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PREDICTION OF SHIP MAGNETIC SIGNATURES THROUGH SEMI-EMPIRICAL METHODS

Georgiana MARIN[1]
Gheorghe SAMOILESCU[2]
Serghei RADU[3]
[1]Lt. Drd. Ing., Research Center for Navy
[2]Cdor. Prof. univ. Dr. Ing., "Mircea cel Batran" Naval Academy
[3]Drd. Ing., Zodiac Company

Abstract - *The issue of assessing and reducing the ship's magnetic signatures, particularly the ones generated by the ferromagnetic materials within its structure, has long been studied through various means, starting from experimental measurements to applying complex mathematical methods. This paper proposes a semi-empirical method which correlates a set of magnetic field measurements with the mathematical model of an equivalent source in order to attain a complete image of the ship's magnetic field distribution and therefore define the danger area outside the ship's hull which would influence the magnetic mines or the surveillance systems.*
Keywords: *magnetic signature, modeling, ship magnetism*

1. Introduction

The vulnerability of surface ships and submarines against magnetic detection is determined by considering all sources generating magnetic fields, among which the ferromagnetic steel used in the construction of the ship's hull has the greatest contribution [1]. The development of ferromagnetic signature prediction models started during World War II, in the scope of determining the surface ships and submarines susceptibility to magnetic mines and surveillance systems [4]. The same models have been used to assess and optimize the efficiency of signature reduction systems in the design stage.

The simple mathematical models used before World War II were limited to the prediction of the ship magnetic field general characteristics at distances a little larges than its width [2]. High fidelity prediction of the magnetic signature near the hull was accomplished only through the use of physical models at convenient scales [4, 5].

Not until the progress of electronic technology has mathematical modeling of surface ships and submarines improved. Rapid processing lead to increasing fidelity of analytical models and the development of numerical tools based on the finite element method (FEM) [6] or the boundary element method (BEM).

Combined with the physical models, the numerical methods can significantly reduce the cost, time and the risks associated with the development of signature reduction systems.

2. Fundamental models and equations

The mathematical models serve for the analytical prediction of signature based on finding solutions to Laplace equation in the appropriate coordinate system defining the ship hull shape or on the numerical FEM and BEM simulations using the detailed geometry of the entire ferromagnetic structure of the ship and its material properties. Once the models are established they can be used to predict the tridimensional magnetic signature, both permanent and induced, considering the ship position on the Earth surface, its heading, the pitch and roll angles and the detecting sensor particularities [1].

Since the ship ferromagnetic fields slowly vary in time, they can be thought of as steady state, so the time dependency in Maxwell equations can be neglected, taking the following expression [3]:

$$\nabla \times \vec{H} = \vec{J} \qquad (1)$$

$$\nabla \cdot \vec{B} = 0 . \qquad (2)$$

When modeling the ship ferromagnetic fields, all magnetic sources are placed on the ship hull or inside it [5]. The outer space, whether the environment is air, seawater or seabed, shall be considered as free of magnetic sources, with a permeability equal to the one of vacuum $\mu_0 = 4\pi \times 10^{-7} H/m$, and

$$\vec{B} = \mu_0 \vec{H} . \qquad (3)$$

The space outside the hull being free of magnetic charges:

$$\nabla \times \vec{H} = 0 \qquad (4)$$

the magnetic field in this region can be represented through a scalar potential Φ_m.

By replacing the magnetic flux density $\vec{B} = -\mu_0 \nabla \Phi_m$ in equation (2), Laplace's equation is obtained:

$$\nabla^2 \Phi_m = 0 \qquad (5)$$

where $\nabla \times \nabla \Phi_m = 0$. The electric current is limited to the conducting cables inside the ship hull, therefore the boundary conditions for the ferromagnetic body are:

$$\left(\vec{H}_2 - \vec{H}_1\right) \times \vec{n} = \vec{J}_S \qquad (6)$$

$$\left(\vec{B}_2 - \vec{B}_1\right) \cdot \vec{n} = 0 \qquad (7)$$

where $\vec{H}_2, \vec{H}_1$ and $\vec{B}_2, \vec{B}_1$ designate the magnetic field intensity and magnetic flux density, respectively, on both sides of the boundary; $\vec{J}_S$ designates the superficial current density on the separation surface between the two environments, and $\vec{n}$ is unit vector normal to the separation surface directed from medium 1 towards medium 2. By applying the boundary conditions a unique solution to Laplace equation is obtained [3].

