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SIMULATIONS OF UNDERWATER VEHICLE DYNAMICS

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Abstract: *This article present a comprehensive analysis about the motion equations of an autonomous rapid underwater vehicle.*
The analysis is based on Newton’s mechanical equation and data’s interpretations includes a lots of coefficients and variables for the vehicle’s movement and form.
Also we will present Closed Loop Scheme and Closed Loop simulations for step inputs and noise input for yaw and step for pitch and roll.
Keywords*:Under water vehicle, Close Loop control, Open Loop Control, Simulink*

1. INTRODUCTION

In this article we present a mathematical model for underwater vehicle and translation and simulation of this model in mathlab 6.5.
The used approach is the particularization of the general differential equations wich describes the movement of the general vehicle body.
In the final part of this article we will present closed loop simulations vehicle attitude for step inputs and noise input for yaw and step for pitch and roll.

2. THE MATHEMATICAL MODEL

When analyzing the motion of rigid body we use 6 DOF (Degrees of Freedom) and 2 fixed main coordinate frames: earth and body fixed.
The orientation of the coordinate frame with respect to another can be expressed by a rotation matrix R^e_b such that

$$v^e = R^e_b v^b \tag{1}$$

where:

$$R^e_b = \begin{bmatrix} i^e_b & j^e_b & k^e_b \end{bmatrix} = \begin{bmatrix} i^{b^T}_e \\ j^{b^T}_e \\ k^{b^T}_e \end{bmatrix} = R^{b^T}_e \tag{2}$$

i_b, j_b, k_b – body versors.

It is customary to describe a rotation matrix by three rotation that are carried out in a predefined order. Let $X_3Y_3Z_3$ be the coordinate system obtained by translating the earth fixed frame $X_eY_eZ_e$ parallel to itself until it’s origin coincides with the origin of the body fixed frame. Then the coordinate system $X_3Y_3Z_3$ is rotated a yaw angle ψ about Z_3 yielding the frame $X_2Y_2Z_2$, this last frame is rotated a pitch angle θ about Y_2 obtaining the frame $X_1Y_1Z_1$, $X_1Y_1Z_1$ is then rotated a roll angle Φ about X_1, yielding the body fixed coordinate frame.

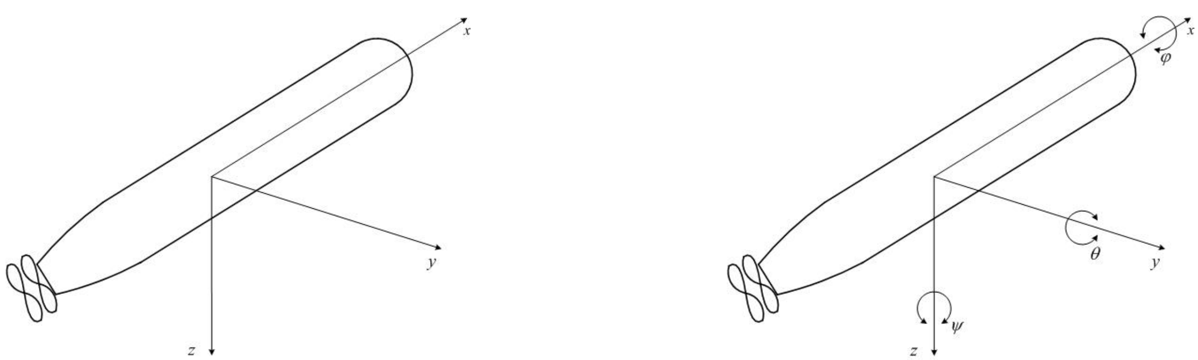

Fig.1 – The body fixed coordinate system, and the ψ, θ, Φ angles.

If the position of “b” with respect to “e” changes with time, then the vector O^e_b is time varying.
The linear velocity of “b” with respect to “e” expressed in the E – frame is:

$$v^e_{b/e} = \dot{O}^e_b \tag{3}$$

$$v^b_{b/e} = R^b_e \cdot v^e_{b/e} = R^b_e \cdot \dot{O}^e_b \tag{4}$$

If the orientation of “b” with respect to “e” changes with time then the rotation matrix R^e_b is time varying. The angular velocity is expressed as:

$$\omega^b_{b/e} = R^b_e \cdot \omega^e_{b/e} \tag{5}$$

However the angular velocity can be related to the derivatives of the roll, pitch and yaw angles:

$$\omega^e_{b/e} = k^e_3\dot{\psi} + j^e_2(\psi)\dot{\theta} + i^e_1(\theta,\psi)\dot{\Phi} = \begin{bmatrix} i^e_1(\theta,\psi) & j^e_2(\psi) & k^e_3 \end{bmatrix} \cdot \begin{bmatrix} \dot{\Phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{6}$$

