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SOME MATHEMATICAL METHODS APPLIED IN NAVAL FRAMEWORK: A CASE STUDY ON CRUISE SHIPS

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Abstract: *This paper aims to highlight how thecanonical-mathematical models provide solutions for the management of different problems occurring in naval framework.*
Keywords: *graph,mathematical model, Hamiltonian path.*

INTRODUCTION

The complexity in the current stage of the management processes in the naval technical or economic domain, the scientifical planning requirements of the activities impose at every level of an institution, company or firm, renouncing to the empirical methods which could lead to ensuitably correlated results with the material and human effort invested in the use of mathematical modern methods.

The use of the mathematical methods in the economic and technical naval practice at every level represents a preoccupation with positive effects in solving the current problems related to the domain.

Using mathematics in solving the matters in the naval branch actually consists of properly accessing the mathematical models which could be said is not very simple but can to help to solved the considered issue.

In these conditions it is necessary that at every level of decision, a large number of information and data should be processed in such a path as to allow a logical thinking for choosing the most suitable alternative.

Thus the mathematical programming methods have appeared to help solve problems of some institutions' optimization activities whose interest domain is transporting.

Designing of a mathematical model has to observe the following stages[1]:

1. Obtaining the descriptive model of the process which requires the following substages:
   - Defining the problem associated to the process;
   - Analysing all information related to the respective process;
   - Anlysing the criteria which describe the wanted objectives;
   - Starting the kind of factors(main or secondary) which the process depends on.
2. Defining and mathematical designing of the descripting model.
3. Studying the model, a stage which deals with the practical solving of the problem according to the model, especially using the computer.

The aim of the current paper is to offer an easy access to some mathematical results frequently used in describing the phenomena from naval transport branch.

We wish that this paper emphasizes the most known economico-mathematical models which could provide solutions for the management problems occuring in the naval domain.

Among these we considered:
- the transport problem regarding optimal length paths;
- the problem of optimal flow in a transport network(a mathematical tool useful for managers and specialists to help them to plan their activities).

THE TRANSPORT PROBLEM REGARDING PATHS IN A GRAPH

„Croaziere Costa" provides a cruise on Mediterranean Sea between Europe and Africa departing from Seville, Spain and arriving in Alexandria, Egypt.
Before the departure, the ship owner has to set the maritime route according to the itinerary. The target is to reach all the ports in the itinerary but with the lowest possible cost. For this reason it should be found the path with the least nautical miles.
Using maps provided byseveralwebsites [3], [4], we can approach the issue as a problem of graphs.

Figure 2.1. graph associated to the itinerary

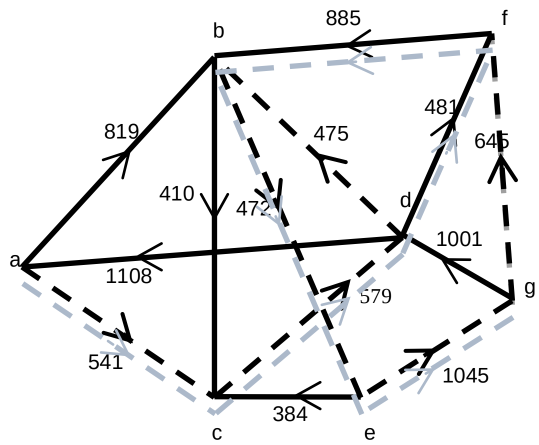


Figure 2.2. Graph attached itinerary

Legend:

a – Spain , Seville e – Tunisia , Tunis
b – France , Marseillef – Greece , Patras
c – Algeria , Algiers g – Egypt , Alexandria
d – Italy , Napoli

In the following it will be presented a method through which all the elementary routes can be found, known as Hamiltonian paths in mathematical terms. This procedure is named „Latine matrix algorithm"[1].

Let (X, Γ) the graph corresponding to the problem.It will be used latin matrix L = (l_{ij}), where

$$l_{ij} = \begin{cases} x_i x_j,\ if\left(x_i\,,x_j\right) \in\ \Gamma \\ 0\,,\ if\ \ (x_i\,,x_j)\ \notin\ \Gamma; \end{cases} \quad i,j = 1,2,\ldots,n$$

and in the following, we obtain, for our problem, the corresponding matrix:

L	**a**	**b**	**c**	**d**	**e**	**f**	**g**
a	**0**	**ab**	**ac**	**0**	**0**	**0**	**0**
b	**0**	**0**	**bc**	**0**	**be**	**0**	**0**
c	**0**	**0**	**0**	**cd**	**0**	**0**	**0**
d	**da**	**db**	**0**	**0**	**0**	**df**	**0**
e	**0**	**0**	**ec**	**0**	**0**	**0**	**eg**
f	**0**	**fb**	**0**	**0**	**0**	**0**	**0**
g	**0**	**0**	**0**	**gd**	**0**	**gf**	**0**

The matrix $\tilde{L}$, is built from L, by removing x_i from the sequence $x_i x_j$, when this exists.

