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SOME REMARKS ON GPS SATELLITES ORBITS

Sergiu L. LUPU[1]
Carmen Elena LUPU[2]
Crețu GABRIEL[3]
[1]Lecturer, Ph.D., Naval Academy, Contanta, Romania
[2]Lecturer, Ph.D., Ovidius Universsity, Contanta, Romania
[3] Cpt., Naval Academy, Contanta, Romania

***Abstract**: NAVSTAR/GPS is a spatial positioning system which answers the questions „What time, what position, and what velocity is it?"quickly, accuately and inexpensively anywhere on the globe. In this paper we present a mathematical form of disruptive forces that affect the orbits of GPS satellites.*

1. INTRODUCTION

In 1973 The United States Department of Defense created a spatial positioning system: NAVSTAR/GPS (Navigation System with Timing And Ranging / Global Positioning System) under the authority of the Joint Program Office (JPO). The standard configuration of the GPS constellation is composed of 24 satellites that provide global coverage. The corresponding scheme entails a layout on 6 quasicircular orbits (e = 0.003), shifted by 60° in the right ascension of the ascending node and with an inclination of 55° towards the equatorial plan. The spare satellites used in different stages are activated while on ground and are mainly used as replacements in case of a possible malfunction.

GPS satellites are spatial vehicles that carry radio-electronic equipment – meant to process and transmit signals towards the terrestrial users, together with an atomic clock, batteries and auxiliary equipment.

2. THE EQUATIONS OF DISRUPTED MOVEMENT

The Keplerian orbit of a satellite is a purely theoretical notion, as it doesn't take into consideration the disruptive factors that really influence its dynamics. In order to describe the real relative movement – therefore disrupted – of the satellite around the attractive body – Earth – we need to sum up all the disruptive accelerations that impact the deviation from the Keplerian movement in the right side of the second order differential equation (homogeneous) of the undisrupted relative movement. Thus we obtain the second order differential equation (non-homogeneous) of the undisrupted relative movement:

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + d\ddot{\vec{r}}, \quad \text{or on the coordinate axes}$$

$$\ddot{x} = -\frac{\mu}{r^3}x + d\ddot{x}, \qquad \ddot{y} = -\frac{\mu}{r^3}y + d\ddot{y}, \qquad \ddot{z} = -\frac{\mu}{r^3}y + d\ddot{z}$$

where $\mu = G \cdot M$ represents Earth's gravitational parameter, whereas $d\ddot{\vec{r}}$ represents the sum of all the disruptive accelerations. For GPS satellites, the module of the central acceleration ($-\mu/r^2$) is 10^4 times bigger than the sum of all these disruptive accelerations.

There are two main methods of determining a disrupted orbit for an artificial satellite:

1. Analytical method, by developing a mathematical model for the disruptive potential, which will help determine the way in which the orbital element varies in time.
2. Numerical method, by developing a model for disruptive accelerations, upon which the non-homogeneous second order differential equation of disrupted movement will integrate directly, providing the satellite's coordinates on every stage.

3. ANALYTICAL METHOD

In order to find the analytical solutions (usually approximate results) we will apply the perturbations theory. To start, we will take into consideration just the homogeneous part of this equation, which will lead to a Keplerian orbit defined by the 6 constant orbital parameters [a, e, i, ω, Ω, τ], notation used below ρ_{io} (i = 1..6), considered at $\tau = t_0$.

Each of the disruptive accelerations $d\ddot{\vec{r}}$ creates time variations $\bar{p}_{i_0}$ for the orbital parameters. Therefore at any given time t, the r_i parameters describe an ellipse called osculating (instantaneous orbit), different than the initial one (Keplerian), defined by the following parameters:

$$p_i = p_{i_0} + \bar{p}_{i_0} \times (t - t_0)$$

It is essential to determine next the $\bar{p}_{i_0}$ variations. To this end, we will compare the disrupted movement (that is the osculating ellipse r_i at time t) with the Keplerian movement (Keplerian ellipse r_{io} at time t_0). Thus we have the following equations for the position vector $\vec{r}$ and the speed vector $\dot{\vec{r}}$ while in disrupted movemen

$$\vec{r} = \vec{r}\left(t, p_i(t)\right)$$

$$\dot{\vec{r}} = \dot{\vec{r}}\left(t, p_i(t)\right)$$

By deriving these relations in relation to time, we get:

$$\dot{\vec{r}} = \frac{\vec{r}}{t} + \sum_{i=1}^{6}\left(\frac{\partial \vec{r}}{\partial p_i} \times \frac{dp_i}{dt}\right)$$

$$\ddot{\vec{r}} = \frac{\dot{\vec{r}}}{t} + \sum_{i=1}^{6}\left(\frac{\partial \dot{\vec{r}}}{\partial p_i} \times \frac{dp_i}{dt}\right) = -\frac{\mu}{r^3}\vec{r} + d\ddot{\vec{r}}$$

the Lagrange planetary equations (EPL), where the disruptive acceleration is replaced by the disruptive potential $\Re$ (or disruptive force function), where $\nabla\Re = d\ddot{\vec{r}}$.

$$\begin{cases}
\dot{a} = \dfrac{2}{na} \cdot \dfrac{\partial \Re}{\partial M} \\[2ex]
\dot{e} = \dfrac{1-e^2}{na^2 e} \cdot \dfrac{\partial \Re}{\partial M} - \dfrac{\sqrt{1-e^2}}{na^2 e} \cdot \dfrac{\partial \Re}{\partial \omega} \\[2ex]
\dot{i} = \dfrac{\cos i}{na^2\sqrt{1-e^2} \cdot \sin i} \cdot \dfrac{\partial \Re}{\partial \omega} - \dfrac{1}{na^2\sqrt{1-e^2} \cdot \sin i} \cdot \dfrac{\partial \Re}{\partial \Omega} \\[2ex]
\dot{\omega} = \dfrac{\sqrt{1-e^2}}{na^2 e} \cdot \dfrac{\partial \Re}{\partial e} - \dfrac{\cos i}{na^2\sqrt{1-e^2} \cdot \sin i} \cdot \dfrac{\partial \Re}{\partial i} \\[2ex]
\dot{\Omega} = \dfrac{1}{na^2\sqrt{1-e^2} \cdot \sin i} \cdot \dfrac{\partial \Re}{\partial i} \\[2ex]
\dot{M} = n - \dfrac{2}{na} \cdot \dfrac{\partial \Re}{\partial a} - \dfrac{1-e^2}{na^2 e} \cdot \dfrac{\partial \Re}{\partial e}
\end{cases}$$

Thus the disruptive force function must be defined through its components (Gauss method):

$$\nabla\Re = \begin{bmatrix} W \\ T \\ S \end{bmatrix}$$

The relations between the Gaussian components of the disruptive force and the time variations for the osculating orbital elements are as follows [2, 11]:

$$\begin{cases}
\dot{a} = \dfrac{2}{n\sqrt{1-e^2}}\left[S\cdot e\cdot\sin v + \left(1+e\cos v\right)\cdot W\right] \\
\dot{e} = \dfrac{\sqrt{1-e^2}}{na^2}\left[S\cdot\sin v + \left(\cos E + \cos v\right)\cdot W\right] \\
\dot{i} = \dfrac{r\cos\left(\omega+v\right)}{na^2\sin i\cdot\sqrt{1-e^2}}\cdot T \\
\dot{\omega} = \dfrac{\sqrt{1-e^2}}{nae}\left[-S\cdot\cos v + \left(1+\dfrac{1}{1+e\cos v}\right)\cdot W\cdot\sin v\right] - \dfrac{r\cos i\sin\left(\omega+v\right)}{na^2\sqrt{1-e^2}\cdot\sin i}\cdot T \\
\dot{\Omega} = \dfrac{r\sin\left(\omega+v\right)}{na^2\sqrt{1-e^2}}\cdot T, \\
\dot{M} = n + \dfrac{1-e^2}{nae}\left[S\cdot\left(\dfrac{-2e}{1+e\cos v}+\cos v\right) - \left(1+\dfrac{1}{1+e\cos v}\right)\cdot W\cdot\sin v\right].
\end{cases}$$

4. DISRUPTIVE FORCE FUNCTIONS

Numerous disruptive forces (accelerations) act on an artificial satellite, and even though they are much smaller (in terms of magnitude) than the central force (acceleration), they do have a significant impact on its dynamics. Judging by their nature we will categorize them to **gravitational** disruptive accelerations and **non-gravitational** disruptive accelerations.

