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CALCULUS AND MATERIALS FOR STIRLING ENGINE`S BOLTER AND REGENATOR

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Abstract: *The performances of the Stirling engine are affected by the convection coefficient and the "X" factor and not only by the variation of the gas quantity from the cylinder with the medium pressure variation. The convection factor indicates that a sensibility study concerning the characteristic parameters is mandatory.*
Keywords: *Stirling, cycle, engine, convection.*

1. INTRODUCTION

We will consider the most common case, which presents the regenerator as a "pressed bolters package". In figure 1 are presented the geometrical characteristics for the bolter:
We can assimilate regenerator's area with a square area equivalent to the regenerator's:

$$A_R = \frac{\pi D_R^2}{4} = a^2 \tag{1}$$

where D_R – regenerator's diameter;

$$a = D_R\sqrt{\frac{\pi}{4}} = \frac{D_R \cdot \sqrt{\pi}}{2} \tag{2}$$

If L is regenerator's length, then:

$$N_s = \frac{L}{2d} \tag{3}$$

where:
- N_s – total bolter number;
- d – bolter's wire diameter

From here results:

$$\left(\text{wires in the equivalent square in X direction}\right) = \frac{a}{b+d} \tag{4}$$

and:

$$\left(\text{bolter`s wires in both directions}\right) = \frac{2a}{b+d} \tag{5}$$

where b is the distance between two bolter's wires

$$\left(\text{bolter`s wires lenght}\right) = \frac{2a^2}{b+d} \tag{6}$$

$$\begin{aligned}&\left(\text{regenerator`s wires lenght}\right) = \\ &\left(\text{bolters number}\right)\cdot\left(\text{bolter's wires lenght}\right) = L_f\end{aligned} \tag{7}$$

$$L_f = \frac{2a^2}{b+d}\cdot\frac{L}{2d} = \frac{L\cdot a^2}{d(b+d)} \tag{8}$$

With these relations we can determine the radius A_R and the weight m_R. Knowing a form of second relation and using it in the 8th relation, results:

$$L_f = \frac{L\cdot\frac{\pi D_R^2}{4}}{d(b+d)} = \frac{\pi D_R^2 \cdot L}{4d(b+d)} \tag{9}$$

The regenerator's area will be:

$$A_R = \pi\cdot d\cdot L_f = \frac{\pi^2\cdot D_R^2\cdot L\cdot d}{4(b+d)\cdot d} \tag{10}$$

meaning:

$$A_R = \frac{\pi^2\cdot D_R^2\cdot L}{4(b+d)} \tag{11}$$

The regenerator's weight will be:

$$m_R = L_f\cdot A_f\cdot\rho_{metal} = \frac{\pi D_R^2 L}{4d(b+d)}\cdot\frac{\pi d^2}{4}\rho_{metal} \tag{12}$$

where:
- A_f – wire's area;
- $\rho_{metal} = \rho_m$ – metal's density

$$m_R = \frac{\pi^2\cdot D_R^2\cdot d\cdot\rho_m\cdot L}{16(b+d)} \tag{13}$$

The ratio $\frac{m_R}{A_R}$ will be:

$$\frac{m_R}{A_R} = \frac{\pi^2\cdot D_R^2\cdot d\cdot\rho_m\cdot L}{16(b+d)}\cdot\frac{4(b+d)}{\pi^2\cdot D_R^2\cdot L} = \frac{d\cdot\rho_m}{4} \tag{14}$$

The relation required to determine the "X" parameter, according with the regenerator's properties will be:

$$X = 1 - \frac{\dfrac{\pi^2\cdot D_R^2\cdot d\cdot\rho_m\cdot L\cdot c_R}{16(b+d)m_g\cdot c_{v_g}}}{\dfrac{n_r\cdot d\cdot\rho_m\cdot c_R}{15\alpha} - 1} \tag{15}$$

In this relation, number 15 is used because the sinusoidal movement is resulted from a quarter rotation, meaning $\frac{60}{4} = 15$.

For accomplishing the operational status for the 15th relation the following two conditions are required:
- determination of the regenerator's convection coefficient;
- bordering the X factor in general optimization scheme for Stirling engine.