$\tilde{L}$	a	b	c	d	e	f	g
a	0	b	c	0	0	0	0
b	0	0	c	0	e	0	0
c	0	0	0	d	0	0	0
d	a	b	0	0	0	f	0
e	0	0	c	0	0	0	g
f	0	b	0	0	0	0	0
g	0	0	0	d	0	f	0

Now in the next it is used a special matrix multiplication named Latin multiplication[1] and written as,"*" , as follows:

a) the multiplying is done rows by columns;

b) instead of the usual multiplicationthe join of elements is used, if these don't repeat or zerois written in the opposite case;

c) instead of the usual addition one considers the groups obtained in b), when we have such groups.

By short we will write L * $\tilde{L}$ = L^2 . Similarly we calculate L^2 * $\tilde{L}$ = L^3, . . . , L^{k-1} * $\tilde{L}$ = L^k. It is noticed that L^k containes all the elementary paths of k length.Consequently, in L^{n-1} matrix all Hamiltonian paths appear. In this case n=7, so we obtain:

L^6	a	b	c	d	e	f	g
a	**0**	**0**	**0**	**0**	**0**	**acdbegf**	**acdfbeg**
b	**0**	**0**	**0**	**0**	**0**	**0**	**0**
c	**0**	**0**	**0**	**0**	**0**	**cdabegf**	**0**
d	**0**	**0**	**0**	**0**	**0**	**0**	**0**
e	**egfbcda**	**0**	**0**	**0**	**0**	**0**	**0**
f	**0**	**0**	**fbegdac**	**0**	**0**	**0**	**0**
g	**gfbecda**	**0**	**0**	**0**	**0**	**0**	**0**

In conclussion the given graph has 6 Hamiltonian paths: d_{1H} = {*a, c, d, b, e, g, f* }, d_{2H} = {*a, c, d, f, b, e, g* }, d_{3H}={*c, d, a, b, e, g, f* }, d_{4H}={*e, g, f, b, c, d, a* }, d_{5H} ={*f, b, e, g, d, a, c* } andd_{6H} ={*g, f, b, e, c, d, a* }.

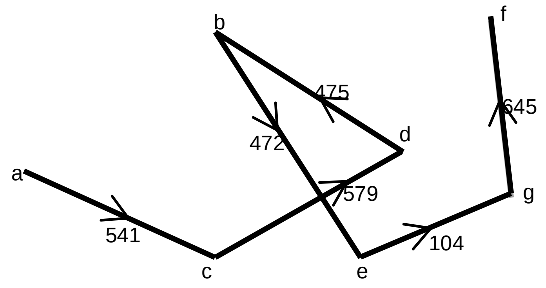
The graph does not have circuits.

Between the six possible paths, we look for the shortest path from the point of view of nautical miles and for this an algorithm will be used to determine the optimal length paths, precisely The Elementary Bellman Algorithm[1]. It is relied on optimization principle: any optimal politic is made of optimal subpolitics.

Through this algorithm to every nodex_i the d_i number is atachedwhich represents the minimum length of the paths from x_1 tox_i measured as the distance between ports.

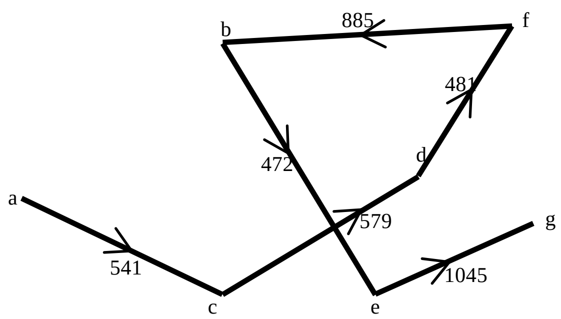
It is considered d_1= 0. Now it is supposed that we want to find d_l ,where the x_l node is the follower of the nodes x_i, x_j andx_k to which numbers d_i , d_j si d_k have already been calculated. Then the minimal d_l length from x_1 tox_l is determined through the formula d_l = min(d_i+ c_{il}, d_j+ c_{jl} , d_k+ c_{kl}) where c_{il} , c_{jl} andc_{kl} are the corresponding capacities of the arcs (x_i,x_l), (x_j,x_l) and respectively (x_k,x_l).

