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ANALITYCAL METHOD FOR CALCULATING STIRLING ENGINES REGENERATOR

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Abstract: *The performances of the Sterling engine are affected by the convection coefficient and the "X" factor and not only by the variation of the gas quantity from the cylinder with the medium pressure variation. The convection factor indicates that a sensibility study concerning the characteristic parameters is mandatory.*
Keywords: *Sterling, cycle, engine, convection*

1. INTRODUCTION

We will consider the most common case, which presents the regenerator as a "pressed bolters package". In figure 1 are presented the geometrical characteristics for the bolter:

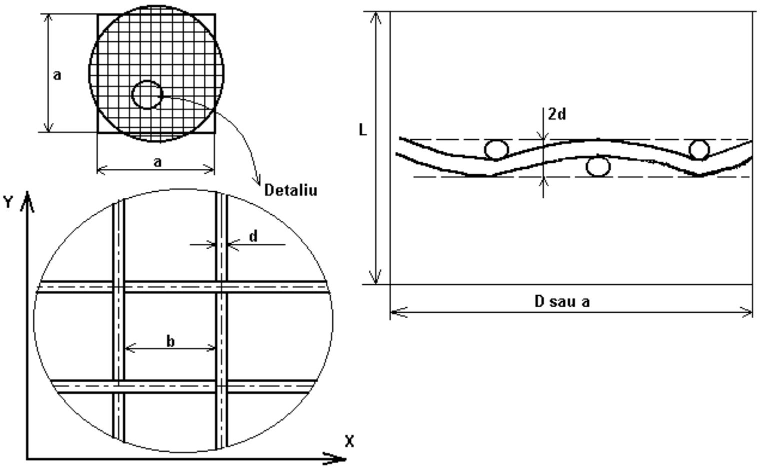

Figure 1. Geometrical characteristics for regenerator's bolters

According to the figure 1, we can assimilate regenerator's area with a square area equivalent to the regenerator's:

$$A_R = \frac{\pi D_R^2}{4} = a^2 \tag{1}$$

where D_R – regenerator's diameter;

$$a = D_R\sqrt{\frac{\pi}{4}} = \frac{D_R \cdot \sqrt{\pi}}{2} \tag{2}$$

If L is regenerator's length, then:

$$N_s = \frac{L}{2d} \tag{3}$$

where:

- N_s – total bolter number;
- d – bolter's wire diameter

From here results:

$$\left(\text{wires in the equivalent square in X direction}\right) = \frac{a}{b+d} \tag{4}$$

and:

$$\left(\text{bolter`s wires in both directions}\right) = \frac{2a}{b+d} \tag{5}$$

where b is the distance between two bolter's wires

$$\left(\text{bolter`s wires lenght}\right) = \frac{2a^2}{b+d} \tag{6}$$

$$\left(\text{regenerator`s wires lenght}\right) = \left(\text{bolters number}\right) \cdot \left(\text{bolter's wires lenght}\right) = L_f \tag{7}$$

$$L_f = \frac{2a^2}{b+d} \cdot \frac{L}{2d} = \frac{L \cdot a^2}{d\left(b+d\right)} \tag{8}$$

With these relations we can determine the radius A_R and the weight m_R. Knowing a form of second relation and using it in the 8th relation, results:

$$L_f = \frac{L \cdot \frac{\pi D_R^2}{4}}{d\left(b+d\right)} = \frac{\pi D_R^2 \cdot L}{4d\left(b+d\right)} \tag{9}$$

The regenerator’s area will be:

$$A_R = \pi \cdot d \cdot L_f = \frac{\pi^2 \cdot D_R^2 \cdot L \cdot d}{4\left(b+d\right) \cdot d} \tag{10}$$

meaning:

$$A_R = \frac{\pi^2 \cdot D_R^2 \cdot L}{4\left(b+d\right)} \tag{11}$$

The regenerator’s weight will be:

$$m_R = L_f \cdot A_f \cdot \rho_{metal} = \frac{\pi D_R^2 L}{4d\left(b+d\right)} \cdot \frac{\pi d^2}{4} \rho_{metal} \tag{12}$$

where:

- A_f – wire’s area;
- $\rho_{metal} = \rho_m$ – metal’s density

$$m_R = \frac{\pi^2 \cdot D_R^2 \cdot d \cdot \rho_m \cdot L}{16\left(b+d\right)} \tag{13}$$

The ratio $\frac{m_R}{A_R}$ will be:

$$\frac{m_R}{A_R} = \frac{\pi^2 \cdot D_R^2 \cdot d \cdot \rho_m \cdot L}{16\left(b+d\right)} \cdot \frac{4\left(b+d\right)}{\pi^2 \cdot D_R^2 \cdot L} = \frac{d \cdot \rho_m}{4} \tag{14}$$

The relation required to determine the “X” parameter, according with the regenerator’s properties will be:

$$X = 1 - \frac{\dfrac{\pi^2 \cdot D_R^2 \cdot d \cdot \rho_m \cdot L \cdot c_R}{16\left(b+d\right) m_g \cdot c_{v_g}}}{\dfrac{n_r \cdot d \cdot \rho_m \cdot c_R}{15\alpha} - 1} \tag{15}$$

In this relation, number 15 is used because the sinusoidal movement is resulted from a quarter rotation, meaning $\frac{60}{4} = 15$.

