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THE MAGNETIC INDUCTION MACHINES' DEGREE OF SATURATION DETERMINING

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Abstract*. The variation shape of the stator current in time is non-sinusoidal for an electric saturated machine. The magnetic field from the machine's air gap has the same variation shape (according to the law of the magnetic circuit).*
Because the rectangular pattern comes from an electric machine powered by a symmetrical phase system that gives a magnetic field for the air gap distributed sinusoidal in space ,modeling a machine with a non-sinusoidal field assumes fragmenting the non-sinusoidal field in harmonics and considering the final product as an assembly of elementary machines that have a sinusoidal distribution of the magnetic field in the air gap.
Keywords: asynchronous machines, magnetic saturation, electrical current

1. Introduction

The issue of magnetic saturation for asynchronous machines.

The number of the elementary machines is equal with the degree (number) of harmonics considered when fragmenting the non-sinusoidal magnetic field in a series of harmonics. (Fig 1)

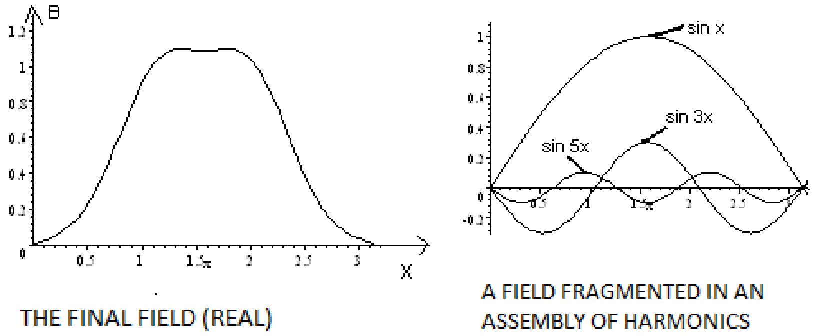

Fig 1.The magnetic field non-sinusoidal distribution in space

2. Determining the saturation degree for the core of the harmonic composition of electrical current.

With the voltage at terminals given U [4],we can calculate the electrical flow within the machine when working with no-load. When modifying the voltage value at terminals [3] we can elevate the magnetization diagram $\psi_m = f\left(I_m\right)$ on the unsaturated and saturated areas. (Fig 2).

The magnetization current *Im* is defined by his components I_{md} and I_{mq}:

$$I_m^2 = I_{md}^2 + I_{mq}^2 \tag{1}$$

Where:

$I_{md} = I_d + I_{dr}$ - the magnetization current corresponding the electrical flow ψ_{md}

$I_{mq} = I_q + I_{qr}$ - the magnetization current corresponding the electrical flow ψ_{mq}

$$\underline{I}_m = I_{md} + jI_{dq} \quad ; \qquad \underline{I}_m = \underline{I}_s + \underline{I}_r$$

I_d Stator current from d axis; I_q Stator current from q axis ; I_{dr} Rotor current from d axis; I_{qr} Rotor current from q axis.

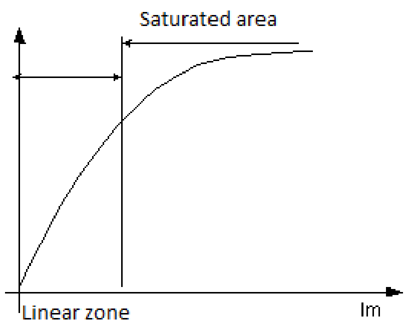

Fig.2 .The magnetization diagram for the asynchronous machine

$\psi_d = \psi_{md} + L_{1\sigma} I_d$ - stator electrical flow from the d axis winding (2)

$\psi_q = \psi_{mq} + L_{1\sigma} I_q$ - stator electrical flow from q axis winding (3)

${\psi_m}^2 = {\psi_{md}}^2 + {\psi_{mq}}^2$ - ψ_m electrical flow (4)

Observation :

