"Mircea cel Batran" Naval Academy Scientific Bulletin, Volume XVII – 2014 – Issue 1
Published by "Mircea cel Batran" NavalAcademy Press, Constanta, Romania

TWO MATHEMATICAL METHODS WITH APPLICATIONS IN NAVAL FRAMEWORK

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Abstract:*The paper aims is to present how two mathematical models, in particular the linear programming and the transport problem, can be used in solving concrete problems in naval framework.*
Key-words:*linear programming,mathematical model, transport problem.*

THE PROBLEM OF THE OPTIMAL MIXING OR NUTRITION

„Croaziere Costa" offers high quality services meant to satisfy even the most challenging client. An important aspect is the one which concerns gastronomy, domain in which the staff has to establish a proper diet and to respect the specialists' advice who say that feeding is considered a fine one if certain substances are offered mainly organic products in well-established minimum quantities, of course here it is necessary that the menu for the passengers to have an optimal amount of nutritious substances.

On the other hand, the travel agent has the purpose of minimizing the costs for the optimal level of the nutritious substances.

Thus the aim for this diet is to contain nutritious substances: proteins, carbohydrates, fats, and vitamins(calcium) in the maximum amounts of 80g, 250g, 80g and respectively 5g reflected in the products: meat, fish, dairy products, fruits and vegetables[2], with the corresponding price per unit 20, 15, 12 respectively 10(the prices are in RON/kg).

We marked with u_{ij} the number of units from substances S_i, i = $\overline{1,4}$, which are found in a unit from the alimentary product A_j , j = $\overline{1,4}$.

Aliment / Substance	A_1 meat	A_2 fish	A_3 dairy products	A_4 fruits/vegetables	Maximum of necessary nutrients
S_1 (proteins)	u_{11}= 3	u_{12}= 2	u_{13}= 0	u_{14}= 3	b_1 = 80
S_2 (carbohydrates)	u_{21}= 1	u_{22}= 2	u_{23}= 2	u_{24}= 0	b_2= 250
S_3 (fats)	u_{31}= 2	u_{32}= 5	u_{33}= 0	u_{34}= 3	b_3 = 80
S_4 (vitamins/ calcium)	u_{41}= 1	u_{42}= 3	u_{43}= 6	u_{44}= 0	b_4 = 5
Aliment prices	c_1= 20	c_2= 15	c_3= 12	c_4= 10	
Consumer units	x_1	x_2	x_3	x_4	

This problem will be dealt with and solved by means of linear programming[1], [5]. As in this case we look for both maximization and minimization of the costs, the duality in the problems of linear programming occur.

The theory of linear programming problems has proven that starting from a linear programming problem whose numerical data are used, we can formulate another linear programming problem which asks for determining the opposite optimal value, and there is a tight connection between the solutions of two linear programming problems.

It is known that the pair of linear programming problems thus built observes a general principle from science, particularly from mathematics, named the duality principle.

Usually the initial linear programming problem is named primal and the second one obtained by duality is named dual.

Considering these, the linear programming problem will be like this:

(max)f(x) = 20x_1 + 15x_2 + 12x_3 + 10x_4

$$\begin{cases} 3x_1 + 2x_2 + 3x_4 \le 80 \\ x_1 + 2x_2 + 2x_3 \le 250 \\ 2x_1 + 5x_2 + 3x_4 \le 80 \\ x_1 + 3x_2 + 6x_3 \le 5 \end{cases} \quad x_i \ge 0 \text{ , i = } \overline{1,4}$$

The dual problem could be transformed as below and we can write:

(min)g(y) = 80y_1 + 250y_2 + 80y_3 +5y_4

$$\begin{cases} 3y_1 + y_2 + 2y_3 + y_4 \ge 20 \\ 2y_1 + 2y_2 + 5y_3 + 3y_4 \ge 15 \\ 2y_2 + 6y_4 \ge 12 \\ 3y_1 + 3y_3 \ge 10 \end{cases} \quad | \text{ * (-1)} \rightarrow$$

$$\begin{cases} -3y_1 - y_2 - 2y_3 - y_4 \le -20 \\ -2y_1 - 2y_2 - 5y_3 - 3y_4 \le -15 \\ -2y_2 - 6y_4 \le -12 \\ -3y_1 - 3y_3 \le -10 \end{cases}$$

$y_i \ge 0$, i = $\overline{1,4}$

Now we can introduce the compensation variable y_5,y_6 , y_7 şi y_8 and the problem becomes:

(min)g(y) = 80y_1 + 250y_2 + 80y_3 +5y_4

$$\begin{cases} -3y_1 - y_2 - 2y_3 - y_4 + y_5 = -20 \\ -2y_1 - 2y_2 - 5y_3 - 3y_4 + y_6 = -15 \\ -2y_2 - 6y_4 + y_7 = -12 \\ -3y_1 - 3y_3 + y_8 = -10 \end{cases} \quad y_i \ge 0 \text{ , i = } \overline{1,8}$$