Because surface ships and submarines length is considerably larger than their width, the prolate spheroidal system is the appropriate coordinate system choice for the tridimensional modeling of their magnetic fields [2].

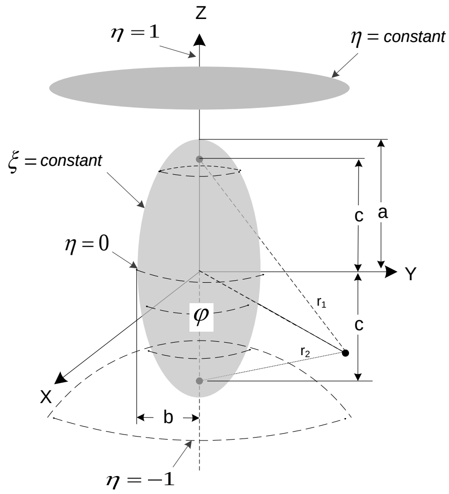

Fig. 1 The prolate spheroidal coordinate system

The solution to Laplace equation in prolate spheroidal coordinates can be computed through the variable separation method and is represented by means of harmonic functions [1, 5].

3. Modeling the ferromagnetic ship

Based on the representation of the ship hull, predictions of the induced magnetization longitudinal component can be performed. One of the main objectives of processing magnetic signature data is representing the ship as a distribution of equivalent sources or magnetizations. These equivalent sources may be used to regenerate the interpolated signatures along a regulate mesh, in order to determine the mine actuation area contour. Moreover, the near field signatures can be extrapolated to the far field ones, having the same equivalent sources model in order to determine the submarine vulnerability to magnetic surveillance systems, underwater or airborne.

The ferromagnetic ship model input data are the magnetic materials geometry of the ship structure, their magnetic permeability and the direction and magnitude of the induced geomagnetic field. These models are used before the construction stage of military and commercial ships to estimate the body induced magnetization dependency of its shape and material properties. The initial assessment of a surface ship or submarine vulnerability to magnetic detection of mines or surveillance systems can be determined in the ship class design stage. Furthermore, the ship geometry or material properties adjusting impact on ship susceptibility to magnetic threats can be referred to construction costs and ship hull performance.

Usually these models are employed for the extrapolation of ship magnetic signatures from the ship measurements performed at sea or the measurements of a scaled model tested in specialized laboratories; for this reason the model is called semi-empirical.

The simplest mathematical model used in ship magnetic signature description is the spherical dipole. The following equations associate the triaxial dipole moments with the magnetic field components:

$$B_x = \frac{\mu_0\left[m_x\left(2x^2 - y^2 - z^2\right) + 3m_y xy + 3m_z xz\right]}{4\pi\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}} \tag{8}$$

$$B_y = \frac{\mu_0\left[3m_x xy + m_y\left(2y^2 - x^2 - z^2\right) + 3m_z yz\right]}{4\pi\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}} \tag{9}$$

$$B_z = \frac{\mu_0\left[3m_x xz + 3m_y yz + m_z\left(2z^2 - x^2 - y^2\right)\right]}{4\pi\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}} \tag{10}$$

where $\left(m_x, m_y, m_z\right)$ represent the longitudinal, transversal and vertical components of the ship spherical dipole moment. Because the ship length is several times its width, prolate spheroidal sources are used in modeling magnetic signatures. Such a prolate spheroidal model generates more accurately the ship magnetic field at ranges much lower than the spherical model.