For the Hamiltonian pathd_{1H}= {*a, c, d, b, e, g, f* } there is the graph:



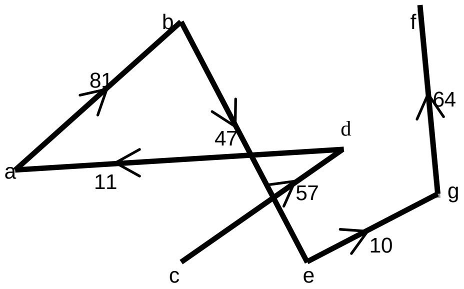
d_a = 0
d_c = min $\{d_a + 541\}$ = 541
d_d = min $\{d_c + 579\}$ = 1120
d_b = min $\{d_d + 457\}$ = 1577,sod_{1H}= 3739 nautical miles
d_e = min $\{d_b + 472\}$ = 2049
d_g = min $\{d_e + 1045\}$ = 3094
d_f = min $\{d_g + 645\}$ = 3739

For the Hamiltonian pathd_{2H}= {*a, c, d, f, b, e, g*} there is the graph:



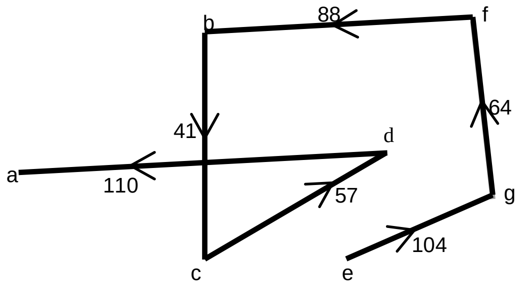
d_a = 0
d_c = min $\{d_a + 541\}$ = 541
d_d = min $\{d_c + 579\}$ = 1120
d_f = min $\{d_d + 481\}$ = 1601, so d_{2H}= 4003 nautical miles.
d_b = min $\{d_f + 885\}$ = 2486
d_e = min $\{d_b + 472\}$ = 2958
d_g = min $\{d_e + 1045\}$ = 4003

For the Hamiltonian pathd_{3H}= {*c, d, a, b, e, f, g*} there is the graph:



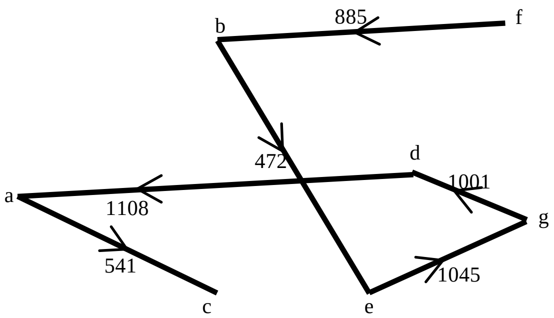
d_c = 0
d_d = min $\{d_c + 579\}$ = 579
d_a = min $\{d_d + 1108\}$ = 1687
d_b = min $\{d_a + 819\}$ = 2506,sod_{3H}= 4668 nautical miles.
d_e = min $\{d_b + 472\}$ = 2978
d_g = min $\{d_e + 1045\}$ = 4023
d_f = min $\{d_g + 645\}$ = 4668

For the Hamiltonian path d_{4H}={*e, g, f, b, c, d, a*} there is the graph:



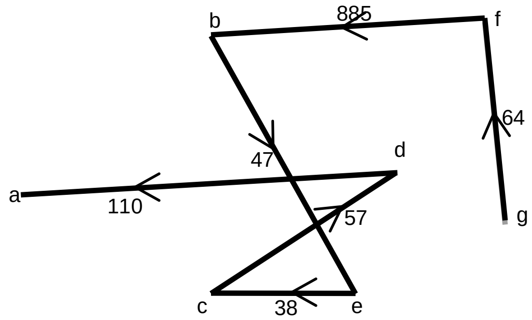
d_e = 0
d_g = min $\{d_e + 1045\}$ = 1045
d_f = min $\{d_g + 645\}$ = 1690
d_b = min $\{d_f + 885\}$ = 2572,sod_{4H} = 4672 nautical miles.
d_c = min $\{d_b + 410\}$ = 2985
d_d = min $\{d_c + 579\}$ = 3564
d_a = min $\{d_d + 1108\}$ = 4672

