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ABOUT POWER CONSUMPTIONS DETERMINATION OF THE TRANSFORMER IN A.C. SWITCHING MODE

Constantin STRÎMBU[1]
Cristian-George CONSTANTINESCU[2]
Marian PEARSICĂ[3]
Ştefan NEDELCU[4]
[1] Şef lucrări dr.ing., Academia Forțelor Aeriene "Henri Coandă', Braşov
[2]Şef lucrări dr.ing., Academia Forțelor Aeriene "Henri Coandă', Braşov
[3]Col. Prof. univ. dr. ing., Academia Forțelor Aeriene "Henri Coandă', Braşov
[4]Prof. univ. dr. mat., Academia Forțelor Aeriene "Henri Coandă', Braşov

Abstract: *This paper introduces least square method based algorithms to obtain the apparent power of the RC loaded transformers operating in a.c. switching mode (the switch is connected in the secondary). In technical literature there is a lack of information concerning the main input data for designing a transformer (the primary, S_1 and secondary, S_2 apparent voltamps). These items are determined as two variable (load circuit and thyristor firing angle) polynomial functions, obtained by using a multiple regression procedure (least squares method based). This function is obtained starting from the results returned by the study of the non-linear transformer model.*
Keywords*: apparent power, least squares method, multiple regression*

1. Introduction

The goal is to obtain an approximating function $f(\xi, \alpha_0)$ *(type, where* α_0 *- firing angle;* $tg(\xi) = \omega RC$*) that could be used for analyzed quantities effective values determination (* $I_{1ef}, I_{2ef}, U_{1ef}, U_{2ef}$ *). Once having these functions, to determine the power consumptions becomes a trivial problem.*

Fig. 1 presents the block diagram of the algorithm used to determine the effective quantities approximating functions. It was implemented as several MathCAD functions. Because of the impossibility to find signals analytical expressions, the authors had to build a database, characterized by:

A. Content

The database includes the analyzed quantities numerical values (i_1, i_2, u_1, u_2*), in steady state regime, along a supplying voltage a.c. cycle, for a number of points big enough. The study was performed for the circuit shown in Fig. 2.*

B. Transformer model

The study [1, 2] was performed considering a non-linear magnetization curve, using a thyristor (SCR) with ideal characteristics and supposing the load circuit parameters are constant. The core magnetization curve was measured experimentally in laboratory conditions. The analysis method consists in the pursuance of the magnetization curve B(H), obtained by laboratory tests on the magnetic circuit. We considered the curve as a succession of little lines, each one characterized by a constant magnetic permeability: $\mu = \frac{dB}{dH}$ *(a piecewise-linearised curve). In this case the study of the non-linear transformer switching becomes a succession of linear transformer ones, presented in [3, 4]. For each line of the linearised curve, one must evaluate the initial values of the electric (currents, voltages) and magnetic (permeability, inductivities) quantities.*