In the scope of equating the spherical dipole magnetic moment *m* to the ellipsoid dipole magnetic moment, the relationship:

$$M = 3m/c^2 \tag{11}$$

is used. Next, the magnetic field components of an ellipsoid dipole with elongation coinciding to the z axis are given:

$$B_x = \frac{3\mu_0}{4\pi c^3}\left\{ m_x\left[\frac{1}{4}\ln\left(\frac{\xi+1}{\xi-1}\right) - \frac{\xi}{2(\xi^2-1)} + \frac{x^2\xi}{r_1r_2(\xi^2-1)^2}\right] + m_y\left[\frac{xy\xi}{r_1r_2(\xi^2-1)^2}\right] + m_z\left[\frac{cx\eta}{r_1r_2(\xi^2-1)}\right]\right\} \quad (12)$$

$$B_y = \frac{3\mu_0}{4\pi c^3}\left\{ m_x\left[\frac{xy\xi}{r_1r_2(\xi^2-1)^2}\right] + m_y\left[\frac{1}{4}\ln\left(\frac{\xi+1}{\xi-1}\right) - \frac{\xi}{2(\xi^2-1)} + \frac{y^2\xi}{r_1r_2(\xi^2-1)^2}\right] + m_z\left[\frac{cy\eta}{r_1r_2(\xi^2-1)}\right]\right\} \quad (13)$$

$$B_z = \frac{3\mu_0}{4\pi c^3}\left\{ m_x\left[\frac{cx\eta}{r_1r_2(\xi^2-1)}\right] + m_y\left[\frac{cy\eta}{r_1r_2(\xi^2-1)}\right] + m_z\left[\frac{-1}{2}\ln\left(\frac{\xi+1}{\xi-1}\right) + \frac{c^2\xi}{r_1r_2}\right]\right\} \quad (14)$$

where m_x, m_y and m_z represent the ellipsoid dipole equivalent sources, (ξ,η,φ) are the system coordinates, given by:

$$\begin{cases} \xi = \dfrac{r_2+r_1}{2c} \\ \eta = \dfrac{r_2-r_1}{2c} \\ \varphi = \dfrac{1}{tg\left(y/x\right)} \end{cases} \quad (15)$$

where:

$$\begin{cases} r_1 = \sqrt{x^2+y^2+(z-c)^2} \\ r_2 = \sqrt{x^2+y^2+(z+c)^2} \\ c = \sqrt{a^2-b^2} \end{cases} \quad (16)$$

and *a* and *b* are the semi-axes of a spheroid with focus in positions ±*c*.

In order to accurately generate the ship magnetic signature at low distances from the ship hull, high order terms have to be included in the spherical or elliptical source expansion. On the other hand, the series expansion equations of spherical and elliptical source harmonic functions are extremely complex; therefore, the less harmonic terms are used in the reproduction of near field signature, less measurements and sensors are required to solve the expanded series.

4. A case study

In the following case, the ship magnetization is represented by eleven vertical spherical magnetic dipoles placed on the ship longitudinal axis. The magnetic sources intensity has been normalized to 1, and the dipoles are distributed along the ship length. The vertical magnetic field component is computed at the normalized depth of 1, below the keel.