For the Hamiltonian pathd_{5H}={*f, b, e, g, d, a, c*} there is the graph:



d_f = 0
d_b = min $\{d_f + 885\}$ = 885
d_e = min $\{d_b + 472\}$ = 1357
d_g = min $\{d_e + 1045\}$ = 2402
d_d = min $\{d_g + 1001\}$ = 3403
d_a = min $\{d_d + 1108\}$ = 4511
d_c = min $\{d_a + 541\}$ = 5052,
sod_{5H}= 5052 nautical miles.

For the Hamiltonian pathd_{6H}={*g, f, b, e, c, d, a* } there is the graph:



d_g = 0
d_f = min $\{d_g + 645\}$ = 645
d_b = min $\{d_f + 885\}$ = 1530
d_e = min $\{d_b + 472\}$ = 2002, so d_{6H}= 4073 nautical miles.
d_c = min $\{d_e + 384\}$ = 2386
d_d = min $\{d_c + 579\}$ = 2965
d_a = min $\{d_d + 1108\}$ = 4073

Thus, the minimum lenght is 4003 nautical miles, and the path which has this length is d_{2H}= dmin= { *a, c, d, f, b, e, g* }. În Figure 2.2, the arcs with the minimum path are drawn with dotted lines.

We wish to mention that the actual distances were taken from the following sources [5], [6], [7].

THE PROBLEM OF DETERMINING THE OPTIMAL FLOW IN A TRANSPORT NETWORK

The provider of the cruise still needs personal to complete the crew, the departments with fewer people being the following: waiter service, bartending, room services and housekeeping. For this they turn to crewing agencies.

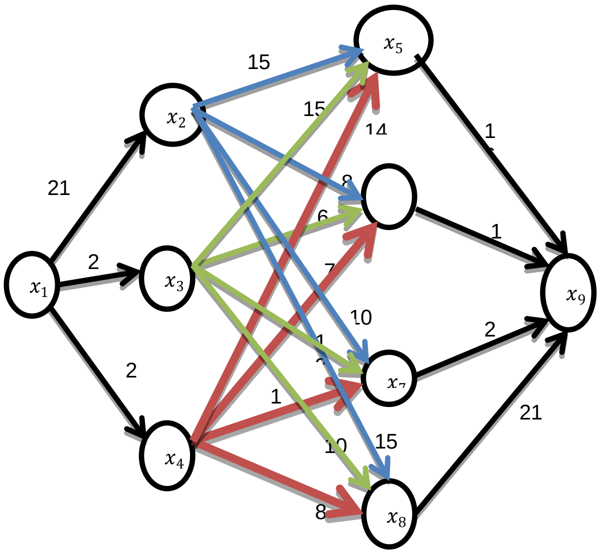
Three crewing agencies A_1,A_2 ,A_3 have 21, 20, respectively 22 people specialized in these domains, some of which heaving 2 or 3 specialization. The employer needs 19 waiters (P_1) , 18 bartenders (P_2) , 20 people for room service (P_3) and 21 housekeepers (P_4).

The possibilities of fulfilling this demand are limited by the availability of each person from the three crewing agencies as it can be seen from the following table:

A_i/A_j	P_1	P_2	P_3	P_4
A_1	15	8	10	15
A_2	15	6	12	10
A_3	14	7	16	8

An optimal plan is required to be determined in such a path as to completely ensured the demand for $P_1$and P_3, and the demand for $P_2$and $P_4$to the greatest extent.

The problem will be transformed into a transport network graph, considering x_1 the starting node x_2, x_3, x_4 the corresponding nodes for the crewing agencies A_1,A_2,A_3, the nodes x_5, x_6, x_7 and $x_8$the correspondingnodes for the four specializations and the ending nodex_9. We can have the following graph for our problem:



Solving the problem means to determine a maximum flow in the transport network from this graph.The Ford–Fulkerson algorithm will be used.

First, we have to determine the flow φ for the network. By attempts, the flow(an aplication φ : Γ → ℝ₊) could be built, observing the following conditions:

1) φ(a) ≤ c(a), for any arc a ∈ Γ ;

2) in any nodex_i ∈ Γ the equality $\sum_{a\in\Gamma^+_{x_i}}$ φ(a)=$\sum_{a\in\Gamma^-_{x_i}}$ φ(a),named conservation feature is satisfied.