The dispersion (leakage) inductances *L1σ* and *L2σ* are not influenced by the saturation degree of the machine. Because of this fact these inductances are considered constant. The rotor dispersion inductance L2σ can be calculated from L (main rotor inductance) and M (mutual inductance) with the relation :

$$L_{2\sigma} = L_2 - M \quad (5)$$

Removing the rotor current I_r results:

$$\underline{\underline{U}}_s = R_1 \cdot \underline{I}_s + j\omega_1\left(\underline{\underline{\psi}}_m + L_{1\sigma}\cdot \underline{I}_s\right);\ 0 = R_2\left(\underline{I}_m - \underline{I}_s\right) + j\omega_1 s\left[\underline{\psi}_m + L_{2\sigma}\left(\underline{I}_m - \underline{I}_s\right)\right] \quad (6)$$

The function $\psi_m = f(I_m)$ can be can be deducted experimental.

$$M_{elmag} = p_1 I_{mag}\left[\underline{I}_s \cdot {\psi_m}^*\right] \quad (7)$$

Or using the axis:

$$U = R_1 I_d - \omega_1 \psi_{mq} - \omega_1 L_{1\sigma} I_q$$

$$0 = R_1 I_q + \omega_1 \psi_{md} + \omega_1 L_{1\sigma} I_d$$

$$0 = R_2\left(I_{md} - I_d\right) - s\omega_1\left[\psi_{mq} + L_{2\sigma}\left(I_{mq} - I_q\right)\right]$$

$$0 = R_2\left(I_{mq} - I_q\right) - s\omega_1\left[\psi_{md} + L_{2\sigma}\left(I_{md} - I_d\right)\right]$$

$$\psi_m = f\left(I_m\right)$$

$$\psi_m = \sqrt{\psi_{md}^2 + \psi_{mq}^2} \quad (8)$$

$$\psi_{md} = \frac{\psi_m}{I_m} I_{md}$$

$$\psi_{mq} = \frac{\psi_m}{I_m} I_{mq}$$

$M_{elmag} = p_1\left(I_q \psi_{md} - I_d \psi_{mq}\right)$ - torque equation

Considering as given the functioning point P on the magnetization diagram $\Psi_m(I_m)$ experimentally risen [2] – the values of the magnetization current I_m and the electrical flow ψ_m–(fig3)

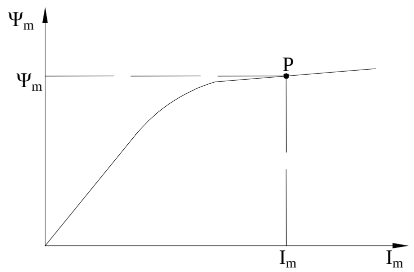

Fig 3. The magnetization diagram

For a motor slip "s " know (imposed speed) solving system of 8 equation we obtain 8 unknowns $U, I_d, I_q, I_{md}, I_{mq}, \psi_{md}, \psi_{mq}$ şi M_{elmag}.

Observation : 1.When neglecting the stator resistance ($R_1 \to 0$) and the leakage $\left(L_{1\sigma} \to 0, L_{2\sigma} \to 0\right)$, we obtain the simplified system :

$$U = -\omega_1 \psi_{mq}$$

$0 = \psi_{md}$, ($\psi_m = \psi_{mq}$ şi $I_{md} = 0$)

$$0 = R_2\left(I_{md} - I_d\right) - s\omega_1\psi_{mq} \quad (9)$$

$$0 = R_2\left(I_{mq} - I_q\right) - s\omega_1\psi_{md}$$

$$\psi_m = \psi_{mq} = f\left(I_m\right)$$

$$M_{elmag} = -p_1 I_d \psi_{mq}$$

or: $$U = -\omega_1\psi_{mq}$$

$$I_d = -\frac{s\omega_1\psi_m}{R_2},$$

$$I_m = I_{mq} = I_q$$

$$I_q = I_m \quad (10)$$

$$\psi_{mq} = \psi_m$$

$$0 = R_2\left(I_{mq} - I_q\right) - s\omega_1\psi_{md}$$

Function: $\psi_m = f\left(I_m\right)$; $M_{elmag} = -p_1 I_d \psi_{mq}$

When using the sinusoidal supply voltage U results an electrical sinusoidal flow and I_m will contain an assembly of harmonics [3] as it results from fig.4