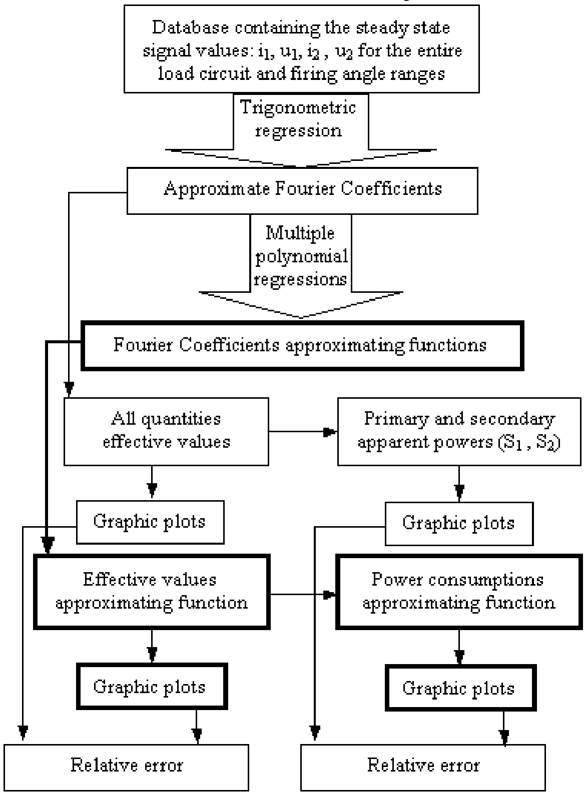

Fig. 1. Block diagram of the analysis

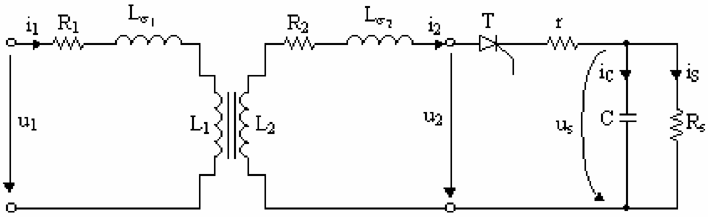

Fig. 2. The voltage rectifier

This model was used owing to the low error level of the theoretical waveforms, comparative with the experimental ones. Fig. 3 shows a comparison between the experimental results and the theoretical ones, returned by the transformer non-linear model.

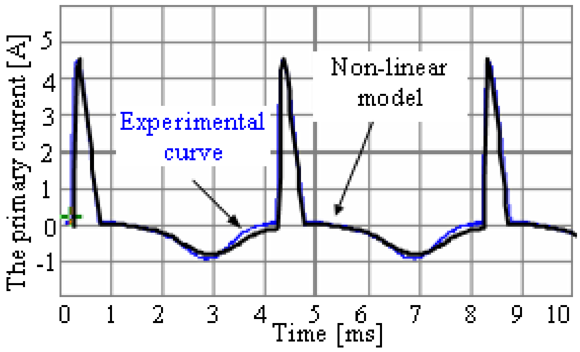

Fig. 3. The primary current, i_1 for $R = 33\Omega,\ \alpha_0 = 120^0$

C. Analyses Method

The transformer switching study was performed considering a wide range for the load circuit (R or ξ) and SCR firing angles values.

2. Switching analysis using harmonic Fourier coefficients approximate values
2.1. Harmonic Fourier Coefficients Approximate Values Determination

Owing the impossibility to determine analytical expressions of the signals characterizing the transformer steady state operation, namely $i_1,\ i_2\ ,\ u_2$, it is also impossible to calculate the harmonic Fourier coefficients exact values. So, the only way is to determine approximate values of these items, using the signals $i_1,\ i_2\ ,\ u_2$ tabled values. The least squares method is proposed as a procedure in this purpose.

The trigonometric Fourier series model function is:

$$x(t) = C_0 + \sum_{k\geq 1} C_k \cos(k\omega t) + \sum_{k\geq 1} S_k \sin(k\omega t) \tag{1}$$

The deviation function is:

$$F(C_0, C_1, C_2, ..., S_1, S_2, ...) = \sum_{i=1}^{m}\left[X_i - \left(C_0 + \sum_{k\geq 1} C_k \cos(k\omega t_i) + \sum_{k\geq 1} S_k \sin(k\omega t_i)\right)\right]^2 \tag{2}$$

where (t_i, X_i) *are the calculated points* $\left(i = \overline{1, m}\right)$.

The least squares method consists in minimizing the deviation function, obtaining the following relations:

$$\frac{\partial F}{\partial C_k} = 0\ , \quad \frac{\partial F}{\partial S_k} = 0 \tag{3}$$

So the problem is to solve a system of (linear) equations, the unknowns being the trigonometric Fourier series coefficients.

Fig. 4, 5 are showing comparisons performed (in the same conditions regarding the load circuit, R or ξ *,, and firing angle,* α_0*) between:*

Analyzed quantity waveform, returned by the study of the transformer a.c. switching regime a);

Analyzed quantity waveform, returned by harmonic Fourier series using the approximate Fourier coefficients. b).