Γ	(x_1,x_2)	(x_1,x_3)	(x_1,x_4)	(x_2,x_5)	(x_2,x_6)	(x_2,x_7)
φ	8	7	7	2	2	2

(x_2,x_8)	(x_3,x_5)	(x_3,x_6)	(x_3,x_7)	(x_3,x_8)	(x_4,x_5)	(x_4,x_6)
2	2	1	2	2	2	2

(x_4,x_7)	(x_4,x_8)	(x_5,x_9)	(x_6,x_9)	(x_7,x_9)	(x_8,x_9)
1	2	6	5	5	6

with the value equal to 22.
It is noticed that φ is incomplete because the paths:

$d_1 = (x_1, x_2, x_5, x_9)$
$d_2 = (x_1, x_2, x_6, x_9)$
$d_3 = (x_1, x_2, x_7, x_9)$
$d_4 = (x_1, x_2, x_8, x_9)$
$d_5 = (x_1, x_3, x_5, x_9)$
$d_6 = (x_1, x_3, x_6, x_9)$
$d_7 = (x_1, x_3, x_7, x_9)$
$d_8 = (x_1, x_3, x_8, x_9)$
$d_9 = (x_1, x_4, x_5, x_9)$
$d_{10} = (x_1, x_4, x_6, x_9)$
$d_{11} = (x_1, x_4, x_7, x_9)$
$d_{12} = (x_1, x_4, x_8, x_9)$

are unsaturated path(a path in the network is said to be saturated if it contains at least a saturated arc and for this, the condition *φ*(*a*) = *c*(*a*) must be fulfilled).

Any flow can turn into a complete one. For this purpose on any unsaturated path, d, from x_1 to x_n the corresponding arc flows are increased by

k = $min_{a\in d}$ (c(a) - φ(a)).

On each of these paths the flow can increased by:

k_1=min(21-8, 15-2, 19-6)=13
k_2=min(21-8, 8-2, 18-5)=6
k_3=min(21-8, 10-2, 20-5)=8
k_4=min(21-8,15-2,21-6)=13
k_5=min(20-7,15-2,19-6)=13
k_6=min(20-7, 6-1, 18-5)=5
k_7=min(20-7,12-2, 20-5)=10
k_8=min(20-7, 10-2, 21-6)=8
k_9=min(22-7,14-2, 19-6)=12
k_{10}=min(22-7, 7-2, 18-5)=5
k_{11}=min(22-7,16-1,20-5)=15
k_{12}=min(22-7, 8-2, 21-6)=6

We obtain the new flow φ_1:

Γ	(x_1,x_2)	(x_1,x_3)	(x_1,x_4)	(x_2,x_5)	(x_2,x_6)	(x_2,x_7)
φ_1	21	20	22	15	2	2

(x_2,x_8)	(x_3,x_5)	(x_3,x_6)	(x_3,x_7)	(x_3,x_8)	(x_4,x_5)	(x_4,x_6)
2	2	6	2	10	2	2

(x_4,x_7)	(x_4,x_8)	(x_5,x_9)	(x_6,x_9)	(x_7,x_9)	(x_8,x_9)
16	2	19	10	20	14

with the value given by 21 + 20 + 22 = 19 + 10 + 20 + 14 = 63.

Because the arc (x_1, x_2) was saturated by increasing it by k_1, the pathsd_2, $d_3$and d_4 have become saturated. Thus the flows on d_2, d_3 and d_4 couldn't be increased .

By increasing by $k_5$the arc (x_5, x_9) cannot be increased because it is already saturated, thus we will increase by k_6 and k_8.

The same happened by increasing by k_{11} the arc (x_1, x_4) has saturated, the paths d_9, d_{10} and d_{12}have become saturated, consequently the flows on d_9, d_{10} and d_{12} couldn't be increased.

It is obvious that the flow φ_1 is complete because on the each of the paths d_1 to d_{12} there is at least one saturated arc.

Now we can go on to the improvement of the flow by using Ford–Fulkerson algorithm. Thus x_1 will be marked with +. As the arcs (x_1, x_2) , (x_1, x_3), (x_2, x_4) are saturated it results that the marking cannot be continnued. Consequently, the nodex_9 cannot be marked. It is implied that the value of the maximum flow is 63.