Applying the dual simplex algorithm[1], [5] we obtain:

		c	80	250	80	5	0	0	0	0
C_b	B	b	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
	p_5	-20	-3	-1	-2	-1	1	0	0	0
	p_6	-15	-2	-2	-5	-3	0	1	0	0
	p_7	-12	0	-2	0	-6	0	0	1	0
	p_8	-10	-3	0	-3	0	0	0	0	1
P_4 →	z_j	0	0	0	0	0	0	0	0	0
P_5 ←	Δ_j	/	80	250	80	5	0	0	0	0
5	p_4	20	3	1	2	1	-1	0	0	0
0	p_6	45	7	1	1	0	-3	1	0	0
0	p_7	108	18	4	12	0	-6	0	1	0
0	p_8	- 10	-3	0	-3	0	0	0	0	1
P_1 →	z_j	100	15	5	10	5	-5	0	0	0
P_8 ←	Δ_j	/	65	245	70	0	5	0	0	0

5	p_4	10	0	1	-1	1	-1	0	0	1
0	p_6	65/3	0	1	-6	0	-3	1	0	7/3
0	p_7	48	0	4	-6	0	-6	0	1	6
80	p_1	10/3	1	0	1	0	0	0	0	-1/3
	z_j	950/3	80	5	75	5	-5	0	0	-65/3
	Δ_j	/	0	245	5	0	5	0	0	65/3

Therefore the standard solution of the problem is:

$$x^{(s)} = \left(\frac{10}{3} , 0 , 0 , 10, 0 , \frac{65}{3} , 48 , 0 \right)$$

For the general problem the solution is: $x^{(g)} = (\frac{10}{3} , 0 , 0 , 10)$ and, (min)g(y) = $\frac{950}{3}$ (RON/person)

The solution for the primal problem is $x^{(p)} = (5, 0 , 0 , \frac{65}{3})$ and, *(max)f(x)* = $\frac{950}{3}$ (RON/person).

THE TRANSPORT PROBLEM RELATED TO MINIMIZING THE COSTS

During the voyage, the cruise ship has to fuel up in different ports along the established route. The ship needs 1100 tons of HFO(P1), 300 tons DIESEL(P2) and 10 tons OIL(P3). The ports they fuel up are from Algeria(F1), Italy(F2), Tunisia(F3) and Egypt(F4). The moment it arrives in the port from Alger, the tank has only 500 tons of HFO and the moment it arrives in the last established port it has only 300 tons of HFO.

The four ports have these products available in quantities b_1 = 500 tons, b_2 = 210 tons, b_3 = 400 tons and b_4 = 300 tons.

The prices of the three products differ from country to country[3] and could be represented like these:

PRODUCT SUPPLIER	HFO ($/tona)	DIESEL ($/tona)	OIL ($/tona)
F1	650	355	300
F2	640	345	310
F3	660	350	290
F4	645	455	295

Therefore, the considered problem is a transport problem with limited capacities[1], [5]. Then the mathematical model of this problem is:

$$\begin{cases} \sum_{j=1}^{n} x_{ij} = a_i , & i = \overline{1,m} \\ \sum_{i=1}^{n} x_{ij} = b_i , & j = \overline{1,n} \\ 0 \le x_{ij} \le d_{ij}, i = \overline{1,n} , j = \overline{1,m} \\ (\min) f = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \end{cases}$$

Or in a table form:

PRODUCT / SUPPLIER	HFO	DIESEL	OIL	AVAILABLE
F1 – Port 3	650 500	355	300	500
F2 – Port 4	640	345	310	210
F3 – Port 5	660	350	290	400
F4 – Port 6	645 300	455	295	300
NECESSARY	1100	300	10	1410 / 1410

From the limit capacities conditions it results that it is necessary that on every line respectively column the inequalities should be verified:

$$\sum_{j=1}^{n} d_{ij} \le a_i \,, i = \overline{1,m}$$

respectively,

$$\sum_{i=1}^{m} d_{ij} \le b_j \,, j = \overline{1,n}$$

When determining an initial solution, because of the limited capacities, there are solutions in which we cannot distribute in boxes all the available and necessary quantities.

To avoid such a situation, it is recomended the following algorithm to occupy the boxes:

a) on the line(column) of the minimum cost the boxes are occupied in the increasing order of prices;
b) it is the same procedure on the column(line) of the last occupied box from the line(column) from a);
c) it is continued in the same way, alternatively, until the available and the necessary are out of stock.

Also, the value of the variable from the box that is occupied it is now chosen as the minimum between the available, necessary and the capacity corresponding to the respective variable. When this minimum equals the respective capacity, the value will be underlined which means that on the line, respectively the column of the box the rest of the variables will not be taken automatically zero. This means that, in general will be occupied more than m+n-1 boxes. A number of m+n-1 variables will be considered not underlined.

PRODUCT / SUPPLIER	HFO		DIESEL		OIL		AVAILABLE
F1 – Port 3	650 500	<u>500</u>	355	0	300	0	500;0
F2 – Port 4	640	0	345	210	310	0	210;0
F3 – Port 5	660	300	350	90	290	10	400;390;90;0
F4 – Port 6	645 300	<u>300</u>	455	0	295	0	300;0
NECESSARY	1100;800;300;0		300;90;0		10;0		1410 / 1410

Now the found solution is degenerated. In our case we obtained four not null solutions(four boxes occupied) and we need 4+3-1 not null so that the solution not be degenerated.