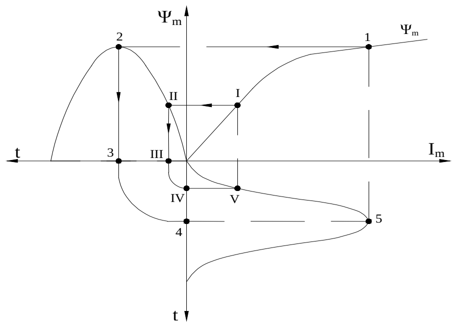

Fig.4.Dependance $I_m(t)$

The system can be resolved in two ways :

a) ψ_m and I_m are given , results voltage $U = -\omega\psi_m$ and current

$$I_d = -\frac{s\omega_1\psi_m}{R_2} \quad (11)$$

With I_d and ψ_{mq} calculated we obtain the electromagnetic torque $M_{elmag} = -p_1 I_d \psi_{mq}$.

b) Knowing the voltage U results the electric flow $\psi_m = -(U / (\omega_1))$ and from the function $\psi_m(I_m)$ we obtain I_m. .The current I_d is calculated from $I_d = -\frac{s\omega_1\psi_m}{R_2}$.

I_q is obtained from $I_m = I_q$ and the electromagnetic torque is $M_{elmg} = p_1 I_d \psi_{mq}$.

When using a sinusoidal voltage U we obtain , due to saturation, a non-sinusoidal current having the following harmonic composition :

$$i_{s(1)} = \sum \sqrt{2} I_k \sin\left(K\omega t + \frac{\pi}{2}\right) \quad (12)$$

$$I_{s(1)} = I_d + jI_{q(1)} \quad (13)$$

Or : $$i_{s(1)} = \sqrt{I_d^2 + I_{q(1)}^2}\sqrt{2}\sin\left[\omega t + arctg\left(\frac{I_{q(1)}}{I_d}\right)\right] \quad (14)$$

$$I_{s(K)} = jI_{q(K)} \quad (15)$$

Knowing spreadsheets where the function $I_m(t)$ has a shape as in the fig.5. we can fragment the function $I_m(t)$:

$$I_m(t) = I_m(1) + I_m(2) + I_m(3) + \ldots$$

And then the stator current $i_s(t)$: $i_s(t) = i_s(1) + i_s(2) + i_s(3) + ...$ (16)

Where : $i_{s(1)} = \sqrt{I_d^2 + I_{q(1)}^2}\sqrt{2}\sin\left[\omega t + arctg\left(\frac{I_{q(1)}}{I_d}\right)\right]$ (17)

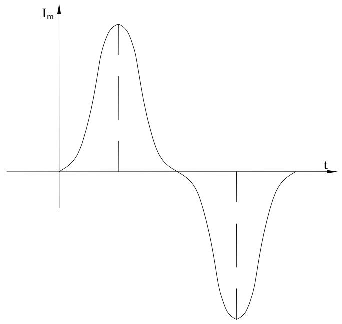

Fig.6. Time variation for magnetizing current

$$I_d = -\frac{s\omega_1\psi_m}{R_2} \quad (18)$$

$$I_m(1) = \frac{4}{T}\int_0^{T/2} I_m(t)\sin(\omega t)dt$$

$$I_m(K) = \frac{4}{T}\int_0^{T/2} I_m(t)\sin(K\omega t)dt$$

Because the function $I_m(t)$ is given by table , a fourrier analysis is required ,discreet with superior harmonics (K≥2) , calculated such:

$$i_m(t) = \sum_{k=2}^{\infty} I_m(k)\sin(Kwt) \quad (19)$$

Specifications :

1.As is well known [1], the non-linear consumer(asynchronous machine) receives active power from the sinusoidal voltage network : $P_1 = UI_s\cos\varphi_1$ a part of which is consumed ,and the other part of active power $\left(\sum_{k=2}^{\infty} P_k\right)$ is guided through harmonics in the supply network ,which means that it goes to other consumers that can be linear or nonlinear.