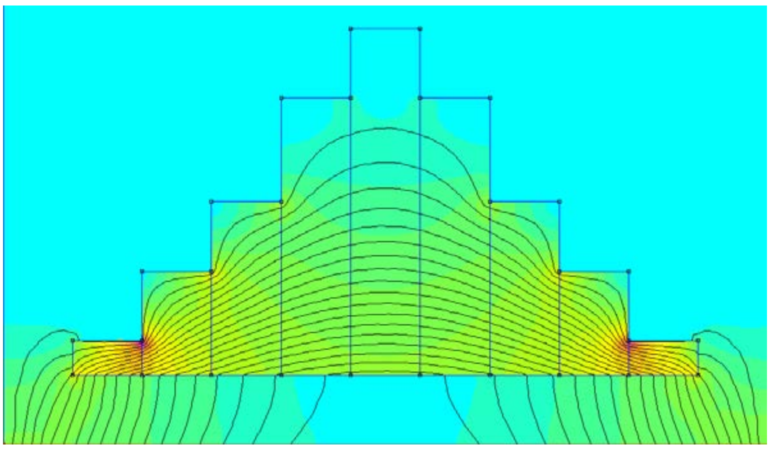

Fig. 2 The dipole distribution along the ship length

The magnetic field computation requires the generation of two matrices. The first one shall contain the eleven dipole moments $[m_z]$, and the second the respective longitudinal positions $[x']$. The column matrix $[m_z]$ elements range from 0 to 1 and back to 0, whereas the longitudinal positions matrix $[x']$ elements vary progressively between -1 and +1 in ten equal steps. The field of each coordinate x_i is computed from the relationship:

$$B_z = \frac{\mu_0\left[3m_x xz + 3m_y yz + m_z\left(2z^2 - x^2 - y^2\right)\right]}{4\pi\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}} \tag{17}$$

by replacing $m_x = m_y = y = 0$ and $z = 1$, in order to obtain:

$$B_i = \sum_{j=1}^{11} c_{i,j} m_j \tag{18}$$

where:

$$c_{i,j} = \frac{\mu_0}{4\pi} \frac{2 - \left(x_i - x'_j\right)^2}{\left[\left(x_i - x'_j\right)^2 + 1\right]^{\frac{5}{2}}} . \tag{19}$$

In its matrix form, relationship (18) can be expressed:

$$\begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,10} & c_{1,11} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,10} & c_{2,11} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{i-1,1} & c_{i-1,2} & \cdots & c_{i-1,10} & c_{i-1,11} \\ c_{i,1} & c_{i,2} & \cdots & c_{i,10} & c_{i,11} \end{bmatrix} \begin{bmatrix} m_1 \\ \vdots \\ m_{11} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{i-1} \\ B_i \end{bmatrix} \tag{20}$$

or in a more concise form:

$$[c] \cdot [m_z] = [B_z] . \tag{21}$$

The vertical component of magnetic field B_z represents the data received from measurements of the respective ship or of a scaled model, performed in specialized laboratories.

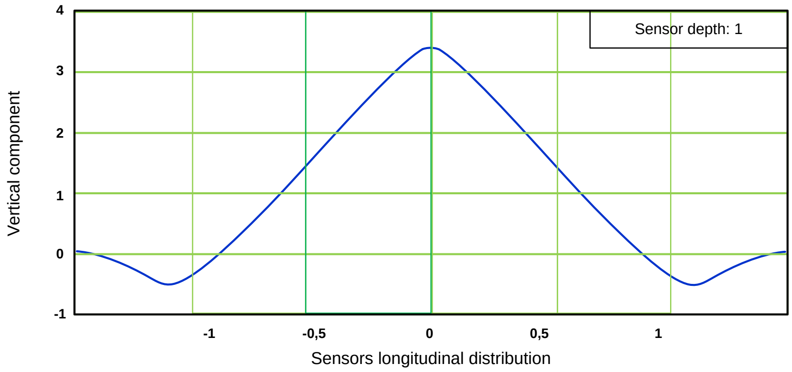

Fig. 3 The vertical component of the ship magnetic signature

5. Conclusions

In the above example, the discrete nature of the dipoles in the total magnetic signature does not render obvious, each source contribution being as if it has been filtered, which can be explained by expressing the solution by means of a convolution between the magnetization distribution and Green function [5].

After determining the magnetic signature spatial distribution and converting it into a temporal simulation for a particular ship speed, the magnetic mine or submarine surveillance system response can be defined. The output data of modeling represents the combatant vulnerability to the above mentioned threats in order to decrease the magnetic signature levels.

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