to identify the adequate protection solutions for on board personnel and equipment against the secondary radiation, but they can be used for other real situations where such an issue appears.*

Introduction

An important problem related to the evaluation of on board personnel protection against the electromagnetic field effects refers to the secondary radiation of the plane metallic structures of the navy ship [1], [3].

Thus, as a result of the antennas radiation field influence the metallic walls or other plane metallic structures on the decks become, in their turn, secondary radiation sources that can be a supplementary danger for on board personnel that performs duties on these decks [4], [5], [8].

The paper presents a calculus model to determine the radiated electromagnetic field and associated power density, emitted by a metallic wall, i.e. the iron wall of the transmitter compartment on command deck of a frigate. For this reason we propose a new method, consisting in the calculus of the parallel-plane shield based on solving the Helmholtz propagation equation, having imposed limit conditions; the metallic wall being considered to belong to a parallelepiped shield of finite dimensions (Fig. 1.1). The idea consists of calculating the value of the magnetic field intensity produced by the induced current within the shield wall, at the limit of its exterior surface. This is considered to be a feedback field of the wall, which can influence in a negative way, in time, the health of the personnel working nearby.

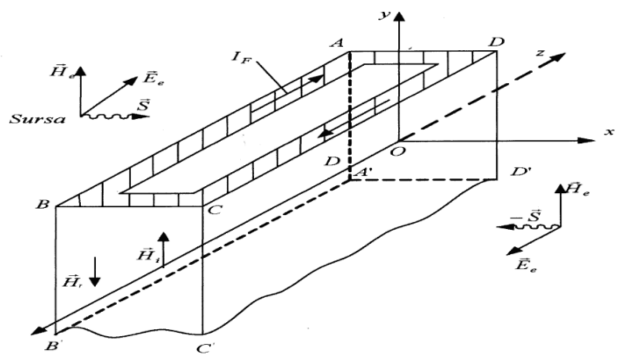

Fig. 1.1. The plan-parallel shield *attacked* by a plane electromagnetic wave: He – the exterior magnetic field of the source; Ee – the exterior electric field of the source; S – the power density of the exterior field; Hi – the interior magnetic field within the shield; Hr – the feedback magnetic field of the shield; -S – the reactive power density; I_F – the induced current in the wall of the shield

Five essential aspects must be specified ab initio:

– the analyzed plane wall, belonging to an transmitter compartment, is considered to be a part of a parallelepiped shield (respectively of a plan-parallel shield) of finite dimension ;

– the computation takes into account a finite dimension wall, in which case;

– the shield extremity effect cannot be disregarded;

– the intensity of the induced electric field does not obey the conservation theorem of tangential components on the external side of the shield[1];

– the displacement current is taken into account in this situation because the incident waves frequency is high (the fields of the radio and radar equipments that *attack* the wall of the shield have usually frequency values between 100 MHz and 10 GHz).

Modeling of the plane wall reaction field uses *Helmholtz's equations* for the plane-parallel shield and has three phases: a. calculation of the field inside the shield; b. calculation of the field outside the shield c. calculation of the field within the wall of the shield.

Calculus of the field inside the shield

Inside the shield s=0 (air – transmitter compartment), $\Gamma_i = j\Gamma_0$ (the propagation constant inside the shield) and the *Helmholtz* equation turns into [1], [2]:

[1] This is because the induced currents in the shield imply the presence of electric charges on the screen surface.

$$\frac{d^2Hy_i}{dx^2} = \Gamma_i^2 \cdot Hy_i; \;\; Hy_i = Hy_i(x); \;\; \Gamma_i = j\Gamma_0 = j\frac{2\pi}{\lambda_0} \tag{1.1}$$

Here $Hy_i(x)$ is the magnetic field intensity inside the shield, which is oriented on the Oy vertical axis.