Having fewer than six occupied boxes it results that the dual variables cannot be determined.

To remove this inconvenience it is used the method of essential zeros. This consists of transforming some free boxes into occupied ones marked 0*, named essential zero. This 0* is placed in the free boxes with minimum costs, which cannot form cycles with boxes already occupied. In the end it is necessary that the total number of occupied boxes to be 4+3-1.

PRODUCT / SUPPLIER	HFO		DIESEL		OIL		AVAILABLE
F1 – Port 3	650		355		300		500;0
	500	500		0		0	
F2 – Port 4	640		345		310		210;0
		0		210		0	
F3 – Port 5	660		350		290		400;390;90;0
		300		90		10	
F4 – Port 6	645		455		295		300;0
	300	300		0		0*	
NECESSARY	1100;800;300;0		300;90;0		10;0		1410 / 1410

Going to the dual problem, there can be found the following optimal conditions for the transport problems with limited capacities:

1) $u_i + v_j \le c_{ij}$, for free boxes (i,j);
2) $u_i + v_j \ge c_{ij}$, for (i,j) boxes with underline variable;

where u_i , i = $\overline{1,m}$ and v_j , j = $\overline{1,n}$ are dual variable u_i and v_j given by system

$$u_i + v_j = c_{ij}$$

and (i,j) represents occupied box index(corresponding to the basic variable).

To find a solution we can take a secondary solution equal with zero and usually we choose $u_1 = 0$.

The values found for $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ are marked on the border of the solution table in a corresponding way (that is why they are also called border values), and the sums $u_i + v_j$ are written in the upper tight corner of the occupied boxes next to the coefficients c_{ij}.

PRODUCT / SUPPLIER	HFO V1=650		DIESEL V2=340		OIL V3=280		AVAILABLE
F1 – Port 3 U1=0	650	650	355	340	300	280	500;0
	500	500		0		0	
F2 – Port 4 U2=5	640	655	345	345	310	285	210;0
	+	0	-	210		0	
F3 – Port 5 U3=10	660	660	350	350	290	290	400;390;90;0
	-	300	+	90		10	
F4 – Port 6 U4=-5	645	645	455	335	295	275	300;0
	300	300		0		0*	
NECESSARY	1100;800;300;0		300;90;0		10;0		1410 / 1410

We move on to verifying the optimal condition for the free boxes. If all the conditions are fulfilled then the initial solution is optimal. If at least one condition is not verified, then we continue with the improving process of the solution.

It can be noticed that in (2,1) box, the optimal condition is not fulfilled because
$u_2 + v_1 = 655 > c_{21} = 640$.

In a cycle we alternatively mark the boxes with + and – starting with the free one. The signs are written in the botton left corner of the box.

To improve the solutions we need to find the minimal, θ, value which x_{ij} take from the boxes marked by – (minus). Then the minimal value θ is added to x_{ij} values, from the boxes marked by + (plus) and is substracted from the values from the boxes marked by – (minus).

Through this process we obtain a new solution for the transport problem, better than the one we started from.

The cycle corresponding to (2,1) box is (2,1),(2,2),(3,2) and (3,1). We mark with + and – . The minimum, θ, value θ = min{210,300}= 210 allows us to write the improved solution given in the table below:

PRODUCT / SUPPLIER	HFO V1=650		DIESEL V2=340		OIL V3=280		AVAILABLE
F1 – Port 3 U1=0	650	650	355	340	300	280	500;0
	500	500		0		0	
F2 – Port 4 U2=-10	640	640	345	330	310	270	210;0
	+	210	-	0		0	
F3 – Port 5 U3=10	660	660	350	350	290	290	400;390;90;0
	-	90	+	300		10	
F4 – Port 6 U4=-5	645	645	455	335	295	275	300;0
	300	300		0		0*	
NECESSARY	1100;800;300;0		300;90;0		10;0		1410 / 1410

We have obtained four not null variables. As there cannot be formed cycles to improve the solution and all the optimal conditions are accomplished it results that the problem has only an optimal solution, that is:

$$X = \begin{pmatrix} 500 & 0 & 0 \\ 210 & 0 & 0 \\ 90 & 300 & 10 \\ 300 & 0 & 0 \end{pmatrix}$$

And the value of the objective function is:

$$\min(f) = 650 \cdot 500 + 640 \cdot 210 + 660 \cdot 90 + 645 \cdot 300 + 350 \cdot 300 + 290 \cdot 10 = 852.700\$$$

Consequently the ship will fuel up with diesel and oil from the port in Tunisia and HFO will be obtained from all the four ports in quantities 500, 210, 90 and 300 tons.

CONCLUSION

We conclude that mathematical models are very efficient, practice on board, managed to confirm this. If these models addresses issues aimed at the smooth running of the voyage, as seen in the present work(providing a proper diet and supply of fuel needed at a convenient price) and if these issues are addressed by interdisciplinary teams that give optimal solution then the success is assured.

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