To justify more easily the Ford–Fulkerson algorithm we introduce the cut notion in a graph.
A cut in a graph G = (X , Γ) is a partition of the set X of nodes in two subsets Y and C(Y), such that x_1∈ Y and x_n∈ C(Y). The value of the cut marked by v(Y / X) is by definition the sum of the arcs capacities with the initial node in Y and the final one in C(Y).

So the cut of minimum capacity is Y = {x_1} with v(Y/X) = 21 + 22 + 20 = 63

The optimal program given by the flow φ_1 can be represented by the table:

Dj \ Cj		x_5 P_1	x_6 P_2	x_7 P_3	x_8 P_4	Number of people available	Number of people employed
x_2	A_1	15	2	2	2	21	21
x_3	A_2	2	6	2	10	20	20
x_4	A_3	2	2	16	2	22	22
Employer requests		19	18	20	21		
Requests satisfied		19	10	20	14		

The values (x_i, x_j) from the table have been read from the optimal flow φ_1 and represents the number of the people from the crewing agencies x_i and the repartition onx_j departments .

To make clear the developing of the problem we present the intermediate situations until we reach the optimal flow φ_1 and we mark by φ_2, φ_3, φ_4, φ_5 the intermediate flows and, φ_6= φ_1.

Γ	(x_1,x_2)	(x_1,x_3)	(x_1,x_4)	(x_2,x_5)	(x_2,x_6)	(x_2,x_7)
φ_2	8	7	7	2	2	2

(x_2,x_8)	(x_3,x_5)	(x_3,x_6)	(x_3,x_7)	(x_3,x_8)	(x_4,x_5)	(x_4,x_6)
2	2	1	2	2	2	2

(x_4,x_7)	(x_4,x_8)	(x_5,x_9)	(x_6,x_9)	(x_7,x_9)	(x_8,x_9)
1	2	6	5	5	6

Γ	(x_1,x_2)	(x_1,x_3)	(x_1,x_4)	(x_2,x_5)	(x_2,x_6)	(x_2,x_7)

φ_3	21	7	7	15	2	2

(x_2,x_8)	(x_3,x_5)	(x_3,x_6)	(x_3,x_7)	(x_3,x_8)	(x_4,x_5)	(x_4,x_6)
2	2	1	2	2	2	2

(x_4,x_7)	(x_4,x_8)	(x_5,x_9)	(x_6,x_9)	(x_7,x_9)	(x_8,x_9)
1	2	19	5	5	6

Γ	(x_1,x_2)	(x_1,x_3)	(x_1,x_4)	(x_2,x_5)	(x_2,x_6)	(x_2,x_7)
φ_4	21	12	7	15	2	2

(x_2,x_8)	(x_3,x_5)	(x_3,x_6)	(x_3,x_7)	(x_3,x_8)	(x_4,x_5)	(x_4,x_6)
2	2	6	2	2	2	2

(x_4,x_7)	(x_4,x_8)	(x_5,x_9)	(x_6,x_9)	(x_7,x_9)	(x_8,x_9)
1	2	19	10	5	6

Γ	(x_1,x_2)	(x_1,x_3)	(x_1,x_4)	(x_2,x_5)	(x_2,x_6)	(x_2,x_7)
φ_5	21	20	7	15	2	2

(x_2,x_8)	(x_3,x_5)	(x_3,x_6)	(x_3,x_7)	(x_3,x_8)	(x_4,x_5)	(x_4,x_6)
2	2	6	2	10	2	2

(x_4,x_7)	(x_4,x_8)	(x_5,x_9)	(x_6,x_9)	(x_7,x_9)	(x_8,x_9)
1	2	19	10	5	6

Γ	(x_1,x_2)	(x_1,x_3)	(x_1,x_4)	(x_2,x_5)	(x_2,x_6)	(x_2,x_7)
φ_6	21	20	22	15	2	2

(x_2,x_8)	(x_3,x_5)	(x_3,x_6)	(x_3,x_7)	(x_3,x_8)	(x_4,x_5)	(x_4,x_6)
2	2	6	2	10	2	2

(x_4,x_7)	(x_4,x_8)	(x_5,x_9)	(x_6,x_9)	(x_7,x_9)	(x_8,x_9)
16	2	19	10	20	14

CONCLUSIONS

In the present study specific issue were dealt with a cruise ship, but mathematical method can easily be used in solving optimization problems of busines establishements, undertakings, trade companies or agricultural concerns.

By means of the theory reduced to the minimum necessary and appropiate examples, richly illustrated, this paper aims to highlight some of the real problems faced by owners when planning a trip and bring mathematical solution, easily applicable.

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