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ON SOME APPROXIMATION PROCESSES IN LOCALLY CONVEX CONEX

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The aim of this paper is to present a constructon of a Korovkin system for a cone of weighted continuous set – valued functions.

§ 1. Preliminaries and notations

- Let (G,V) be a separated locally convex cone such that G is a linear space. We shall consider:

(1) $CConv(G) = \{A \subset G \mid \emptyset \neq A \in CConv(G,\downarrow)$, *compact in the upper topology on* $G\}$,

which becomes a locally convex cone, as a subcone of the full locally convex cone $\overline{(Conv(G),V)}$, where $\overline{V} = \{\overline{v} \mid \overline{v} = (v^-, v) \in V\}$.

It's not difficult to verify that $(CConv(G),\overline{V})$ is a M -uniformly up-directed cone, $\vee$-semilattice and all its elements are bounded.

Recall that there's a natural embedding $j: G^* \to (CConv(G))^*,\ j(\mu) = \overline{\mu}$, where $\overline{\mu}(A) = sup\{\mu(a) \mid a \in A\},\ A \in CConv(G)$.

Let (2) $M = \{\overline{\mu} \in (CConv(G))^* \mid \mu \in G^*\}$. Then, M has the following properties:

1. $(\forall)\, v \in V,\ M \cap \overline{v}^0,\ \overline{v}^*$ – compact;
2. $(\forall)\, A,B \in CConv(G),\ (\forall)\, v \in V$ pentru care $(\exists)\, \rho > 1$ a.i. $A \leq B + \rho\overline{v}$, $(\exists)\, \overline{\mu} \in M \cap \overline{v}^0$ a.i. $\overline{\mu}(A) > \overline{\mu}(B) + 1$.
3. $\overline{\mu}(co\{a_1,...,a_n\}) = \bigvee_{i=0}^{n} \mu(a_i),\ a_i \in G,\ i = \overline{1,n}$.

- Let X be a locally compact Hausdorff space and w, a weight on X.

Now, we shall consider:

(3) $C^w(X;CConv(G)) = \{f \in C_s(X;CConv(G)) \mid (\forall)\, v \in V,\ (\exists)\, J \subset X$, compact such that $f \leq \overline{v}_w$ and $0 \leq f + \overline{v}_w$ on $X \setminus Y\}$ endowed with abstract neighborhood system $\overline{V}_w = \left\{\overline{v}_w \mid \overline{v}_w = \left\{\frac{v}{w}\right\},\ v \in V\right\}$ and

(4) $M_X^w = \left\{\overline{\mu}_x \mid \overline{\mu}_x \in (C^w(X;CConv(G)))^*,\ \mu \in M,\ x \in X\right\}$.

Then, it can easily be proved that (3) and (4) inherit the same properties as (1) and (2).

§ 2. A Korovkin system for $C^w(X;CConv(G))$

- Firstly, we consider

(5) $F^w(X;CConv(G)) = \{f \in C^w(X;CConv(G)) \mid (\exists)\, \varphi_i \in C^w(X;G)$ of finite rank, $i = \overline{1,n}$ such that, $(\forall)\, x \in X,\ f(x) = \overline{co}\{\varphi_1(x),...\varphi_n(x)\}\} \left(\overset{not}{=} F^w\right)$.

- Prop. 1: F^w is a sup-stable subcone of $C^w(X;CConv(G),\overline{V}_w)$.
- Cor. 2: F^W is an M -uniformly up-directed cone and $\vee$ -semilattice.

Prop. 3: If $f \in C^w(X;CConv(G),\ \forall\ \overline{v} \in V,\ \mu \in G^*,\ x \in X,\ \exists\ \overline{g} \in F^w \cap \overline{v}_w$ such that $\overline{\mu}_x(f) = \overline{\mu}_x(g)$

Dem. prop.3Conformly with the definition of $\overline{\mu}_X$ and of the continuity of $f(x), \exists\ \overline{a} \in f(x)$ such that. $\mu(a) = \overline{\mu}_x(f)$.

Conformly of the definition of C^W, $\exists\ \overline{Y} \in X$ compact such that $f(x) \leq \overline{v}_W$ and $0 \leq f(x) + \overline{v}_W,\ \forall\ \overline{x} \in X \setminus Y$.

For $y \in Y$, we choose $a_y \in f(y)$. (in particularly, $a_x = a$).

Conformly with the continuity of $f,\ \exists\ \overline{U}_y = \overset{o}{U}_y \in V(y)$ such that. $\overline{a}_y \leq f(z) + \frac{\overline{v}}{1+w(y)}$ and $w(z) \leq w(y) + 1,\ \forall\ \overline{z} \in U_y$; so we have: $\overline{a}_y \leq f(z) + \frac{\overline{v}}{w(y)+1} \leq f(z) + \frac{\overline{v}}{w(z)},\ \forall\ \overline{z} \in U_y$.

- We define $g = \sum_{i=1}^{n} \alpha_i a_{y_i}$,where $\alpha_i\ _{i=\overline{1,n}}$ is a unit-partition $U_{y_i}\ _{i=\overline{1,n}}$ of X. g verify the conditions:

$\overline{g(y)} \leq f(y) + \overline{v},\ \forall\ \overline{y} \in X$ because $\alpha_1(x) = 1$ and $\alpha_i(x) = 0,\ i = \overline{1,n} \Rightarrow g(X) C < a_y, \ldots, a_{y_n}$

- .
- To prove the main result of § 2, we use a consequence of a result due to Keimel and Roth(3):
- Prop.4: Let (G, V) be locally convex cone with all elements bounded and $C \subset G$, a subcone.

Then: $Sup_C(G^*) = Sub_C(G^*) = \overline{G}^s$.

Theorem 5: $F^w(X;CConv(G))$ is a lower-Korovkin system for $C^w(X;CConv(G))$, Dem.Th 5:Follow the prop.3 and the definition of inf-envelope we $\overline{\mu}_x(f) = \breve{f}$ and so $f \in Sub_{F^w}(M_X^W)$.

Because M_X^W is strictly- separating, $Ex(C_W^0 \setminus C^*) \subset M_X^W$ and from F^W, $M-u-d-$directed $f \in Sub_{F^W}\left((C^W)^*\right) = (C^W)^s$, conformly prop.4

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