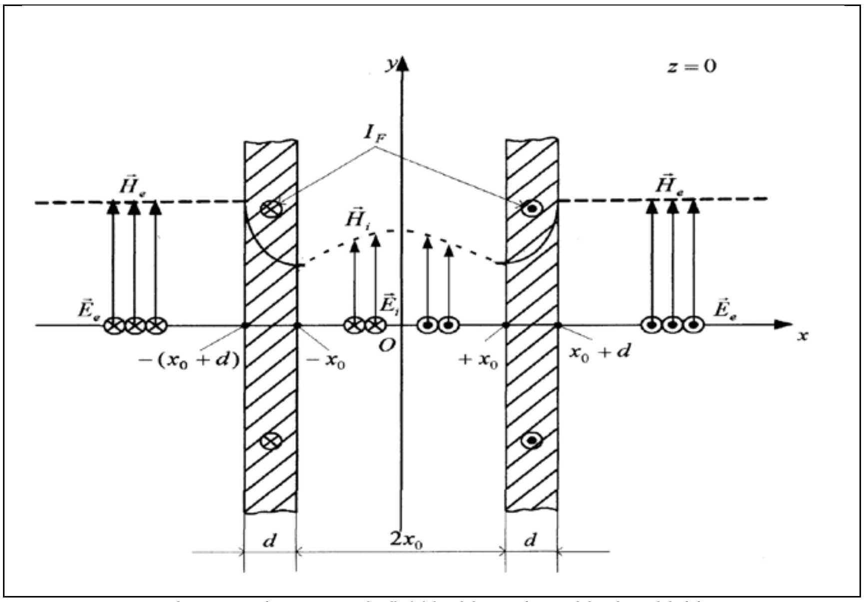

Fig.1.1 – The magnetic field inside and outside the shield

The solution of this equation is:

$$Hy_i(x) = C_1 \cdot e^{j\Gamma_0 x} + C_2 \cdot e^{-j\Gamma_0 x}, \;\; x \le x_0 \tag{1.2}$$

where x_0 is the distance from the centre of the shield (of the transmitters compartment) to the internal wall.

For practical reasons of symmetry it can be expressed as follows:

$$Hy_i(x) = Hy_i(-x). \; C_1 = C_2 = C \tag{1.3}$$

The inner magnetic field becomes:

$$Hy_i(x) = 2C \cdot \cos(\Gamma_0 \cdot x). \;\; x \le x_0 \tag{1.4}$$

The electric field inside the shield is calculated, being oriented on Oz axis.

Because $\overline{E} = Ez(x)\overline{k}$, it is calculated:

$$\nabla \times \overline{E} = -\overline{j}\left(\frac{\partial Ez(x)}{\partial x}\right). \tag{1.5}$$

On the other hand it can be written the Faraday`s law:

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}, \tag{1.6}$$

which in the complex calculation is formulated:

$$\nabla \times \overline{E} = -j\omega By_i\,\overline{j} = -j\omega\mu Hy_i\,\overline{j}. \tag{1.7}$$

By comparing relations (1.5) and (1.7) we obtain:

$$\frac{dEz(x)}{dx} = j\omega\mu Hy_i(x), \tag{1.8}$$

or:

$$dEz_i(x) = j\omega\mu_0 Hy_i(x)dx. \tag{1.9}$$

Relation (1.9) is valid in any of the three areas: the interior of the shield, the wall of the shield, the exterior of the shield (as a consequence of a general law).

Inside the shield $\mu = \mu_0$ and the equation (1.9) become:

$$dEz_i = j\omega\mu_0 Hy_i(x)dx. \tag{1.10}$$

Designating the magnetic field in the shield center (on the longitudinal axis) with $Hy_i(0)$, we obtain:

$$Hy_i(0) = 2C = Hi_0 \text{ (constant)} \tag{1.11}$$

and:

$$Hy_i(x) = Hi_0 \cdot \cos(\Gamma_0 \cdot x) \tag{1.12}$$

respectively.

Relation (1.12) describes the non-uniformity of the inner magnetic field, which has a stationary wave character, as opposed to the case of disregarding the displacement current, in which this field is constant.

