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THE MATHEMATICAL MODEL TO DETERMINE THE UNDERWATER EXPLOSIONS DIRECTION AND DISTANCE

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Abstract: *This report presents the triangulation of the underwater explosion source. The analysis is based on the time-delay measurement the underwater acoustic wave, deriving the range and the direction to the underwater source of explosion. The mathematical model is simulated for different values of the time-delay at three sensors. It was built a practical demonstrator, which gave the possibility to verify in real environment the mathematical model.*

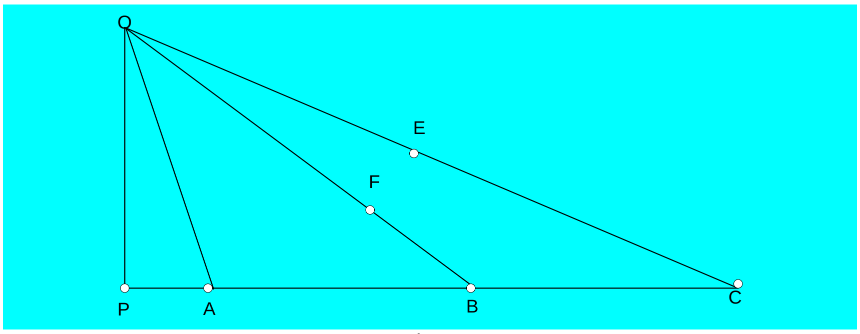

Fig. 1

The hydro-acoustic sensors are placed in the A, B, C points, at d_o range; the event take place in O point. We trace a perpendicular in P point.

OA=OF=OE and represents the range covered by the wave from $t_{0,}$ moment when the event took place and $t_{1,}$ moment when the signal was received from A point. The wave will cover the range FB in the time T_B, which is $T_B=t_2-t_{1,}$ where t_2 is the time when the wave came in the B point, so $FB=T_B*v$, where v is the speed of wave in the water, speed known either from the hydro-acoustic prognosis or approximated at 1450m/s.

The wave will cover the range EC in the time T_C, which is $T_C=t_3-t_{1,}$ where t_3 is the time when the wave came in the point C, so $EC=T_C*v$.

We can write the following relations:

$(OP)^2=(OA)^2-(PA)^2$
$(OP)^2=(OF+FB)^2-(AB+PA)^2$
$(OP)^2=(OE+EC)^2-(AB+BC+PA)^2$

he unknown of the system are: OP, OA, PA.
Knowing the sides, in OPA triangle, sin A=OP/OA.
So the range and the direction are determinates. We observe that if the event is in the left of the hydro-acoustic sensors line, the wave came firstly in point C.
In this case:

$(OP)^2=(OC)^2-(PC)^2$
$(OP)^2=(OF+FB)^2-(CB+PC)^2$
$(OP)^2=(OE+EC)^2-(CB+BC+PC)^2$

The unknown of the system are: OP, OC, PC.
Knowing the sides, in OPC triangle, sin C=OP/OC.

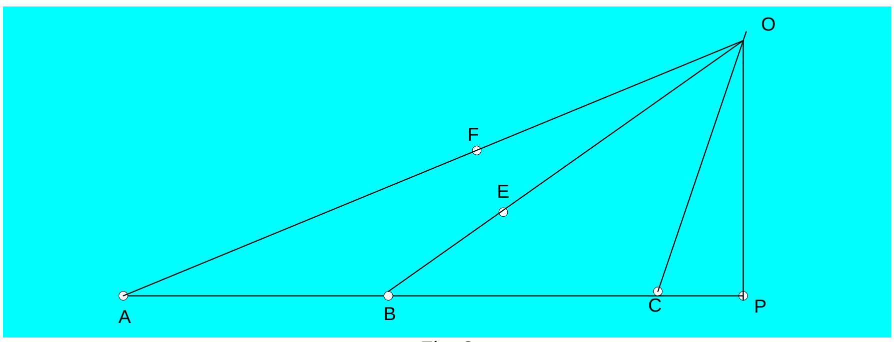

Fig. 2

The resolution of the system is simple:

$(OP)^2=(OA)^2-(PA)^2$
$(OP)^2=(OF+FB)^2-(AB+PA)^2$
$(OP)^2=(OE+EC)^2-(AB+BC+PA)^2$

We decompose,

$OP^2=OA^2-PA^2$
$OP^2=OF^2+2*OF*FB+FB^2-AB^2-2*AB*PA-PA^2$

Considering AB=BC of known value, we would analyze the minimum value for AB

$OP^2=OE^2+2*OE*EC+EC^2-4*AB^2-4*AB*PA-PA^2$

Replacing in the last two equations:

$OA^2-\underline{PA}^2=OF^2+2*OF*FB+FB^2-AB^2-2*AB*PA-\underline{PA}^2$
$OA^2-\underline{PA}^2=OE^2+2*OE*EC+EC^2-4*AB^2-4*AB*PA-\underline{PA}^2$

Or:

$OA^2 = OF^2+2*OF*FB+FB^2-AB^2-2*AB*PA$
$OA^2 = OE^2+2*OE*EC+EC^2-4*AB^2-4*AB*PA$

But OA=OF=OE, so:

$OA^2 = OA^2+2*OA*FB+FB^2-AB^2-2*AB*PA$
$OA^2 = OA^2+2*OA*EC+EC^2-4*AB^2-4*AB*PA$

The unknown of the system are OP, OA, PA. By simplification the system became:

$2*OA*FB+FB^2-AB^2-2*AB*PA=0$
$2*OA*EC+EC^2-4*AB^2-4*AB*PA=0$

Result PA:

$$PA = \frac{FB^2 + 2\cdot OA\cdot FB - AB^2}{2\cdot AB}$$

$$OA = \frac{2FB^2 + 2AB^2 - EC^2}{2\cdot EC - 4\cdot FB}$$

Where AB is the range between the sensors, B=T_B*v, EC=T_C*v, with $T_B=t_2-t_1$ and $T_C=t_3-t_1$, t_1 is the time when the signal was received by the sensor from the A point, t_2 is the time when the wave came in the B point, t_3 is the time when the wave came in the C point. The results of the simulation are presented down.

INITIAL DATA TRIANGULATION PROGRAM

No.	X	y	z	v	d	t0	t1	t2	t1-t0	t2-t0
Event at the left										
1	20	-24	31.241	1400	2	0.0223	0.0234	0.0245	0.0011	0.0022
2	35	-30	46.0977	1400	2	0.0329	0.0338	0.0348	0.0009	0.0019
3	60	-41	72.6705	1400	2	0.0519	0.0527	0.0535	0.0008	0.0016
4	95	-52	108.301	1400	2	0.0773	0.0780	0.0787	0.0006	0.0014
5	450	-132	468.961	1400	2	0.3349	0.3353	0.3357	0.0004	0.0008
6	1200	-450	1281.6	1400	5	0.9154	0.9166	0.9179	0.0012	0.0025
7	2400	-870	2552.82	1400	5	1.8234	1.8246	1.8258	0.0012	0.0024
8	5000	-2400	5546.17	1400	5	3.9615	3.9630	3.9646	0.0015	0.0030
9	12400	-6000	13775.3	1400	10	9.8395	9.8426	9.8457	0.0031	0.0062
10	25000	-12000	27730.8	1400	10	19.807	19.810	19.813	0.0030	0.0061
Event at the right										
No.	X	y	z	v	d	t0	t1	t2	t1-t0	t2-t0
11	20	24	31.241	1400	2	0.0223	0.0212	0.0202	0.0010	0.0021
12	35	30	46.0977	1400	2	0.0329	0.0320	0.0311	0.0008	0.0017
13	60	41	72.6705	1400	2	0.0519	0.0511	0.0503	0.0007	0.0015
14	95	52	108.301	1400	2	0.0773	0.0766	0.0760	0.0006	0.0013
15	450	132	468.961	1400	2	0.3349	0.3345	0.3341	0.0003	0.0007
16	1200	450	1281.6	1400	5	0.9154	0.9141	0.9129	0.0012	0.0024
17	2400	870	2552.82	1400	5	1.8234	1.8222	1.8210	0.0012	0.0024
18	5000	2400	5546.17	1400	5	3.9615	3.9600	3.9584	0.0015	0.0030
19	12400	6000	13775.3	1400	10	9.8395	9.8364	9.8333	0.0031	0.0062
20	25000	12000	27730.8	1400	10	19.807	19.804	19.801	0.0030	0.0061

Table 1

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