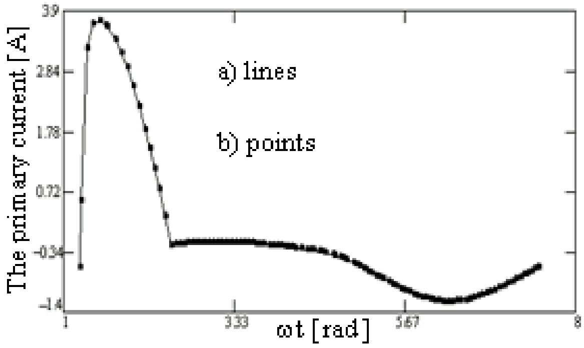

Fig. 4. Current i_1 *returned: a) by the transformer a.c. switching regime study; b) by using the approximate harmonic Fourier series*

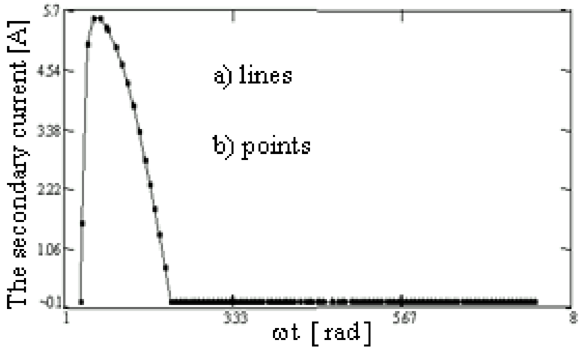

Fig. 5. Current i_2 *returned: a) by the transformer a.c. switching regime study; b) by using the approximate harmonic Fourier series*

2.2. Apparent voltamps determination

Primary and secondary apparent voltamps determination imposes previously to obtain the currents and voltages RMS values. The harmonic Fourier series attached to transformer currents and voltages are:

$$u(t) = U_0 + \sum_{n=1}^{\infty} U_n \cos(n\omega_0 t + \varphi_{un}) \tag{4}$$

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \cos\left(n\omega_0 t + \varphi_{in}\right) \qquad (5)$$

A periodical signal x(t) is characterized by its RMS value:

$$X_{ef} = \sqrt{\frac{1}{T}\int_T x^2(t)dt} \qquad (6)$$

Taking into consideration (4) and (5) in (6) it results:

$$U_{ef} = \sqrt{U_0^2 + \sum_{n=1}^{\infty}\frac{U_n^2}{2}}, \quad I_{ef} = \sqrt{I_0^2 + \sum_{n=1}^{\infty}\frac{I_n^2}{2}} \qquad (7)$$

The instantaneous power consumption is defined as:

$$p(t) = u(t)\cdot i(t) \qquad (8)$$

When studying the transformers the goal is to find the apparent voltamps (power absorption in the primary winding and power delivery in the secondary one).

The apparent voltamps is defined as:

$$S = U_{ef} I_{ef} \qquad (9)$$

As mentioned above, it's impossible to find exactly the harmonic Fourier coefficients corresponding to the transformer steady state signals, namely i_1, i_2, u_2 *. That's why the approximate ones will be used. MathCAD functions were built to compute the RMS values and the apparent voltamps.*

In Fig. 6 … 9 are presented the apparent voltamps as functions of load (R) and firing angle, α_0.