For accomplishing the operational status for the 15th relation the following two conditions are required:

- determination of the regenerator’s convection coefficient, α;
- bordering the X factor in general optimization scheme for Stirling engine

2. THE REGENERATOR’S CONVECTION COEFFICIENT EVALUATION

The following evaluation is based upon the description of the relation between the similitude criteria for the regenerator indicated by Organ [Thermodynamics and Gas dynamics of Stirling Cycle Machine, pag 113]:

$$N_{ST} \cdot N_{\Pr}^{\frac{2}{3}} = \frac{1{,}25}{\sqrt{N_{\text{Re}}}} \tag{16}$$

According to the Romanian scientific standards the relation will be written as:

$$St \cdot \Pr^{\frac{2}{3}} = \frac{1{,}25}{\sqrt{\mathrm{Re}_D}} \qquad (17)$$

relation used for a pore – ratio with following values:

$$\varepsilon_p = 0{,}602 \div 0{,}832 \qquad (18)$$

where ε_p is the porosity

Incopera indicates a similar relation [1, page 292]:

$$St \cdot \Pr^{\frac{2}{3}} = \frac{0{,}79}{\varepsilon_p \cdot \mathrm{Re}^{0{,}576}} \qquad (19)$$

where:
Re – Reynolds criterion;
St – Stanton criterion;
Pr – Prandtl number

The two formulas would be almost identical if the porosity formula, used in the second relation would be $\varepsilon_p = \frac{0{,}79}{1{,}25} = 0{,}63$, included in the first formula domain.

We will consider the Incopera formula first, continuing with the Organ formula, because a comparative study would be valuable in order to assign the possible implications for Stirling engines:

$$\mathrm{Re}_D = \frac{D \cdot \rho \cdot \overline{w}}{\mu} = \frac{D_R \cdot \overline{w}}{\nu} \qquad (20)$$

where:
- $\overline{W}$ is the medium speed.
- μ dynamic viscosity
- υ kinematic viscosity

The coefficient Re_D is determined for medium speed $\overline{W}$ and D_R, the diameter for regenerator’s empty shell.

$$St = \frac{\alpha}{\rho \cdot \overline{w} \cdot c_p} \qquad (21)$$

and

$$\Pr = \frac{\nu}{a} = \frac{\nu}{\lambda / \left(\rho \cdot c_p\right)} = \frac{\rho \cdot c_p \cdot \nu}{\lambda} \qquad (22)$$

α – convection coefficient;
λ – conduction coefficient
ρ – density
c_p – constant pressure specific heat

$$\varepsilon_p = \frac{V_{total,\mathrm{Reg}} - V_{fire,site}}{V_{total,\mathrm{Reg}}} = 1 - \frac{V_{fire,site}}{V_{total,\mathrm{Reg}}} \qquad (23)$$

with $V_{fire,site}$ – the volume of bolter’s wire.

Developing the presented relations, we can obtain:

$$\varepsilon_p = 1 - \frac{\pi \cdot d}{4\left(b + d\right)} \qquad (24)$$

Using equation number 21 in 9 and 23 statements results:

$$\frac{\alpha}{\rho \cdot \overline{w} \cdot c_p} \cdot \Pr^{\frac{2}{3}} = \frac{0{,}79}{\left[1 - \frac{\pi d}{4\left(b+d\right)}\right] \cdot \mathrm{Re}^{0{,}576}} \qquad (25)$$

This relation makes the determination of the α coefficient depending of Pr and Re criterions:

$$\alpha = \frac{0{,}79 \rho \overline{w} c_p}{\left[1 - \frac{\pi d}{4\left(b+d\right)}\right] \cdot \Pr^{\frac{2}{3}} \cdot \mathrm{Re}^{0{,}576}} \qquad (26)$$

The 26th relation may be developed as following:

$$\alpha = \frac{0{,}79\rho \cdot \overline{w} \cdot c_p}{\left[1 - \frac{\pi \cdot d}{4\left(b+d\right)}\right] \cdot \left(\frac{\rho \cdot c_p \cdot \nu}{\lambda}\right)^{\frac{2}{3}} \cdot \left(\frac{D_R \cdot \overline{w}}{\nu}\right)^{0{,}576}} \qquad (27)$$

equivalent with:

$$\alpha = \frac{0{,}79\rho^{\frac{1}{3}} \cdot \overline{w}^{0.424} \cdot c_p^{\frac{1}{3}} \cdot \lambda^{\frac{2}{3}}}{\left[1 - \frac{\pi \cdot d}{4\left(b+d\right)}\right] \cdot \nu^{0{,}09} \cdot D_R^{0{,}576}} \qquad (28)$$

The α coefficient must be determined while the Stirling cycle’s characteristics are monitorised, especially gas properties ρ, c_p, λ, υ, determined for the average temperature, between T_3 and T_4 (T_H and T_L) for the average gas pressure.