Integrating (10) equation after replacing $Hy_i(x)$ from equation (1.12), the inner electric field becomes:

$$Ez_i(x) = j\frac{\omega\mu_0}{\Gamma_0} Hi_0 \cdot \sin(\Gamma_0 x) + C. \tag{1.13}$$

Because the equation is linear and the incident wave on the shield is considered to be harmonic, the inner field must be harmonic too, therefore:

$$C = 0 \tag{1.14}$$

and:

$$Ez_i(x) = j\frac{\omega\mu_0}{\Gamma_0} Hi_0 \sin(\Gamma_0 x) \tag{1.15}$$

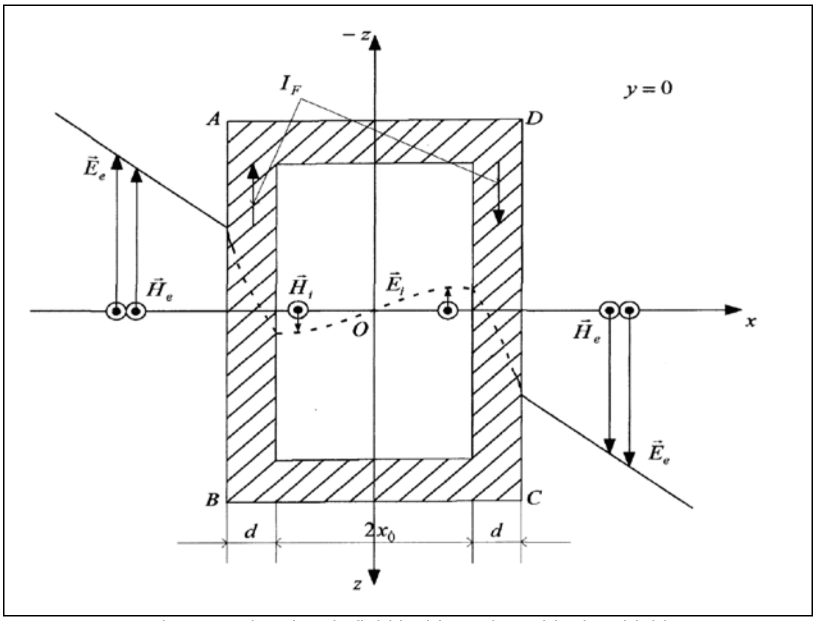

Fig.1.2 – The electric field inside and outside the shield

Noticing that:

$$\frac{\mu_0}{\Gamma_0} = \frac{\mu_0}{\frac{2\pi}{\lambda_0}} = \frac{\mu_0}{2\pi} c_0 \cdot T = \frac{\mu_0}{2\pi f} c_0 = \frac{\mu_0}{\omega} \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\omega} \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{Z_0}{\omega}, \quad (1.16)$$

where:

$Z_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$; $Z_0 = 120\pi = 377(\Omega)$ is the free space impedance (inner space of the shield); $c_0 = 3 \cdot 10^8 m/s$ is the propagation speed of the wave in free space (air), we obtain:

$$Ez_i(x) = j\omega \frac{Z_0}{\omega} Hi_0 \sin(\Gamma_0 x) = jZ_0 Hi_0 \sin(\Gamma_0 x). \quad x \le x_0 \quad (1.17)$$

Calculus of the field outside the shield

Starting from the *Helmholtz* equation:

$$\frac{d^2 Hy_e}{dx^2} = \Gamma_e^{\,2} \cdot Hy_e,\ \sigma = 0;\ \ \Gamma_e = \Gamma_i = j\Gamma_0 \quad (1.18)$$

it results that:

$$Hy_e(x) = C_1 e^{j\Gamma_0 x} + C_2 e^{-j\Gamma_0 x},\ |x| \ge x_0 + d \quad (1.19)$$

where *d* is the thickness of the wall shield.

Because the field is not of infinite value, respectively:

$$\lim_{x \to \infty} Hy_e(x) = finite, \quad (1.20)$$

it results that:

$$C_1 = 0. \quad (1.21)$$

Thus:

$$Hy_e(x) = C_2 e^{-j\Gamma_0 x} \quad (1.22)$$

and: $|Hy_e(x)| = C_2 = H_e$ (constant).

The magnetic field *He* emitted by the source is the far field ($r >> \lambda / 2\pi$, according to EMC terminology), having a constant value at distance from the shield, known through measurements.