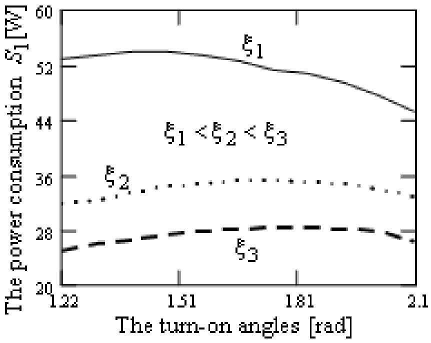

Fig. 6. Level curves $\xi = ct$ *of the primary power consumption vs. the turn-on angles, where* $\xi = a\tan(\omega RC)$

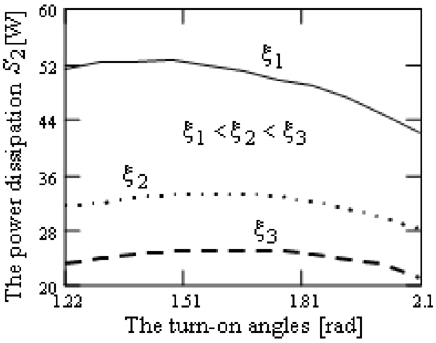

Fig. 7. Level curves $\xi = ct$ *of the secondary power dissipation vs. the turn-on angles, where* $\xi = a\tan(\omega RC)$

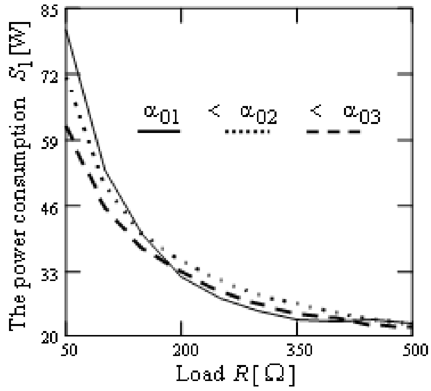

Fig. 8. Level curves $\alpha_0 = \mathrm{ct}$ *of the primary power consumption vs. load* $\xi = \mathrm{a}\tan(\omega RC)$

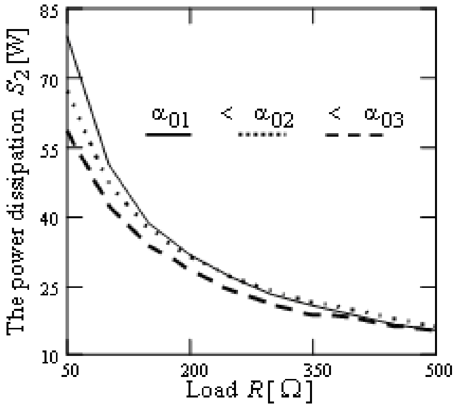

Fig. 9. Level curves $\alpha_0 = \mathrm{ct}$ *of the secondary power dissipation vs. load* $\xi = \mathrm{a}\tan(\omega RC)$

The apparent powers (primary S1 and secondary S2) have a decreasing variation function of load, the firing angle, α_0*, being constant. The difference* $S_1 - S_2$ *is a decreasing function of load capacitive character (R is shrinking, the capacitor being constant).*

3. Analysis of the transformer a.c. switching regime using harmonic Fourier coefficients approximating functions

3.1. Harmonic Fourier Coefficients Approximate Values Determination

In this section a procedure to compute harmonic Fourier coefficients approximation functions will be presented. It will return a signal function of type:

$$x(t) = A_0(\alpha_0, \xi) + \sum_{k \geq 1} A_k(\alpha_0, \xi)\cos(k\omega t + \varphi_k(\alpha_0, \xi)) \qquad (10)$$

where $x(t)$ *is* $i_1(t)$ *,* $i_2(t)$ *and* $u_2(t)$ *.*

The approximate harmonic Fourier coefficients A_k *and phase-shifts* φ_k *(returned by the trigonometric regression) model function are two variable polynomials (multiple regression). Because of the difficulty to guess the best fitting model function, the following procedure is proposed [4, 5].*

The database is (re)arranged as a matrix whose first columns are containing the variables (α_0 *and* ξ *in this case). In the last column the function values (*A_k *and* φ_k *in this case) are inserted.*

Polynomials coefficients corresponding to the last variable (α_0 *in this case), the others being constant, are computed. Vectors containing the polynomials coefficients associated to each combination of constant variables values (only* ξ *in this case) are resulting in this way.*

The vectors returned by the previous step will be considered as dependent functions of the next regression, which will be applied *considering the next variable (ξ in this case) as independent variable.*

The previous step is followed till exhausting all variables. This isn’t performed in this case, the function being a two-variable one.