2. The current component from the longitudinal axis I_d is determined by the electrical flow ψ_m and from this reason it is also called the flow dependent component.

The function $I_m(\psi_m)$ can be approximated by polynomial (11th degree) expression[5] :

$$i_m(t) = a_1\psi_m + a_2\psi_m^2 + a_3\psi_m^3 + a_4\psi_m^4 + a_5\psi_m^5 + a_6\psi_m^6 + a_7\psi_m^7 + a_8\psi_m^8 + a_9\psi_m^9 + a_{10}\psi_m^{10} + a_{11}\psi_m^{11} \quad (20)$$

Because the supply voltage has a sinusoidal variation ,the electrical flow ψ_m, can be written: $\psi_m(t) = A\sin(\omega_1 t)$

where : $A = \frac{U}{\omega_1}$ and the current expression $i_m(t)$ becomes :

$$\begin{aligned} i_m(t) &= a_1A\sin(\omega t) + a_2A^2\sin^2(\omega t) + a_3A^3\sin^3(\omega t) + a_4A^4\sin^4(\omega t) + a_5A^5\sin^5(\omega t) + \\ &+ a_6A^6\sin^6(\omega t) + a_7A^7\sin^7(\omega t) + a_8A^8\sin^8(\omega t) + a_9A^9\sin^9(\omega t) + a_{10}A^{10}\sin^{10}(\omega t) + \\ &+ a_{11}A^{11}\sin^{11}(\omega t) = \end{aligned}$$

$$= a_1A\sin(\omega t) + \frac{a_2A^2}{2}[1 - \cos(2\omega t)] + \frac{a_3A^3}{4}[3\sin(\omega t) - \sin(3\omega t)] +$$

$$+ \frac{a_4A^4}{8}[3 - 4\cos(2\omega t) + \cos(4\omega t)] + \frac{a_5A^5}{16}[10\sin(\omega t) - 5\sin(3\omega t) + \sin(5\omega t)] +$$

$$+\frac{a_6A^6}{32}\left[10-15\cos(2\omega t)+6\cos(4\omega t)-\cos(6\omega t)\right]+$$

$$+\frac{a_7A^7}{64}\left[35\sin(\omega t)-21\sin(3\omega t)+7\sin(5\omega t)-\sin(7\omega t)\right]+ \quad (22)$$

$$+\frac{a_8A^8}{128}\left[35-56\cos(2\omega t)+28\cos(4\omega t)-8\cos(6\omega t)+\cos(8\omega t)\right]+$$

$$+\frac{a_9A^9}{256}\left[126\sin(\omega t)-84\sin(3\omega t)+36\sin(5\omega t)-9\sin(7\omega t)+\sin(9\omega t)\right]+$$

$$+\frac{a_{10}A^{10}}{512}\left[126-210\cos(2\omega t)+120\cos(4\omega t)-45\cos(6\omega t)+10\cos(8\omega t)-\cos(10\omega t)\right]+$$

$$=B_1\sin(\omega t)+B_3\sin(3\omega t)+B_5\sin(5\omega t)+B_7\sin(7\omega t)+B_9\sin(9\omega t)+B_{11}\sin(11\omega t)+$$

$$+B_2\cos(2\omega t)+B_4\cos(4\omega t)+B_6\cos(6\omega t)+B_8\cos(8\omega t)+B_{10}\cos(10\omega t)+B_0$$