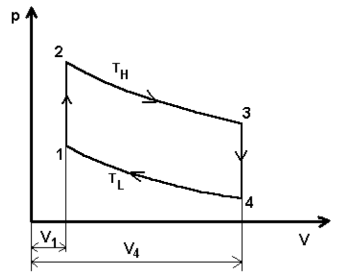

Figure 2. The Stirling cycle in p – V coordinates

$$T_{m,g} = \frac{T_3 + T_4}{2}; \quad p_{med,g} = \frac{1}{2}\left(\frac{p_3 + p_4}{2} + \frac{p_1 + p_2}{2}\right); \quad \rho = \frac{\rho_m}{RT_m} \qquad (29)$$

where: T_{mg} gas average temperature;
$p_{med,g}$ gas average pressure;
ρ_m average density.

3. **DENSITY EVALUATION**

In figure 2 is presented the Stirling cycle. Considering ε, the compression ratio, $\varepsilon = \frac{V_4}{V_1}$ and $\tau = \frac{T_H}{T_L}$, the temperature ratio we will determine the values for medium pressure, medium temperature and medium density.

The medium pressure is indicated for most of the real engines, so we consider that is necessary to determine it:

$$p_{med} = \frac{p_1 + p_2 + p_3 + p_4}{4} = \frac{\varepsilon p_4 + \tau\varepsilon p_4 + \tau p_4 + p_4}{4} = \frac{\left(\varepsilon+1\right)\cdot\left(\tau+1\right)\cdot p_4}{4} \qquad (30)$$

The average temperature is:

$$T_m = \frac{T_3 + T_4}{2} = \frac{T_H + T_L}{2} = \frac{T_L}{2}\left(\tau+1\right) = \frac{T_o}{2}\left(\tau+1\right) \qquad (31)$$

and average density:

$$\rho_m = \frac{p_m}{RT_m} = \frac{2p_4\left(\varepsilon+1\right)\cdot\left(\tau+1\right)}{4T_L\left(\tau+1\right)} = \frac{p_4\left(\varepsilon+1\right)}{2RT_L} = \frac{\rho_4}{2}\left(\varepsilon+1\right) \qquad (32)$$

The α coefficient will be:

$$\alpha = \frac{\frac{0{,}79}{2}\rho_4^{\frac{1}{3}} \cdot \left(\varepsilon+1\right)^{\frac{1}{3}} \cdot \overline{w}^{0{,}424} \cdot c_p^{\frac{1}{3}} \cdot \lambda^{\frac{2}{3}}}{\left[1 - \frac{\pi \cdot d}{4\left(b+d\right)}\right] \cdot \nu^{0{,}09} \cdot D_R^{0{,}576}} \qquad (33)$$

For perfect gases the equation is:

$$\rho_4^{\frac{1}{3}} c_p^{\frac{1}{3}} = \left[\frac{p_4}{RT_L} \cdot \frac{KR}{K-1}\right]^{\frac{1}{3}} = \left[\frac{p_4}{T_L} \cdot \frac{K}{K-1}\right]^{\frac{1}{3}} \qquad (34)$$

so the value is not affected by the gas nature.

At the end, are determined the following directions of study:

- first, depending on the initial pressure and temperature (p_0, T_0), (p_0=p_4 and T_0=T_L):

$$\alpha = \frac{0{,}395\left(p_4 / RT_L\right)\cdot\left(\varepsilon+1\right)\cdot \overline{w}^{0{,}424}\cdot c_p\cdot \nu^{0{,}576}}{\left[1-\frac{\pi}{4\left(\frac{b}{d}+1\right)}\right]\cdot D_R^{0{,}576}\cdot \Pr^{\frac{2}{3}}} \qquad (35)$$

- second, using the average pressure and temperature (p_{med} and T_L):

$$\alpha = \frac{0{,}395\left(4p_{med} / RT_L\right)\cdot \overline{w}^{0{,}424}\cdot c_p\cdot \nu^{0{,}576}}{\left(1+\tau\right)\cdot\left[1-\frac{\pi}{4\left(\frac{b}{d}+1\right)}\right]\cdot D_R^{0{,}576}\cdot \Pr^{\frac{2}{3}}} \qquad (36)$$

As a conclusion we may underline that the α coefficient depends on the geometric characteristics of the engine, and on the gas physical properties.

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