4.1 The Disruptive Force Function of the Non-central Earth Gravitational Field

The general form of Earth’s gravitational potential is represented by infinite series of spherical harmonic functions, and when defined in polar coordinates (r,φ,λ) will be [8]:

$$V = \frac{\mu}{r}\left\{1 - \sum_{\ell=2}^{\infty}\left(\frac{a_e}{r}\right)^{\ell} J_{\ell}P_{\ell}\left(\sin\varphi\right) - \sum_{\ell=2}^{\infty}\sum_{m=1}^{\ell}\left(\frac{a_e}{r}\right)^{\ell}\left[C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda\right]P_{\ell m}\left(\sin\varphi\right)\right\}$$

Notations used by Kaula [10].

The first term (μ/r) represents the potential (V_0)of spherical Earth whereas its gradient [grad (μ/r) = - μ $\vec{r}$ /r^3] is the central acceleration in Keplerian movement.

Thus the disruptive force function corresponding to the non-centrality ($\Re_T$) will be obtained by making the difference:

$\Re_T = V - V_0 =$

$$= -\frac{\mu}{r}\left\{\sum_{\ell=2}^{\infty}\left(\frac{a_e}{r}\right)^{\ell} J_{\ell}P_{\ell}\left(\sin\varphi\right) - \sum_{\ell=2}^{\infty}\sum_{m=1}^{\ell}\left(\frac{a_e}{r}\right)^{\ell}\left[C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda\right]P_{\ell m}\left(\sin\varphi\right)\right\}$$

4.2 The Disruptive Force Function of the gravitational attraction exerted by the Sun and the Moon

The attraction exerted by the Sun and the Moon represents the second most important gravitational influence over the movement of artificial satellites. In order to evaluate it, we will take into account the following simplifying hypothesis:

1) The Moon and the Sun have similar effects, therefore will be treated analogously
2) We will neglect Sun and Moon movement during one complete revolution of the artificial satellite
3) Perturbations caused by the planets part of the Solar System will be considered null.

The Disruptive Force Function of the gravitational attraction exerted by the Sun and the Moon expressed by osculating orbital elements was proved by several authors [7, 13] and is analogous to the expression of the non-centrality force function:

$$\Re_{L-S} = \frac{\mu'}{r'}\sum_{\ell=2}^{\infty}\left(\frac{a}{r'}\right)^{\ell}\sum_{m=0}^{\ell}\sum_{p=0}^{\ell}\sum_{q=-\infty}^{\infty}F_{\ell mp}\left(i\right)\cdot G_{\ell pq}\left(e\right)\cdot S^{*}_{\ell mpq}\left(\omega,\Omega,M\right)$$

where:

$$S^{*}_{\ell mpq} = \left|\begin{matrix} A_{\ell m} \\ -B_{\ell m}\end{matrix}\right|_{\ell-m\ \text{impar}}^{\ell-m\ \text{par}}\cos\left[\left(\ell-2p\right)\omega + \left(\ell-2p+q\right)M + m\Omega\right] +$$

$$+\left|\begin{matrix} B_{\ell m} \\ A_{\ell m}\end{matrix}\right|_{\ell-m\ \text{impar}}^{\ell-m\ \text{par}}\sin\left[\left(\ell-2p\right)\omega + \left(\ell-2p+q\right)M + m\Omega\right]$$

$A_{\ell m}$ şi $B_{\ell m}$ are the coefficients of the spherical harmonics expressed in absolute equatorial coordinates (α′, δ′) of the third body:

$$A_{\ell m} = \frac{\left(\ell-m\right)!}{\left(\ell+m\right)!}\cdot\varepsilon_m\cdot P_{\ell m}\left(\sin\delta'\right)\cos m\alpha',$$

$$B_{\ell m} = \frac{\left(\ell-m\right)!}{\left(\ell+m\right)!}\cdot\varepsilon_m\cdot P_{\ell m}\left(\sin\delta'\right)\sin m\alpha'.$$

ε_m =1 if m=0 and ε_m=0 if m>0,
function $F_{\ell mp}(i)$ is the inclination function [10],
function $G_{\ell pq}$ (e) is the eccentricity function [10]

4.3 The Disruptive Force Function of the direct solar radiation pressure

When considering the non-gravitational forces that influence the movement of an artificial satellite in general and particularly the movement of a GPS satellite, the pressure exerted by the solar radiation plays an important part. Musen [13] was one of the first who insisted on taking into consideration this disruptive force, as well as Parkinson et al. (1960) and it was taken into account during the study of Vanguard 1. Only after considering this disruptive acceleration (leaving aside the gravitational ones already mentioned) are we able to obtain an adequate report between calculations and observations.