The exterior magnetic field becomes:

$$Hy_e(x) = H_e \cdot e^{-j\Gamma_0 x}.\ |x| \ge x_0 + d. \quad (1.23)$$

The exterior magnetic field in the proximity of the shield is affected by the feedback field due to the extremity effect (finite dimension shield).

The exterior electric field becomes (by comparison to (1.10)):

$$dEz_e(x) = j\omega\mu_0 Hy_e(x)\mathrm{d}x. \quad (1.24)$$

Integrating (24) equation we obtain:

$$Ez_e(x) = \int j\omega\mu_0 H_e \mathrm{d}x = -\frac{\omega\mu_0}{\Gamma_0} H_e \cdot \mathrm{e}^{-j\Gamma_0 x} + C \cdot \quad (1.25)$$

The field being harmonic, *C=0* and results:

$$Ez_e(x) = -Z_0 He \cdot e^{-j\Gamma_0 x};\ H_e = ct..\ |x| \ge x_0 + d \quad (1.26)$$

Calculus of the field within the wall of the shield

The propagation equation (*Helmholtz*) is in this case:

$$\frac{d^2 Hy_p}{dx^2} = \Gamma^2{}_P \cdot Hy_p, \quad \Gamma = \Gamma_p ; \Gamma_p = \sqrt{j\omega\sigma\mu} = \frac{1+j}{\delta} ; \delta = \sqrt{\frac{2}{\omega\sigma\mu}} \tag{1.27}$$

and has the following solution:

$$Hy_p(x) = Ae^{\Gamma_p x} + Be^{-\Gamma_p x}, \; x_0 \le |x| \le x_0 + d , \tag{1.28}$$

with *s≠0,* $\Gamma_p \neq 0$, where $Hy_p(x)$ is the magnetic field within the wall of the shield. The electric field within the wall of the shield is in the form: $Ez_p(x) = Z_e\left(Ae^{\Gamma_p x} - Be^{-\Gamma_p x}\right) = \frac{\Gamma_p}{\sigma}\left(Ae^{\Gamma_p x} - Be^{-\Gamma_p x}\right); Z_s = \frac{\Gamma_p}{\sigma};$

$$|Z_s| = \left|\frac{\Gamma_p}{\sigma_{Fe}}\right| = \frac{\sqrt{\omega\sigma_{Fe}\mu_{Fe}}}{\sigma_{Fe}} = \sqrt{\frac{\omega\mu_{Fe}}{\sigma_{Fe}}} = \sqrt{\frac{\omega\mu_0\mu_{rFe}}{\sigma_{Fe}}}$$

$$Ez_p(x) = Z_s\left(Ae^{\Gamma_p x} - Be^{-\Gamma_p x}\right), \tag{1.29}$$

where *Zs* is the impedance of the shield (*intrinsic impedance*), respective an impedance that characterizes the behavior of the shield against the radiation field) [1], [3].

The determination of the constants is done by introducing initial conditions for the tangential components of the electric and magnetic field:

$$H_t(x_0)^- = H_t(x_0)^+ ; \tag{1.30}$$

$$H_t(x_0 + d)^- = H_t(x_0 + d)^+ ; \tag{1.31}$$

$$E_t(x_0)^- = Et(x_0)^+ . \tag{1.32}$$

Note 1: In this case (of taking into account the extremity effect) there can not be introduced similar constraints for $Et(x_0 + d)^+$ - respectively for the outer electric field, as already.

In the above equations:

$H_t(x_0)^-$ - the field Hyi (x), for x=x0;

$H_t(x_0)^+$ - the field Hyp (x), for x=x0;

$H_t(x_0 + d)^-$ - the field Hyp(x), for x=x0+d;

$H_t(x_0 + d)^+$ - the field Hye(x), for x=x0+d;

$E_t(x_0)^-$ - the field Ezi (x), for x=x0;

$E_t(x_0)^+$ - the field Ezp (x), for x=x.