2. THE REGENERATOR'S CONVECTION COEFFICIENT EVALUATION

The following evaluation is based upon the description of the relation between the similitude criteria for the regenerator indicated by Organ [Thermodynamics and Gas dynamics of Stirling Cycle Machine, pag 113]:

$$N_{ST}\cdot N_{\mathrm{Pr}}^{\frac{2}{3}} = \frac{1{,}25}{\sqrt{N_{\mathrm{Re}}}} \tag{16}$$

According to the Romanian scientific standards the relation will be written as:

$$St\cdot\mathrm{Pr}^{\frac{2}{3}} = \frac{1{,}25}{\sqrt{\mathrm{Re}_D}} \tag{17}$$

relation used for a pore – ratio with following values:

$$\varepsilon_p = 0{,}602 \div 0{,}832 \tag{18}$$

where ε_p is the porosity.

Incopera indicates a similar relation [1, page 292]:

$$St \cdot \Pr^{\frac{2}{3}} = \frac{0{,}79}{\varepsilon_p \cdot \mathrm{Re}^{0{,}576}} \tag{19}$$

The two formulas would be almost identical if the porosity formula, used in the second relation would be $\varepsilon_p = \frac{0{,}79}{1{,}25} = 0{,}63$, included in the first formula domain.

We will consider the Incopera formula first, continuing with the Organ formula, because a comparative study would be valuable in order to assign the possible implications for Stirling engines:

$$\mathrm{Re}_D = \frac{D \cdot \rho \cdot \overline{w}}{\mu} = \frac{D_R \cdot \overline{w}}{\nu} \tag{20}$$

where:

- $\overline{W}$ is the medium speed.,μ dynamic viscosity, υ kinematic viscosity.

The coefficient Re_D is determined for medium speed $\overline{W}$ and D_R, the diameter for regenerator's empty shell.

$$St = \frac{\alpha}{\rho \cdot \overline{w} \cdot c_p} \tag{21}$$

and

$$\Pr = \frac{\nu}{a} = \frac{\nu}{\lambda / (\rho \cdot c_p)} = \frac{\rho \cdot c_p \cdot \nu}{\lambda} \tag{22}$$

$$\varepsilon_p = \frac{V_{total,\mathrm{Reg}} - V_{fire,site}}{V_{total,\mathrm{Reg}}} = 1 - \frac{V_{fire,site}}{V_{total,\mathrm{Reg}}} \tag{23}$$

with $V_{fire,site}$ – the volume of bolter's wire.

Developing the presented relations, we can obtain:

$$\varepsilon_p = 1 - \frac{\pi \cdot d}{4(b+d)} \tag{24}$$

Using equation number 21 in 9 and 23 statements results:

$$\frac{\alpha}{\rho \cdot \overline{w} \cdot c_p} \cdot \Pr^{\frac{2}{3}} = \frac{0{,}79}{\left[1 - \frac{\pi d}{4(b+d)}\right] \cdot \mathrm{Re}^{0{,}576}} \tag{25}$$

This relation makes the determination of the α coefficient depending of Pr and Re criterions:

$$\alpha = \frac{0{,}79 \rho \overline{w} c_p}{\left[1 - \frac{\pi d}{4(b+d)}\right] \cdot \Pr^{\frac{2}{3}} \cdot \mathrm{Re}^{0{,}576}} \tag{26}$$

The 26th relation may be developed as following:

$$\alpha = \frac{0{,}79 \rho \cdot \overline{w} \cdot c_p}{\left[1 - \frac{\pi \cdot d}{4(b+d)}\right] \cdot \left(\frac{\rho \cdot c_p \cdot \nu}{\lambda}\right)^{\frac{2}{3}} \cdot \left(\frac{D_R \cdot \overline{w}}{\nu}\right)^{0{,}576}} \tag{27}$$

equivalent with:

$$\alpha = \frac{0{,}79 \rho^{\frac{1}{3}} \cdot \overline{w}^{0.424} \cdot c_p^{\frac{1}{3}} \cdot \lambda^{\frac{2}{3}}}{\left[1 - \frac{\pi \cdot d}{4(b+d)}\right] \cdot \nu^{0{,}09} \cdot D_R^{0{,}576}} \tag{28}$$

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