The pressure of light is the mechanical action of the solar light upon the satellite body and it widely influences the orbits of satellites with high flying altitudes, such as GPS satellites.

The following hypothesis will be considered:
1) The Sun’s parallax will be neglected,
2) Earth’s movement around the Sun is uniform,
3) The solar flux is constant along the orbit, with the exception of the shadow arches,
4) The disruptive force function is not influenced by the satellite’s shape, but by the area-mass ratio,
5) The indirect effect (albedo) will be neglected.

Under the conditions above, the disruptive acceleration introduced by direct solar pressure will be:

F = k’·(A/m)·q

where:

k’ = satellite’s reflectivity constant. For calculations we will consider k = 1 if the reflection or the absorption of solar light by the surface of the satellite is total, and k’ = 1.$_{44}$ if the reflection is diffuse,

A = the area of the transversal section of the satellite, perpendicular on the direction of the disruptive force.

m = satellite’s mass,

q = the ratio between the solar constant and the speed of light.

4.4 Relativistic effects

The disruptive acceleration introduced by the relativistic effect is a consequence of artificial satellites moving inside the Earth’s gravitational field. In a simplified expression (taking into account only the second harmonic) [9]:

$$\begin{cases} \ddot{x} = -\dfrac{3\mu^2 a\left(1-e^2\right)}{r^5 c^2} x \\ \ddot{y} = -\dfrac{3\mu^2 a\left(1-e^2\right)}{r^5 c^2} y \\ \ddot{z} = -\dfrac{3\mu^2 a\left(1-e^2\right)}{r^5 c^2} z \end{cases}$$

For average values of the geocentrical vector radius (r), large semi-axis (a) and the eccentricity (e) of the GPS satellites’ orbits, the disruptive acceleration caused by relativistic effects will have a size order of $3\cdot10^{-10}$ m·s^{-2}.

5. NUMERICAL METHOD

In order to calculate the GPS satellites’ orbits, two methods will be used. The first one is based on the analytical solutions of the Lagrange planetary equations, expressed in the terms of Keplerian orbital elements terms. The second method is based on the numerical solution of the second order differential equation of the disruptive relative movement:

$$\ddot{\underline{r}} = -\frac{\mu}{r^3}\underline{r} + \frac{\partial\Re}{\partial r}$$

The numerical solution of the GPS satellites’ orbits is based on the direct numerical integration of the second order differential equations of the disrupted relative movement.

If the initial conditions are defined (mainly the position $\underline{x}_0$ and the speed $\dot{\underline{x}}_0$ considered at launching time t_0), the equations can be integrated numerically. The Keplerian orbit will be cross-referenced as well. Thus only the small differences between the total acceleration and the central acceleration will have to be integrated. As a result, the precise position will be given by the sum of the (incremental) growth $d\underline{x}$ and the position vector on the referenced ellipse. The second order differential equations usually transform into a system of 2 first order differential equations, such as:

$$\dot{\underline{x}}(t) = \dot{\underline{x}}(t_0) + \int_{t_0}^{t}\left[d\dot{\underline{x}}(t_0) - \frac{\mu}{r^3(t_0)}\underline{x}(t_0)\right]dt$$

$$\underline{x}(t) = \underline{x}(t_0) + \int_{t_0}^{t}\dot{\underline{x}}(t_0)dt$$

The numerical integration of this system is made by applying a fourth order Runge Kutta algorithm.

Fourth order Runge Kutta method was used to calculate the rectangular coordinates and the osculating orbital elements of GPS satellites, by integrating numerically the equations of disrupted movement, considering on a selective basis the perturbations produced by:
1) The non-centrality of Earth’s gravitational field;
2) Sun and Moon’s gravitational attraction;
3) Direct solar radiation pressure;
4) Relativistic effects.

For the initial conditions:

Tab. 1 Initial conditions

X_0 = 2017,873929	Y_0 = -15394,807277	Z_0 = 21652,716838	[Km]
V_{X_0} = 3,740049	V_{Y_0} = 0,911161	V_{Z_0} = 0,306443	[Km/s^2]
X_S = 146514888,8718	Y_S = 32514335,1508	Z_S = -8699485,9205	[Km]
X_L = 379821,1117	Y_L = 25054,9574	Z_L = 53622,5347	[Km]

We obtained the following values for the rectangular coordinates, speeds and osculating orbital elements over a timeframe of 4 hours.