Taking these constraints into account, the following relations are obtained:

$$Hi_0 \cos(\Gamma_0 x_0) = Ae^{\Gamma_p x_0} + Be^{-\Gamma_p x_0} ; \tag{1.33}$$

$$Ae^{\Gamma_p(x_0+d)} + Be^{-\Gamma_p(x_0+d)} = He \cdot e^{-j\Gamma_0(x_0+d)} ; \tag{1.34}$$

$$jZ_0 Hi_0 \sin(\Gamma_0 x_0) = Z_s\left(Ae^{\Gamma_p x_0} - Be^{-\Gamma_p x_0}\right), \tag{1.35}$$

or, rearranging the terms:

$$Ae^{\Gamma_p x_0} + Be^{-\Gamma_p x_0} = Hi_0 \cdot \cos(\Gamma_0 x_0) ; \tag{1.36}$$

$$Ae^{\Gamma_p(x_0+d)} + Be^{-\Gamma_p(x_0+d)} = He \cdot e^{-j\Gamma_0(x_0+d)} ; \tag{1.37}$$

$$Ae^{\Gamma_p x_0} - Be^{-\Gamma_p x_0} = j\frac{Z_0}{Z_s} Hi_0 \cdot \sin(\Gamma_0 x_0) = K_0 \cdot Hi_0 \cdot \sin(\Gamma_0 x_0), \tag{1.38}$$

where they noticed:

$$K_0 = \frac{\mu_0 \Gamma_p}{\mu\Gamma_0} \; K_0 = j\frac{Z_0}{Z_s} . \tag{1.39}$$

There are three unknown quantities in relations (1.36), (1.37),) (1.38), namely *A, B, Hi₀* , where Hi=*ct* (the value of the magnetic field in the centre of the screen).

We specify that the equation (1.35) is obtained based on the conservation theorem of tangential components of the electric field at the surface between internal wall of the shield and the free space [1], [2]:

$$Ez_{pt} = Ez_{it} . \tag{1.40}$$

Note 2: The application of the theorem is possible due to the consideration that the parallelepiped shield is completely closed, so that there are no other electric charges on the inner side of the shield.

The solutions of the system consisting in equations (1.36), (1.37), (1.38) are:

$$A=\frac{H_e e^{-\Gamma_p x_0 - j\Gamma_0(x_0+d)}\left[\cos(\Gamma_0 x_0)+j\frac{Z_0}{Z_s}\sin(\Gamma_0 x_0)\right]}{\cos(\Gamma_0 x_0)\left(e^{\Gamma_p d}+e^{-\Gamma_p d}\right)+j\frac{Z_0}{Z_s}\sin(\Gamma_0 x_0)\left(e^{\Gamma_p d}-e^{-\Gamma_p d}\right)};$$

$$B=\frac{H_e e^{\Gamma_p x_0 - j\Gamma_0(x_0+d)}\left[\cos(\Gamma_0 x_0)-j\frac{Z_0}{Z_s}\sin(\Gamma_0 x_0)\right]}{\cos(\Gamma_0 x_0)\left(e^{\Gamma_p d}+e^{-\Gamma_p d}\right)+j\frac{Z_0}{Z_s}\sin(\Gamma_0 x_0)\left(e^{\Gamma_p d}-e^{-\Gamma_p d}\right)};$$

$$H_{i_0}=\frac{2H_e e^{-j\Gamma_0(x_0+d)}}{\cos(\Gamma_0 x_0)\left(e^{\Gamma_p d}+e^{-\Gamma_p d}\right)+j\frac{Z_0}{Z_s}\sin(\Gamma_0 x_0)\left(e^{\Gamma_p d}-e^{-\Gamma_p d}\right)}.$$

Calculus of the induced current density within the shield wall

According to the electrical conduction law – local form, the expression of the current density induced in the wall of the shield by the incident electromagnetic waves is:

$$\overline{Jz_p}(x)=\bar{k}Jz_p(x)=\sigma\overline{E}z_p=\bar{k}\sigma Ez_p(x), \quad (1.41)$$

where the electric field intensity within the shield wall is given by the equations (1.29).