This procedure is applied to find approximate harmonic Fourier coefficients approximating functions of α_0 and ξ, corresponding to each quantity needed by apparent voltamps determination, as already mentioned above.

It means that the approximate harmonic Fourier coefficients approximating functions are like the following:

$$f(\xi,\alpha_0) = b_{00} + b_{01}\xi + b_{02}\xi^2 \ldots + b_{05}\xi^5 + \left(b_{10} + b_{11}\xi + b_{11}\xi^2 \ldots + b_{15}\xi^5\right)\alpha_0 +$$
$$+ \left(b_{20} + b_{21}\xi + b_{21}\xi^2 \ldots + b_{25}\xi^5\right)\alpha_0^2 + \ldots + \left(b_{40} + b_{41}\xi + b_{42}\xi^2 \ldots + b_{45}\xi^5\right)\alpha_0^4 \quad (11)$$

The $b_{i,j}$ coefficients were determined for each analyzed quantity (i_1, i_2, u_2), the biasing voltage u1 being sinusoidal. The polynomials degrees were established by using an optimizing procedure, whose goal is to minimize the deviation function.

Another procedure was developed use the polynomial coefficients, namely to compute the approximation function value in a given point, $(\xi,\ \alpha_0)$. The relations (11) were used to compute analyzed quantities RMS values and then the apparent voltamps S1 and S2.

*Fig. 10 and 11 are showing the RMS values as functions of load (10 values - **x** axis) and (11 values - **y** axis).*

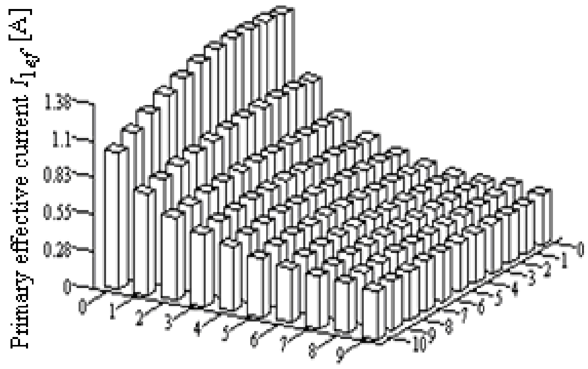

Fig. 10. The primary effective current vs. the load $\xi = a\tan(\omega RC)$ *and turn-on angles*

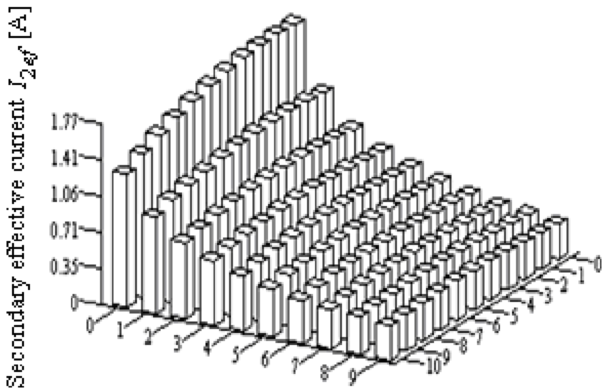

Fig. 11. The secondary effective current vs. the load $\xi = a\tan(\omega RC)$ *and turn-on angles*

Fig. 12 and 13 are showing the power consumptions as functions of the same variables.