Tab. 2 Ephemerides values calculated 4 hours time

00:00:00	X =-16,2702	Vx =3,8272	$\Omega =$ 16,920	$\omega =$ -100,583
	Y =-15526,5014	Vy =0,4134	*i* =54,727	M =154,853
	Z =21652,7029	Vz =0,3064	*e* =0,00355	a =26558,874
00:15:00	X =2425,8676	Vx =3,8327	$\Omega =$ 16,919	$\omega =$ -107,974
	Y =-15215,1157	Vy =0,4529	*i* =54,727	M =162,590
	Z =21743,2188	Vz =-0,1055	*e* =0,00362	a =26556,918
00:30:00	X =4869,5214	Vx =3,7943	$\Omega =$ 16,919	$\omega =$ -116,636
	Y =-15030,7748	Vy =0,4888	*i* =54,727	M =170,242
	Z =21463,3137	Vz =-0,5156	*e* =0,00369	a =26555,104
00:45:00	X =7266,8831	Vx =3,7124	$\Omega =$ 16,918	$\omega =$ -126,475
	Y =-14972,9626	Vy =0,5262	*i* =54,727	M =177,825
	Z =20817,8747	Vz =-0,9167	*e* =0,00375	a =26553,458
01:00:00	X =-1228,0675	Vx =3,8441	$\Omega =$ 16,919	$\omega =$ -106,528
	Y =-15341,2399	Vy =-0,3586	*i* =54,727	M =161,183
	Z =21754,3684	Vz =-0,0304	*e* =0,00361	a =26557,265
01:15:00	X =1208,7705	Vx =3,8219	$\Omega =$ 16,919	$\omega =$ -114,961
	Y =-15645,3951	Vy =-0,3170	*i* =54,727	M =168,848
	Z =21541,7820	Vz =-0,4413	*e* =0,00368	a =26555,422
01:30:00	X =3582,4296	Vx =3,7571	$\Omega =$ 16,918	$\omega =$ -124,593
	Y =-16066,2011	Vy =-0,2657	*i* =54,727	M =176,438
	Z =20962,3097	Vz =-0,8446	*e* =0,00374	a =26553,742
01:45:00	X =5848,4114	Vx =3,6520	$\Omega =$ 16,918	$\omega =$ -127,315
	Y =-16587,1013	Vy =-0,1998	*i* =54,727	M =176,008
	Z =20025,8810	Vz =-1,2334	*e* =0,00380	a =26552,241
02:00:00	X =-4764,8181	Vx =3,6310	$\Omega =$ 16,919	$\omega =$ -107,500
	Y =-14644,2430	Vy =-1,3099	*i* =54,727	M =162,133
	Z =21748,1699	Vz =-0,0813	*e* =0,00362	a =26557,030
02:15:00	X =-2491,8759	Vx =3,6150	$\Omega =$ 16,919	$\omega =$ -116,086
	Y =-15565,1417	Vy =-1,2618	*i* =54,727	M =169,787
	Z =21490,0052	Vz =-0,4916	*e* =0,00369	a =26555,206
02:30:00	X =-314,4974	Vx =3,5606	$\Omega =$ 16,918	$\omega =$ -125,857
	Y =-16580,0206	Vy =-1,1930	*i* =54,727	M =177,371
	Z =20865,8480	Vz =-0,8934	*e* =0,00375	a =26553,547
02:45:00	X =1729,0450	Vx =3,4715	$\Omega =$ -127,134	$\omega =$ -126,840
	Y =-17660,8966	Vy =-1,0996	*i* =175,656	M =175,077
	Z =19886,3795	Vz =-1,2801	*e* =0,00380	a =26552,069
03:00:00	X =-8462,0568	Vx =3,1633	$\Omega =$ 16,919	$\omega =$ -107,332
	Y =-12864,0205	Vy =-2,2124	*i* =54,727	M =161,971
	Z =21749,6313	Vz =-0,0726	*e* =0,00362	a =26557,070
03:15:00	X =-6505,9208	Vx =3,1611	$\Omega =$ 16,919	$\omega =$ -115,892
	Y =-14344,1799	Vy =-2,1624	*i* =54,727	M =169,626
	Z =21499,2637	Vz =-0,4830	*e* =0,00368	a =26555,243
03:30:00	X =-4665,8597	Vx =3,1272	$\Omega =$ 19,918	$\omega =$ -125,639
	Y =-15890,8677	Vy =-2,0825	*i* =54,727	M =177,211
	Z =20882,7442	Vz =-0,8851	*e* =0,00375	a =26553,580
03:45:00	X =-2971,6601	Vx =3,0660	$\Omega =$ 16,918	$\omega =$ -126,921
	Y =-17467,1825	Vy =-1,9698	*i* =54,727	M =175,237
	Z =19910,6253	Vz =-1,2721	*e* =0,00380	a =26552,098
04:00:00	X =-11510,5064	Vx =2,4807	$\Omega =$ 16,918	$\omega =$ -107,332
	Y =-10227,3003	Vy =-2,9575	*i* =54,727	M =161,970
	Z =21749,6313	Vz =-0,0726	*e* =0,00362	a =26557,070