If we want

$$\omega^b_{b/e} = R^b_e \cdot \omega^e_{b/e} = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\theta & c\theta s\theta \\ 0 & -s\theta & c\theta c\Phi \end{bmatrix} \begin{bmatrix} \dot{\Phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{7}$$

where

$$\begin{aligned} c\Phi &= \cos(\Phi) \\ s\Phi &= \sin(\Phi) \\ c\theta &= \cos(\theta) \\ s\theta &= \sin(\theta) \end{aligned} \tag{8}$$

If we want the roll, pitch and yaw values, the relation is:

$$\begin{bmatrix} \dot{\Phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s\Phi t\theta & c\Phi t\theta \\ 0 & c\Phi & -s\Phi \\ 0 & s\Phi/c\theta & c\Phi/c\theta \end{bmatrix} \cdot \omega^b_{b/e} \tag{9}$$

where

$$t\theta = \tan(\theta) \tag{10}$$

The following vectors are used to describe the general motion of a marine vehicle in 6 DOF (Degrees of Freedom):

$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \tag{11}$$

where

$$\eta_1 = O_b^e = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{12}$$

$$\eta_2 = \begin{bmatrix} \Phi \\ \theta \\ \psi \end{bmatrix} \tag{13}$$

η is the generalized position vector of the B – frame with respect to E – frame with coordinates in the E – frame, more in detail x, y, z are the positions of O_b respectively along the x_e, y_e, z_e axes, and Φ, θ, ψ are the roll, pitch and yaw angle of the B – frame with respect to E – frame.
The general velocity vector v of the B – frame with respect to E – frame is:

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{14}$$

where

$$v_1 = v_{b/e}^b = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{15}$$

$$v_2 = \omega_{b/e}^b = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{16}$$

The vector v is related to the derivative of η by the matrix J(η):

$$\dot{\eta} = J(\eta)v \tag{17}$$

where

$$J(\eta) = \begin{bmatrix} J_1(\eta_2) & 0_3 \\ 0_3 & J_2(\eta_2) \end{bmatrix} \tag{18}$$

0_3 is a 3 by 3 zero matrix
$J_1(\eta_2)$ is R_b^e (the transpose of R_e^b)
and $J_2(\eta_2)$ is the matrix (9).

3. CLOSED LOOP SCHEME

In final section, a simple attitude linear controller will be designed for the linear model obtained. The controller will then be tried on the complete nonlinear model.
The Linear Quadratic Regulator is one of the easiest controllers to be designed, and, being also among the best in term of both performance and robustness, it is a very good choice for testing the closed loop behavior of our nonlinear model. Having in the workspace the matrices a,b,c,d of the linear model considered in the previous section, the synthesis of the feedback controller requires only a couple of command lines:
The above command line selects the Q and R weight matrices, the fact that the attitude states are weighted with a value of “20” compared to the value “1” of the remaining states and inputs, reflects our will to track attitude commands.
Finally, the above command line computes the matrix K to be connected as a negative state feedback.