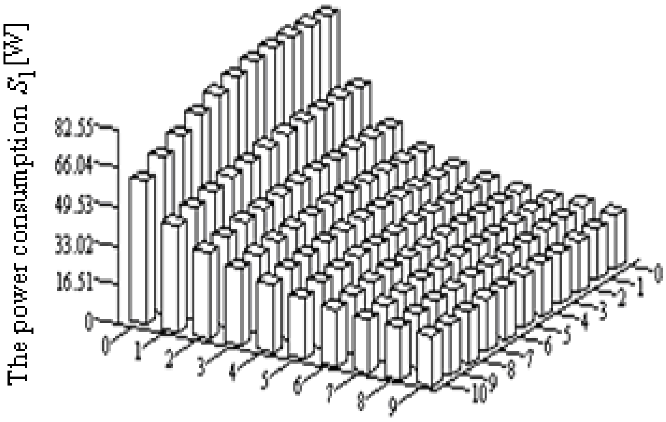

Fig. 12. The primary power absorption vs. the load $\xi = a\tan(\omega RC)$ *and turn-on angles*

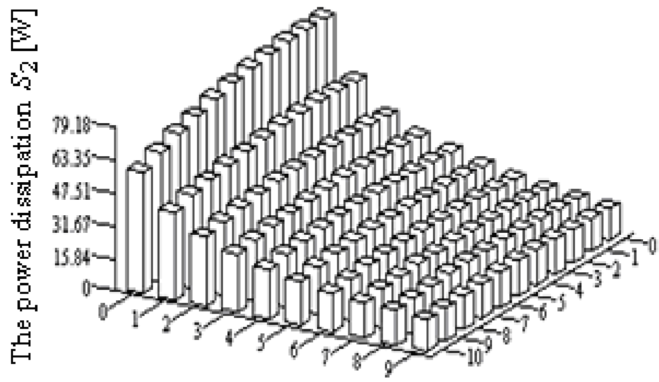

Fig. 13. The secondary power delivery vs. the load $\xi = a\tan(\omega RC)$ *and turn-on angles*

The polynomial multiple regression was implemented as a MathCAD function.

3.2. Comparison between the results returned by using the approximate harmonic Fourier coefficients and their approximating functions

Comparing the results returned by analyzing the periodical a.c. switching regime of the transformer using approximate harmonic Fourier coefficients and the corresponding approximating functions, relative errors were provided.
Fig. 14…17 are showing the relative error level corresponding to the voltages and currents RMS values.