6. CONCLUSIONS

Numerical data concerning the precise ephemeride values of the GPS satellites may be obtained on the National Geodetic Survey website http://www.ngs.noaa.gov/orbits/.

Această lucrare calculează pe baza unui modul software elementele de efemeridă ale orbitei perturbate ale unui satelit GPS ca urmare a acțiunii însumate a forței perturbatoare datorată necentralității câmpului gravitațional al Pământului, a atracției gravitaționale a Soarelui şi a Lunii, a presiunii radiației solare directe şi a efectelor relativiste. Avantajul acestei metode constă în faptul că se poate controla precizia coeficienților, a formulelor, a parametrilor şi a erorilor rezultate din trunchierea numerică. Mai mult, se pot trage concluzii privind dinamica erorilor orbitale ale sateliților GPS. Valorile numerice ale elementelor de efemeridă sunt date pentru 4 ore pornind de la datele inițiale şi ora 00:00:00 UTC.

O importanță deosebită trebuie acordată formulelor şi valorilor coeficienților care intră în modelarea forțelor perturbatoare.

REFERENCES

[1] Adrian Drăguşan, Sergiu Lupu, "Methods for Representing the Gravitational Potential", Romanian Astronomical Journal, Vol. 16, Supplement, p. 1-282, Bucharest, 2006
[2] Brouwer, D., Clemence, G.M. "Methods of Celestial mechanics", Academic Press, New York and London 1961
[3] Cojocaru S., "Global Positioning System GPS/GLONASS-present and future" The Cluj Academical Days, Cluj Napoca 1996
[4] Cojocaru, S., "The Influence of Odd Zonal Harmonics in the Motion of an Artificial Satellite", The Romanian Astronomical Journal, 1, 1997.
[5] Cojocaru, S., "Contributions to the study of GPS satellite orbital errors due to the terrestrial gravitational potential", Doctoral dissertation, 397/30-06-1997 Astronomical Institute, Bucharest, defended at 11-04-1998.
[6] Cojocaru, S., "Perturbed motion of an artificial satellite in the non-central Earth's gravitational field", Astronomical Institute Library, Bucharest 1996.
[7] Giacaglia, G.E.O., "Lunar perturbation on artificial satellites of the Earth", SAO Spec. Rep 352, Cambridge USA 1973
[8] Heiskanen W. A., Moritz H., "Physical Geodesy", W.H. Freeman and Company, San Francisco, 1967
[9] Hofmann-Wellenhof, B., Lichtenegger, H., Collins, J., "GPS- Theory and Practice", second edition, Springer-Verlag, New York 1993
[10] Kaula, W.M., "Theory of satellite geodesy", Blaisdell Publ.Co., Waltham 1966.
[11] Levallois J., J., Kovalevsky J., "Geodesie generale", Tome IV, Eyrolles, Paris 1971
[12] Musen, P., "On the motion of a satellite in an asymmetrical gravitational field", Journ. Geophys., 1960
[13] Nakiboglu, S.M., E.J. Krakiwski, K.P. Schwarz, B. Buffet, B. Wanless, "A multi-sation, multi-pass approach to Global Positioning System improvement and precise positioning", Geodetic Survey of Canada, Rep. 85-003, Ottawa 1985