Where:

$$B_1=a_1A+\frac{3a_3A^3}{4}+\frac{5a_5A^5}{8}+\frac{35a_7A^7}{64}+\frac{63a_9A^9}{128}+\frac{231a_{11}A^{11}}{512}$$

$$B_3=-\frac{a_3A^3}{4}-\frac{5a_5A^5}{16}-\frac{21a_7A^7}{64}-\frac{21a_9A^9}{64}-\frac{165a_{11}A^{11}}{512}$$

$$B_5=\frac{5a_5A^5}{16}+\frac{7a_7A^7}{64}+\frac{9a_9A^9}{64}+\frac{165a_{11}A^{11}}{1024}$$

$$B_7=-\frac{a_7A^7}{64}-\frac{9a_9A^9}{256}-\frac{55a_{11}A^{11}}{1024}$$

$$B_9=\frac{a_9A^9}{256}+\frac{11a_{11}A^{11}}{1024} \qquad B_{11}=-\frac{a_{11}A^{11}}{1024}$$

$$B_2=-\frac{a_2A^2}{2}-\frac{a_4A^4}{2}-\frac{15a_6A^6}{32}-\frac{7a_8A^8}{16}-\frac{105a_{10}A^{10}}{256}$$

$$B_4=\frac{a_4A^4}{8}+\frac{3a_6A^6}{16}+\frac{7a_8A^8}{32}+\frac{15a_{10}A^{10}}{64}$$

$$B_6=-\frac{a_6A^6}{32}-\frac{a_8A^8}{16}-\frac{45a_{10}A^{10}}{512} \quad (23)$$

$$B_8=\frac{a_8A^8}{128}+\frac{5a_{10}A^{10}}{256} \qquad B_{10}=-\frac{a_{10}A^{10}}{512}$$

$$B_0=\frac{a_2A^2}{2}+\frac{3a_4A^4}{8}+\frac{5a_6A^6}{16}+\frac{35a_8A^8}{128}+\frac{63a_{10}A^{10}}{256}$$

The current harmonic composition :

$$i_m(t)=B_0+B_1\sin(\omega t)+B_2\cos(2\omega t)+B_3\sin(3\omega t)+B_4\cos(4\omega t)+B_5\sin(5\omega t)+$$
$$+B_6\cos(6\omega t)+B_7\sin(7\omega t)+B_8\cos(8\omega t)+B_9\sin(9\omega t)+B_{10}\cos(10\omega t)+B_{11}\sin(11\omega t) \quad (24)$$

Determining B_k coefficients (K=1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11).As it can be observed from B_K coefficients expression ,it is required determining coefficients a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}.
The a_k coefficients are determined from the experimental diagram $I_m(\psi_m)$

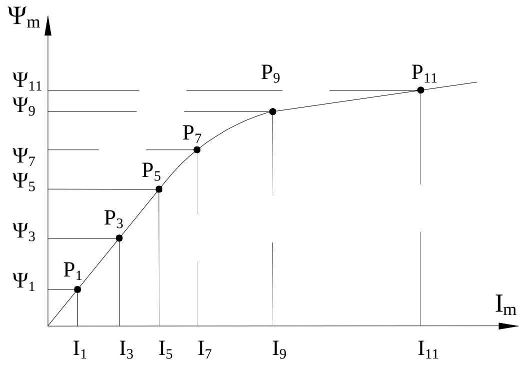

Fig.7. Setting points on the experimental diagram $\psi_m(I_m)$

The following algebric system is formed:

$$\begin{aligned}
I_1 &= a_1\psi_1 + a_2\psi_1^2 + a_3\psi_1^3 + a_4\psi_1^4 + a_5\psi_1^5 + a_6\psi_1^6 + a_7\psi_1^7 + a_8\psi_1^8 + a_9\psi_1^9 + a_{10}\psi_1^{10} + a_{11}\psi_1^{11} \\
I_2 &= a_1\psi_2 + a_2\psi_2^2 + a_3\psi_2^3 + a_4\psi_2^4 + a_5\psi_2^5 + a_6\psi_2^6 + a_7\psi_2^7 + a_8\psi_2^8 + a_9\psi_2^9 + a_{10}\psi_2^{10} + a_{11}\psi_2^{11} \\
I_3 &= a_1\psi_3 + a_2\psi_3^2 + a_3\psi_3^3 + a_4\psi_3^4 + a_5\psi_3^5 + a_6\psi_3^6 + a_7\psi_3^7 + a_8\psi_3^8 + a_9\psi_3^9 + a_{10}\psi_3^{10} + a_{11}\psi_3^{11} \\
I_4 &= a_1\psi_4 + a_2\psi_4^2 + a_3\psi_4^3 + a_4\psi_4^4 + a_5\psi_4^5 + a_6\psi_4^6 + a_7\psi_4^7 + a_8\psi_4^8 + a_9\psi_4^9 + a_{10}\psi_4^{10} + a_{11}\psi_4^{11} \quad (25) \\
I_5 &= a_1\psi_5 + a_2\psi_5^2 + a_3\psi_5^3 + a_4\psi_5^4 + a_5\psi_5^5 + a_6\psi_5^6 + a_7\psi_5^7 + a_8\psi_5^8 + a_9\psi_5^9 + a_{10}\psi_5^{10} + a_{11}\psi_5^{11} \\
I_6 &= a_1\psi_6 + a_2\psi_6^2 + a_3\psi_6^3 + a_4\psi_6^4 + a_5\psi_6^5 + a_6\psi_6^6 + a_7\psi_6^7 + a_8\psi_6^8 + a_9\psi_6^9 + a_{10}\psi_6^{10} + a_{11}\psi_6^{11} \\
I_7 &= a_1\psi_7 + a_2\psi_7^2 + a_3\psi_7^3 + a_4\psi_7^4 + a_5\psi_7^5 + a_6\psi_7^6 + a_7\psi_7^7 + a_8\psi_7^8 + a_9\psi_7^9 + a_{10}\psi_7^{10} + a_{11}\psi_7^{11} \\
I_8 &= a_1\psi_8 + a_2\psi_8^2 + a_3\psi_8^3 + a_4\psi_8^4 + a_5\psi_8^5 + a_6\psi_8^6 + a_7\psi_8^7 + a_8\psi_8^8 + a_9\psi_8^9 + a_{10}\psi_8^{10} + a_{11}\psi_8^{11} \\
I_9 &= a_1\psi_9 + a_2\psi_9^2 + a_3\psi_9^3 + a_4\psi_9^4 + a_5\psi_9^5 + a_6\psi_9^6 + a_7\psi_9^7 + a_8\psi_9^8 + a_9\psi_9^9 + a_{10}\psi_9^{10} + a_{11}\psi_9^{11} \\
I_{10} &= a_1\psi_{10} + a_2\psi_{10}^2 + a_3\psi_{10}^3 + a_4\psi_{10}^4 + a_5\psi_{10}^5 + a_6\psi_{10}^6 + a_7\psi_{10}^7 + a_8\psi_{10}^8 + a_9\psi_{10}^9 + a_{10}\psi_{10}^{10} + a_{11}\psi_{10}^{11} \\
I_{11} &= a_1\psi_{11} + a_2\psi_{11}^2 + a_3\psi_{11}^3 + a_4\psi_{11}^4 + a_5\psi_{11}^5 + a_6\psi_{11}^6 + a_7\psi_{11}^7 + a_8\psi_{11}^8 + a_9\psi_{11}^9 + a_{10}\psi_{11}^{10} + a_{11}\psi_{11}^{11}
\end{aligned}$$

- we must resolve the equation system to find the following unknown a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11};
- we must calculate for a certain given value of the coefficient $A = \dfrac{U}{\omega_1}$ the value of the B_k coefficients

3. Conclusions

Having as a basis the patterns mentioned in specialty books , there were created patterns that take into account the magnetic saturation. The mathematic patterns built were analyzed taking in consideration certain applications.

The limits of the asynchronous machine parameters were analyzed in case of a stabile functioning .The evaluation of the magnetic saturation gives ample solutions for resolving a voltage supply problem ,or a variable frequency problem.

The equations of the rectangular pattern ,built from the ones already existing ,as it result, presents simplicity and consistency, advantages that makes the rectangular pattern particularly useful ,when thinking a simpler representation and in the same time comprehensive and accurate .

Determining asynchronous machine parameters on a catalog data basis , and no-load tests is a simple method that implies the experimental part (no-load at rated voltage),which doesn't rise any problems (even with high-power machines),and the calculation (doesn't rise any problems either) being suitable to the situation when catalog car data are known.

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