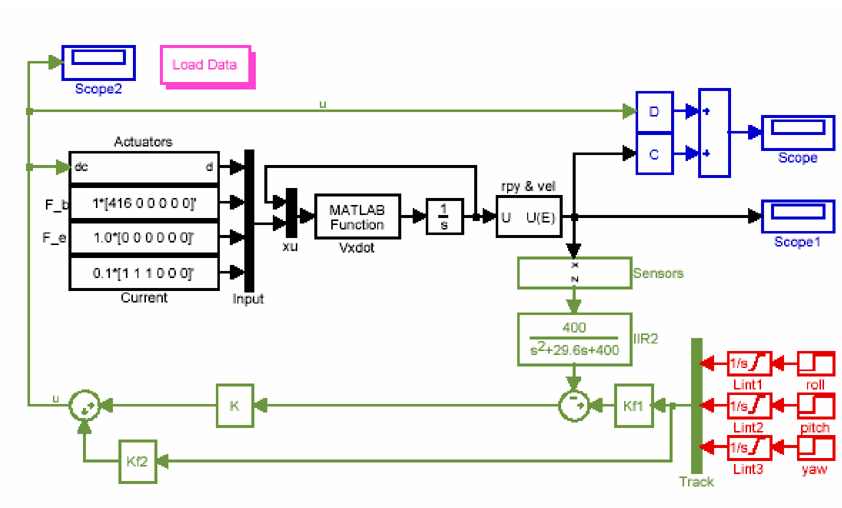

Fig. 2 Closed Loop Scheme

The nonlinear model starts from the same input and initial condition previously considered for the linearization. As a disturbance, a slow and constant sea current is considered, and some noise is added to each available measurement. To increase the realism of the simulation, a model of the actuators with delays, position limiter and rate limiter is included:

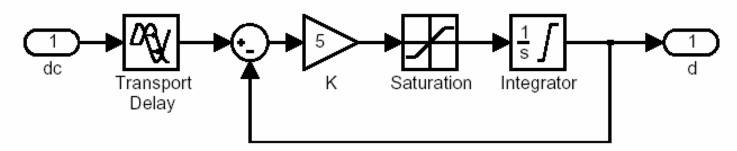

Fig. 3 Actuators model

Along the feedback path, the signal is quantized, the initial condition is subtracted, and only the available measures are selected.

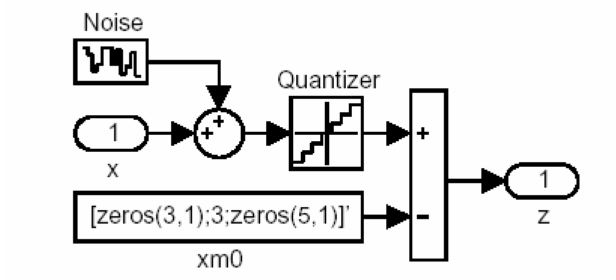

Fig. 4 Sensors model

4. ATTITUDE AND SET POINT LQR ATTITUDE CONTROL

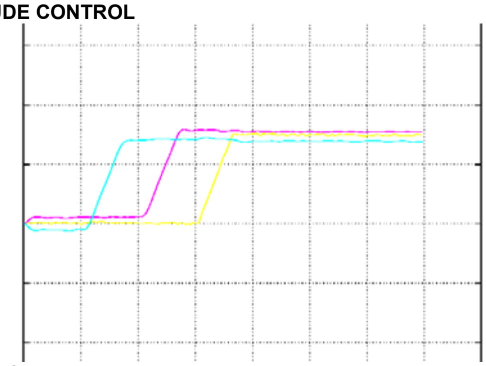

Fig. 5 Closed Loop simulation vehicle attitude: step inputs

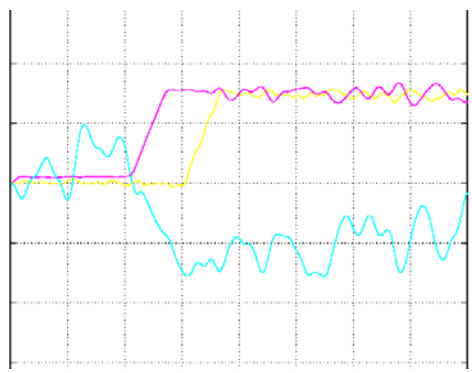

Fig. 6 Closed Loop simulation vehicle attitude: noise input for yaw and step for pitch and roll.

5. CONCLUSIONS

In this article, a mathematical model of movement of an underwater vehicle has been developed. The model started from study of frames. The successive step was kinematics and dynamics involved in a 6 DOF (degree of freedom) rigid body modeling problem. In the final section we presented Closed Loop Scheme and Closed Loop simulations for step inputs and noise input for yaw and step for pitch and roll.

6. REFERENCES

[1] Burlacu, P., *Contribuții la sinteza sistemelor de control automat cu aplicații în tehnica armamentului subacvatic,* Doctoral Thesis, Tehnical University Cluj Napoca, 2006.
[2] Giampeiro Campa, Mario Inocenti(1999), *Model of an underwater vehicle*, University of Pisa, 50 p.
[3] Papoulias, Fotis(1994), *Nonlinear Dynamics and Bifurcations in Autonomous Vehicle Guidance and Control*, Workshop on future research directions in underwater robotics.