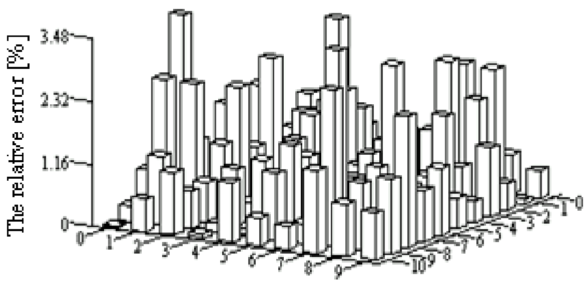

Fig. 14. The relative error of I_{1ef}

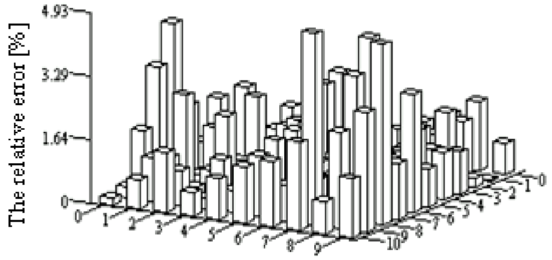

Fig. 15. The relative error of I_{2ef}

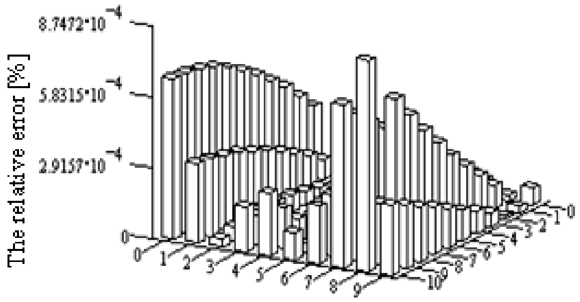

Fig. 16. The relative error of U_{1ef}

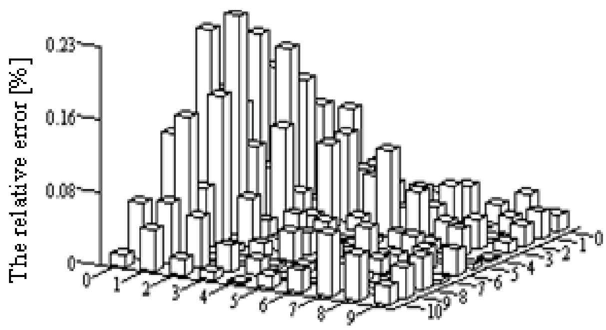

Fig. 17. The relative error of U_{2ef}

Fig. 18 and 19 are showing the relative error level corresponding to the apparent voltamps.

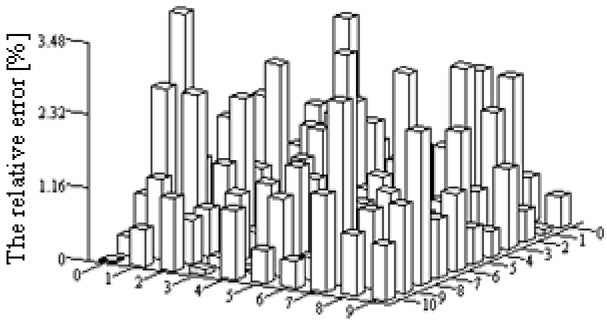

Fig. 18. The relative error of the primary power absorption

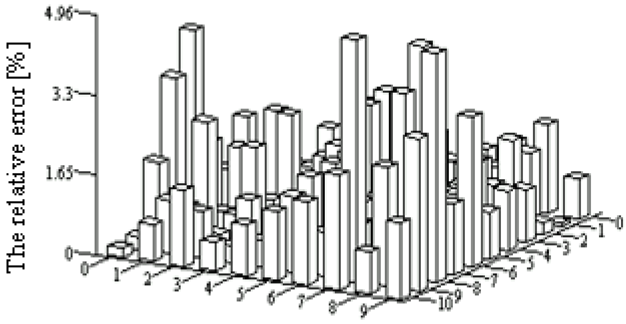

Fig. 19. The relative error of the secondary power delivery

4. Conclusions

Based on approximate harmonic Fourier coefficients determination, the RMS values (I_{1ef} , I_{2ef} , U_{1ef} , U_{2ef}) were computed, as well as the primary and secondary apparent voltamps.
The method used (trigonometric regression) provided satisfactorily results, as proved by Fig. 4 and 5, which are proving negligible errors between the signals given by the harmonic Fourier series and the original ones. So, the numerical values obtained in this way can be used as a database to determine approximating functions.
The approximating function determination is based on a (polynomial) multiple regression algorithm. The function obtained in this way (11) is considered as being suitable to be applied for transformer primary and secondary apparent voltamps determining.
The low error level between the quantities obtained by using the approximate Fourier coefficients values and the ones obtained by using the approximating functions can sustain the correctness of this affirmation (Fig. 14…19).
In order to be sure that (11) can be extrapolated as a formula for transformers apparent powers determination, it is necessary to increase the input database, so that it becomes representative for the maximum possible range of the (capacitive) load circuit and firing angle.
The results allowed the obtaining of a numerical method for computing the current and voltage effective values, respectively, powers and power factor values applied for pollution harmonic rate characterization [8, 9]. Power converters presence inevitable means harmonics in power supply current, and consecutively, distortion factor weak. Each type of converter has its specific harmonic spectrum. Power processing at a poor power factor may distortion the power supply voltages waveforms, disturbing in this way the operation of other equipments powered in parallel by the same mains.
The developed method presented in this paper allowed the analyses of the electromagnetic pollution phenomena regarding laser beam processing equipment, and particularly, in case of the converters power supplies components [8, 9, 10], laser being a nonlinear load with a strong capacitive character.

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