We obtain:

$$\overline{Jz_p}(x)=\bar{k}\sigma\cdot\frac{\Gamma_p}{\sigma}\left(\mathrm{A}e^{\Gamma_p x}-Be^{-\Gamma_p x}\right)=\bar{k}\cdot\Gamma_p\left(\mathrm{A}e^{\Gamma_p x}-Be^{-\Gamma_p x}\right). \quad (1.42)$$

On the other hand the feedback magnetic field created within the shield wall by the induced current is:

$$\overline{Hy_{pf}}(x)=\bar{j}Hy_{pf}(x). \quad (1.43)$$

It results that the only the non-zero components of the $\nabla\times\overline{Hy_f}$ are directed along the Oz axis, perpendicular on Hy direction. According to the first equation of Maxwell, it can be written:

$$\nabla\times\overline{Hy_{pf}}(x)=\overline{Jz_p}(x)=\bar{k}Jz_p(x), \quad (1.44)$$

where by extension we obtain:

$$\nabla\times\overline{Hy_{pf}}(x)=-\bar{k}\frac{\partial Hy_{pf}(x)}{\partial x}. \quad (1.45)$$

From the equations (1.42), (1.43), (1.44) result:

$$-\frac{\partial Hy_{pf}(x)}{\partial x}=\Gamma_p\left(\mathrm{A}e^{\Gamma_p x}-Be^{-\Gamma_p x}\right), \quad (1.46)$$

or:

$$dHy_{pf}(x)=-\Gamma_p\left(\mathrm{A}e^{\Gamma_p x}-Be^{-\Gamma_p x}\right)dx. \quad (1.47)$$

By integrating the equation (1.47) we obtain the feedback magnetic field of the shield:

$$Hy_{pf}(x)=-(Ae^{\Gamma_p x}+Be^{-\Gamma_p x})+C, \quad (1.48)$$

Based on previous reasoning $C=0$ and the equation (1.48) becomes:

$$Hy_{pf}(x)=-(Ae^{\Gamma_p x}+Be^{-\Gamma_p x}). \quad (1.49)$$

Determination of the feedback electromagnetic power density by the shield

The components of the feedback magnetic field and electric field, Hy_{pf} and Ez_p, are determined at the exterior wall of the shield, for $x=x_0+d$, by relations (1.29), respectively (1.49):

$$\mathrm{Ez_{pe}}(\mathrm{x}_0+\mathrm{d})=\frac{\Gamma_p}{\sigma}\left(\mathrm{A}e^{\Gamma_p(x_0+d)}-Be^{-\Gamma_p(x_0+d)}\right)=Z_s\left(\mathrm{A}e^{\Gamma_p(x_0+d)}-Be^{-\Gamma_p(x_0+d)}\right); \quad (1.50)$$

$$Hy_{pf}(x)=-(Ae^{\Gamma_p(x_0+d)}+Be^{-\Gamma_p(x_0+d)}). \quad (1.51)$$

where Zs is the intrinsic impedance of the shield.

Thereby, the module of the feedback electromagnetic power density (Poynting's Vector) radiated by the shield will be:

$$Sx_{pe}=Ez_{pe}(x_0+d)\cdot Hy_{pfe}(x_0+d),$$

or:

$$Sx_{pe}=-Z_s\left(\mathrm{A}^2e^{2\Gamma_p(x_0+d)}-B^2e^{-2\Gamma_p(x_0+d)}\right). \ \text{(W/m}^2\text{)} \quad (1.52)$$

Replacing the unknown A, B, Hi_0 with their previously calculated values we finally obtain:

$$Sx_{pe}=-Z_sH_e^2\left\{e^{2\left[\Gamma_p d-j\Gamma_0(x_0+d)\right]}\left[\cos(\Gamma_0 x_0)+j\frac{Z_0}{Z_s}\sin(\Gamma_0 x_0)\right]^2-\right\} \quad (1.53)$$

$$\left\{-e^{-2\left[\Gamma_p d - j\Gamma_0 (x_0+d)\right]}\left[\cos\left(\Gamma_0 x_0\right) - j\frac{Z_0}{Z_s}\sin\left(\Gamma_0 x_0\right)\right]^2\right\}.$$

It is necessary to compare the obtained value of the feedback electromagnetic power density of the wall with the admitted limit value of exposure for the onboard personnel, according the standards [6], [7], to establish if are or not necessary adequate safety measures.

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